Internal Convection: Fully Developed Flow

**Laminar Flow in Circular Tube: Analytical**
- local Nusselt number is constant in fully develop region
  - depends on surface thermal condition
- constant heat flux \( Nu_D = \frac{hD}{k} = 4.36 \)
- constant surface temperature \( Nu_D = \frac{hD}{k} = 3.66 \)

**Turbulent Flow in a Circular Tube: Empirical**
- smooth surface & fully turbulent \((Re_D > 10,000)\): Dittus-Boelter Correlation
  \[
  Nu_D = 0.023Re_D^{3/4}Pr^n \rightarrow \begin{cases} 
    n = 0.3 & (T_s < T_m) \\
    n = 0.4 & (T_s > T_m) 
  \end{cases}
  \]
- rough surface & transitional turbulent \((Re_D = 3000)\): Gnielinski Correlation
  \[
  Nu_D = \frac{(f/8)(Re_D - 1000)Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)} \quad \text{smooth surface} \\
  f = (0.790 \ln Re_D - 1.64)^{-2}
  \]
Internal Convection: Fully Developed Flow

- **Non-Circular Tubes**
  - use *hydraulic diameter* as the characteristic length

\[ D_h = \frac{4A_c}{P} \]

- since the local convection coefficient varies around the periphery of a tube, approaching zero at its corners, correlations for the fully developed region are associated with convection coefficients averaged over the periphery of the duct.

- **Laminar Flow**
  - local *Nusselt number* is *constant* but a function of *duct geometry* and *surface thermal condition*

- **Turbulent Flow**
  - use Dittus-Boelter correlation or Gnielinski correlation as a first approximation for any surface thermal condition
Internal Convection: Fully Developed Flow

Table 8.1 Nusselt numbers and friction factors for fully developed laminar flow in tubes of differing cross section

\[ Nu_D = \frac{hD_h}{k} \]

<table>
<thead>
<tr>
<th>Cross Section</th>
<th>( b/a )</th>
<th>(Uniform ( q_s'' ))</th>
<th>(Uniform ( T_s ))</th>
<th>( f \cdot Re_{D_h} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle</td>
<td>—</td>
<td>4.36</td>
<td>3.66</td>
<td>64</td>
</tr>
<tr>
<td>Square</td>
<td>1.0</td>
<td>3.61</td>
<td>2.98</td>
<td>57</td>
</tr>
<tr>
<td>Rectangular</td>
<td>1.43</td>
<td>3.73</td>
<td>3.08</td>
<td>59</td>
</tr>
<tr>
<td>Rectangular</td>
<td>2.0</td>
<td>4.12</td>
<td>3.39</td>
<td>62</td>
</tr>
<tr>
<td>Rectangular</td>
<td>3.0</td>
<td>4.79</td>
<td>3.96</td>
<td>69</td>
</tr>
<tr>
<td>Rectangular</td>
<td>4.0</td>
<td>5.33</td>
<td>4.44</td>
<td>73</td>
</tr>
<tr>
<td>Rectangular</td>
<td>8.0</td>
<td>6.49</td>
<td>5.60</td>
<td>82</td>
</tr>
<tr>
<td>Heated</td>
<td>( \infty )</td>
<td>8.23</td>
<td>7.54</td>
<td>96</td>
</tr>
<tr>
<td>Insulated</td>
<td>( \infty )</td>
<td>5.39</td>
<td>4.86</td>
<td>96</td>
</tr>
<tr>
<td>Triangle</td>
<td>—</td>
<td>3.11</td>
<td>2.49</td>
<td>53</td>
</tr>
</tbody>
</table>

Internal Convection: Entry Region

• The manner in which the **Nusselt number decays from inlet to fully developed conditions** for laminar flow depends on the nature of *thermal and velocity boundary layer development* in the entry region, as well as the *surface thermal condition*
  – affects the **average Nusselt number** across the length of the tube
  – typically analyze two *laminar* cases: combined & thermal entry length

\[
Nu_{Dh} = \frac{hL}{k}
\]
Internal Convection: Laminar Entry Region

- **Average Nusselt Numbers**

**Thermal Entry Length**
- simplifying assumption that the thermal boundary layer develops in the presence of a *fully developed velocity profile* \( \Rightarrow \) unheated starting length (USL)
  - also applicable for uniform velocity inlets for \( \text{Pr} \gg 1 \)
  - Hausen correlation

\[
\overline{\text{Nu}}_D = 3.66 + \frac{0.0668(D/L)\text{Re}_D \text{Pr}}{1 + 0.04[(D/L)\text{Re}_D \text{Pr}]^{2/3}}
\]

**Combined Entry Length**
- thermal and velocity boundary layers develop concurrently from uniform profiles at the inlet
  - Whitaker and Sieder & Tate correlations

laminar flow in circular tube with uniform surface temperature

\[
\left[(D/L)\text{Re}_D \text{Pr}\right]^{1/3} (\mu/\mu_s)^{0.14} \geq 2 \Rightarrow \overline{\text{Nu}}_D = 1.86\left[(D/L)\text{Re}_D \text{Pr}\right]^{1/3} (\mu/\mu_s)^{0.14}
\]

\[
\left[(D/L)\text{Re}_D \text{Pr}\right]^{1/3} (\mu/\mu_s)^{0.14} < 2 \Rightarrow \overline{\text{Nu}}_D = 3.66
\]
Internal Convection: Turbulent Entry Region

• **Entry Region**
  – the effects of the entry and surface thermal conditions are less for turbulent flows and may be *neglected*

• **Average Nusselt Number**
  – the *turbulent* average Nusselt number is a function of the tube geometry and the *fully developed Nusselt number*
  – circular tube, irrespective of surface thermal condition

\[
\begin{align*}
\text{short tubes} & \quad (L/D) < 60 \Rightarrow \overline{Nu_D} \approx \overline{Nu_{D,fd}} \left[ \frac{1 + C}{(L/D)^m} \right] \\
\text{long tubes} & \quad (L/D) > 60 \Rightarrow \overline{Nu_D} \approx \overline{Nu_{D,fd}} \quad C \approx 1 \; ; \; m \approx 2/3
\end{align*}
\]
Internal Convection: Non-Circular Entry Region

- **Laminar flow**
  - average Nusselt number depends strongly on aspect ratio, entry region and surface thermal conditions
  - no single correlation available & must refer to literature

- **Turbulent flow**
  - as a first approximation typically apply correlations for a circular tube

For all internal flows, the effect of variable properties on the average Nusselt number may be considered by evaluating all properties at the average mean temperature:

$$\bar{T}_m = \frac{T_{m,i} + T_{m,o}}{2}$$

- most accurate solutions often require iteration on the fluid properties
Internal Convection: Concentric Annulus

- **Special Case: Concentric Tube Annulus**
  - fluid flow through region formed by concentric tubes
  - convection heat transfer may be from or to:
    - inner surface of outer tube
    - outer surface of inner tube
  - standard surface thermal conditions (uniform temperature or heat flux) typically considered
  - convection coefficients associated with each surface
    - correlations available in the literature for fully developed laminar and turbulent flow

\[
q''_i = h_i(T_{s,i} - T_m) \Rightarrow Nu_i = \frac{h_i D_h}{k}
\]

\[
q''_o = h_o(T_{s,o} - T_m) \Rightarrow Nu_o = \frac{h_o D_h}{k}
\]

\[D_h = D_o - D_i\]