

```
> restart;
> p := (z,w) -> z^3+w^3-1;
```

$$p := (z, w) \rightarrow z^3 + w^3 - 1 \tag{1}$$

```
> f := unapply(-diff(p(1+exp(I*t), w), t)/diff(p(1+exp(I*t), w), w), t, w);
```

$$f := (t, w) \rightarrow -\frac{I(1 + e^{It})^2 e^{It}}{w^2} \tag{2}$$

```
> N:=100;
u:=1.0;
t:=-evalf(Pi);
h:= evalf(2*Pi/N);
for j from 1 to N do
u:= u + f(t,u)*h;
t:=t+h;
od;

N:= 100
u := 1.0
t := -3.141592654
```

```
> u;
```

$$-0.4994017002 + 0.8664065008 I \tag{4}$$

```
> evalf(exp(2*Pi*I/3));
```

$$-0.5000000000 + 0.8660254040 I \tag{5}$$

```
> (-.5000000000+.8660254040*I)^3;
```

$$1.0000000001 - 3.233738859 \cdot 10^{-10} I \tag{6}$$

Unfortunately, the builtin Ruge-Kutta is confused by the complex numbers (I guess?)

```
> with(Student[NumericalAnalysis]):
> t:='t';
RungeKutta(diff(w(t), t) = f(t,w(t)), w(-Pi) = 1.0, t = evalf(Pi),
submethod = rk4);

t := t
-0.6667 + 0.7579 I
```

So let's be more systematic now

```
> restart;
> p := (z,w) -> z^3+w^3-1;
```

$$p := (z, w) \rightarrow z^3 + w^3 - 1 \tag{8}$$

```
> omega:=evalf(exp(2*Pi*I/3));
N:=100;
h:= evalf(2*Pi/N);

omega := -0.5000000000 + 0.8660254040 I
N := 100
```

```
> for k from 1 to 3 do
f := unapply(-diff(p(omega^k+exp(I*t), w), t)/diff(p(omega^k+exp(I*t), w), w),
t, w);
for m from 1 to 3 do
u:=omega^m;
if k=1 then tt:= -evalf(Pi/3)
elif k=2 then tt:= evalf(Pi/3)
elif k=3 then tt:= -evalf(Pi)
end if;
for j from 1 to N do
u:= u + f(tt,u)*h;
```

```
tt:=tt+h:
od:
print (omega^k, omega^m, u) :
od:
od:
```

```
-0.5000000000 + 0.8660254040 I, -0.5000000000 + 0.8660254040 I, -0.5006291993 - 0.8656977886 I
-0.5000000000 + 0.8660254040 I, -0.5000000004 - 0.8660254040 I, 1.000030879 - 0.0007087069736 I
-0.5000000000 + 0.8660254040 I, 1.0000000001 - 3.233738859 10-10 I, -0.4994016819 + 0.8664064995 I
-0.5000000004 - 0.8660254040 I, -0.5000000000 + 0.8660254040 I, -0.5006291980 - 0.8656977908 I
-0.5000000004 - 0.8660254040 I, -0.5000000004 - 0.8660254040 I, 1.000030888 - 0.0007087066195 I
-0.5000000004 - 0.8660254040 I, 1.0000000001 - 3.233738859 10-10 I, -0.4994016980 + 0.8664065041 I
1.0000000001 - 3.233738859 10-10 I, -0.5000000000 + 0.8660254040 I, -0.5006291929 - 0.8656977945 I
1.0000000001 - 3.233738859 10-10 I, -0.5000000004 - 0.8660254040 I, 1.000030877 - 0.0007087024967 I
1.0000000001 - 3.233738859 10-10 I, 1.0000000001 - 3.233738859 10-10 I, -0.4994016972 + 0.8664064990 I
```

(10)