What are the analytic one forms on a curve?

To make everything concrete, what are the analytic one forms on $x^3 + 1 - y^3$?

Idea consider $\frac{dx}{y}$ on the curve $y = 0$ and $\frac{dy}{y}$ on $y = 0$.

We can "take the residue to get rid of the denominator" giving $dx$.

More generally $\frac{p(x, y)}{y} \, dx \, dy$ where $p(x, y)$ is not identically 0 on any given $p(x, 0) \, dx$.

Is this well defined?
$\mathbb{P}^2$ consists of points $[z_0, z_1, z_2]$ $(z_0, z_1, z_2) \neq (0, 0, 0)$

and for $\lambda \neq 0$ $[\lambda z_0, \lambda z_1, \lambda z_2] = [z_0, z_1, z_2]$

3 patches covering are

$[1, x, y]$ on overlap $[1, x, y] = [u, 1, v]$

$[u, 1, v]$ on overlap $[u, 1, v] = [a, b, 1] = \left[ \frac{a}{b}, 1, \frac{1}{b} \right]$

$[a, b, 1]$ on overlap $[a, b, 1] = [1, x, y]$

$= \left[ \frac{1}{y}, \frac{x}{y}, 1 \right]$
\[ p(x, y, z) = \frac{1}{1 - y^3 + x} \]

\[ d = \text{length} \]

\[ d(x, y, z) = \sqrt{x^2 + y^2 + z^2} \]

\[ p(x, y) = p(y, y) \]

\[ \text{Note:} \quad p(1, x, y) = p(0, y, x) \]

\[ \text{When} \quad p(x, y, z) \equiv 0 \]

\[ \text{and} \quad p(1, y, x) \quad \text{on the path} \]

\[ p(1, y, x) = 0 \]

\[ \text{Conserve} \quad \text{on} \quad \text{MP} \]

\[ \frac{dX}{dy} \]

\[ \frac{dY}{dy} \]
\[
\frac{dx dy}{p(1,x,y)} = \frac{d(u) d(v)}{p(u,1,v)} u^d
\]

\[
= - \frac{du}{u^2} \frac{dv}{u} u^d = \frac{(dv du) u^{d-3}}{p(u,1,v)}
\]

\[
dx dy \rightarrow \frac{dv du}{u^3}
\]

\[
\sum_{x,y} g(x,y) \frac{dx dy}{p(1,x,y)} \text{ well defined}
\]

\[
dx g = d-3
\]

\[
\frac{dx dy}{x^3-y^3+1} = w \cdot (3x^2 dx - 3y^2 dy)
\]

"we want to use x as a "global coordinate"

\[
w = \frac{dx}{(x^3-y^3+1)(-3y^2)} \text{ in for}
\]
Residue on C in \([1, \infty, y]\)

\[
\frac{dx}{dp(0, x, y)}
\]

\[
\frac{dx}{-3y^2} = \frac{dx}{-3\sqrt[3]{(x^3 + 1)^2}}
\]

No zeroes

\[x = e^{\frac{1}{3}y^3}\]

More generally, q p homogeneous

\[V(p) \text{ smooth,} \quad \deg p = d, \quad \deg q_f = d - 3\]

\[\frac{dx}{dq(1, x, y)} \text{ is an analytic form on } C\]