Homework Assignment # 2

1. Show that \( \mathbb{Z}/pq \) is isomorphic to \( \mathbb{Z}/p \oplus \mathbb{Z}/q \) if and only if \( p \) and \( q \) are relatively prime.

2. a) Show that if \( X_1 \) and \( X_2 \) are compact connected surfaces, then
\[
\chi(X_1 \# X_2) = \chi(X_1) + \chi(X_2) - 2.
\]
b) Calculate the Euler characteristic of all compact connected surfaces.

3. a) Calculate the homology groups of all compact connected surfaces.
b) Can the Euler characteristic of a compact connected surface be expressed in terms of its homology groups?
c) Looking at your calculation of \( H_2(X) \) for compact connected surfaces \( X \), what do you observe?

Remarks:

(1) Later we will define the Euler characteristic of more general topological spaces in terms of their homology groups. We will generalize the description of the Euler characteristic as an alternating sum of the number of vertices, edges and faces to an important class of spaces known as “CW complexes”.

(2) Towards the end of the semester we will prove that your observation about \( H_2(X) \) for compact connected manifolds of dimension 2 generalizes to a statement about \( H_n(X) \) for compact connected manifolds of dimension \( n \).