

```
> with(LinearAlgebra);
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[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm, BilinearForm, CARE, CharacteristicMatrix, CharacteristicPolynomial, Column, ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix, CompressedSparseForm, ConditionNumber, ConstantMatrix, ConstantVector, Copy, CreatePermutation, CrossProduct, DARE, DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, FromCompressedSparseForm, FromSplitForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic, GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, KroneckerProduct, LA_Main, LUdecomposition, LeastSquares, LinearSolve, LyapunovSolve, Map, Map2, MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower, MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply, NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot, PopovForm, ProjectionMatrix, QRdecomposition, RandomMatrix, RandomVector, Rank, RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation, RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues, SmithForm, SplitForm, StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis, SylvesterMatrix, SylvesterSolve, ToeplitzMatrix, Trace, Transpose, TridiagonalForm, UnitVector, VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip]
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(1)

```
> P:=Matrix(3);
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$$P := \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(2)

```
> a:=1/2:b:=3/10:c:=1/5:
d:=2/5:e:=2/5:f:=1/5:
g:=1/4:h:=7/20:i:=2/5:
P(1,1):=a:
P(1,2):=b:
P(1,3):=c:
P(2,1):=d:
P(2,2):=e:
P(2,3):=f:
P(3,1):=g:
P(3,2):=h:
P(3,3):=i:
P;
Determinant(P);
```

$$\begin{bmatrix} \frac{1}{2} & \frac{3}{10} & \frac{1}{5} \\ \frac{2}{5} & \frac{2}{5} & \frac{1}{5} \\ \frac{1}{4} & \frac{7}{20} & \frac{2}{5} \end{bmatrix}$$

$$\frac{1}{50}$$

(3)

> **A:= MatrixInverse(IdentityMatrix(3)-s*P) ;**

$$A := \begin{bmatrix} -\frac{1}{2} \frac{9s^2 - 80s + 100}{s^3 - 16s^2 + 65s - 50} & \frac{5}{2} \frac{s(-6 + s)}{s^3 - 16s^2 + 65s - 50} & \frac{s}{s^2 - 6s + 5} \\ \frac{1}{2} \frac{s(-40 + 11s)}{s^3 - 16s^2 + 65s - 50} & -\frac{5}{2} \frac{3s^2 - 18s + 20}{s^3 - 16s^2 + 65s - 50} & \frac{s}{s^2 - 6s + 5} \\ -\frac{1}{2} \frac{s(4s + 25)}{s^3 - 16s^2 + 65s - 50} & \frac{5}{2} \frac{s(-7 + 2s)}{s^3 - 16s^2 + 65s - 50} & -\frac{4s - 5}{s^2 - 6s + 5} \end{bmatrix}$$

(4)

> **sP:=unapply(A,s) ;**

> **sP(s) ;**

$$\begin{bmatrix} -\frac{1}{2} \frac{9s^2 - 80s + 100}{s^3 - 16s^2 + 65s - 50} & \frac{5}{2} \frac{s(-6 + s)}{s^3 - 16s^2 + 65s - 50} & \frac{s}{s^2 - 6s + 5} \\ \frac{1}{2} \frac{s(-40 + 11s)}{s^3 - 16s^2 + 65s - 50} & -\frac{5}{2} \frac{3s^2 - 18s + 20}{s^3 - 16s^2 + 65s - 50} & \frac{s}{s^2 - 6s + 5} \\ -\frac{1}{2} \frac{s(4s + 25)}{s^3 - 16s^2 + 65s - 50} & \frac{5}{2} \frac{s(-7 + 2s)}{s^3 - 16s^2 + 65s - 50} & -\frac{4s - 5}{s^2 - 6s + 5} \end{bmatrix}$$

(5)

> **g:=taylor(sP(s)(1,2),s,11) ;**

$$g := \frac{3}{10}s + \frac{17}{50}s^2 + \frac{173}{500}s^3 + \frac{347}{1000}s^4 + \frac{17359}{50000}s^5 + \frac{173607}{500000}s^6 + \frac{1736103}{5000000}s^7 + \frac{3472219}{10000000}s^8$$

$$+ \frac{173611079}{500000000}s^9 + \frac{1736111047}{5000000000}s^{10} + O(s^{11})$$

(6)

> **coeff(convert(g,polynom),s^6) ;**

$$\frac{173607}{500000}$$

(7)

> **X:=Matrix(3) :**

for j from 1 to 3 do for k from 1 to 3 do

X(j,k):=limit((1-s)*sP(s)(j,k),s=1) ;

od:

od:

X;

$$\begin{bmatrix} \frac{29}{72} & \frac{25}{72} & \frac{1}{4} \\ \frac{29}{72} & \frac{25}{72} & \frac{1}{4} \\ \frac{29}{72} & \frac{25}{72} & \frac{1}{4} \end{bmatrix}$$

(8)

```
> P:=Matrix(4):
P(1,2):=p: P(1,4):=1-p:
P(2,1):=1-p: P(2,3):=p:
P(3,2):=1-p: P(3,4):=p:
P(4,1):=p: P(4,3):=1-p:
P;
```

$$\begin{bmatrix} 0 & p & 0 & 1-p \\ 1-p & 0 & p & 0 \\ 0 & 1-p & 0 & p \\ p & 0 & 1-p & 0 \end{bmatrix}$$

(9)

```
> A:= MatrixInverse(IdentityMatrix(4)-s*P):
sP:=unapply(A,s):
sP(s);
```

$$\left[\left[\begin{array}{l} -\frac{2p^2s^2-2ps^2+1}{4p^2s^4-4ps^4-4p^2s^2+s^4+4ps^2-1}, -\frac{s(2p^2s^2-3ps^2+s^2+p)}{4p^2s^4-4ps^4-4p^2s^2+s^4+4ps^2-1}, \\ -\frac{(2p^2-2p+1)s^2}{4p^2s^4-4ps^4-4p^2s^2+s^4+4ps^2-1}, -\frac{s(2p^2s^2-ps^2-p+1)}{4p^2s^4-4ps^4-4p^2s^2+s^4+4ps^2-1} \end{array} \right], \right. \\ \left[\begin{array}{l} -\frac{s(2p^2s^2-ps^2-p+1)}{4p^2s^4-4ps^4-4p^2s^2+s^4+4ps^2-1}, -\frac{2p^2s^2-2ps^2+1}{4p^2s^4-4ps^4-4p^2s^2+s^4+4ps^2-1}, \\ -\frac{s(2p^2s^2-3ps^2+s^2+p)}{4p^2s^4-4ps^4-4p^2s^2+s^4+4ps^2-1}, -\frac{(2p^2-2p+1)s^2}{4p^2s^4-4ps^4-4p^2s^2+s^4+4ps^2-1} \end{array} \right], \\ \left[\begin{array}{l} -\frac{(2p^2-2p+1)s^2}{4p^2s^4-4ps^4-4p^2s^2+s^4+4ps^2-1}, -\frac{s(2p^2s^2-ps^2-p+1)}{4p^2s^4-4ps^4-4p^2s^2+s^4+4ps^2-1}, \\ -\frac{2p^2s^2-2ps^2+1}{4p^2s^4-4ps^4-4p^2s^2+s^4+4ps^2-1}, -\frac{s(2p^2s^2-3ps^2+s^2+p)}{4p^2s^4-4ps^4-4p^2s^2+s^4+4ps^2-1} \end{array} \right], \\ \left. \left[\begin{array}{l} -\frac{s(2p^2s^2-3ps^2+s^2+p)}{4p^2s^4-4ps^4-4p^2s^2+s^4+4ps^2-1}, -\frac{(2p^2-2p+1)s^2}{4p^2s^4-4ps^4-4p^2s^2+s^4+4ps^2-1}, \\ -\frac{s(2p^2s^2-ps^2-p+1)}{4p^2s^4-4ps^4-4p^2s^2+s^4+4ps^2-1}, -\frac{2p^2s^2-2ps^2+1}{4p^2s^4-4ps^4-4p^2s^2+s^4+4ps^2-1} \end{array} \right] \right]$$

(10)

```
> sP1:=Matrix(4):
for j from 1 to 4 do for k from 1 to 4 do
sP1(j,k):=limit((1-s)*sP(s)(j,k),s=1);
od:
od:
sP1;
```

[

$$\begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{bmatrix}$$

(11)