Name: _____

Math 126, Final Exam Instructor: _____
May 6, 2004

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Good Luck!

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3.	(a)	(b)	(c)	(d)	(e)	17.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)	18.	(a)	(b)	(c)	(d)	(e)
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12.	(a)	(b)	(c)	(d)	(e)						
13.	(a)	(b)	(c)	(d)	(e)						
14.	(a)	(b)	(c)	(d)	(e)						

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Multiple Choice

1.(6 pts.) Let $f(x) = x^5 - x^3 + 2x$. Find $(f^{-1})'(2)$.

(a) 1/4 (b) 1/70 (c) 4 (d) 1/5 (e) 70

2.(6 pts.) Solve the following equation for x.

$$7^x 5^x = 5\, e^2 \ .$$

(a) $x = \dfrac{\ln 5 + 2}{\ln 12}$ (b) $x = \dfrac{2}{\ln 35}$ (c) $x = \dfrac{\ln 5 + 2}{\ln 35}$

(d) $x = \dfrac{2 \ln 7}{\ln 35}$ (e) There is no solution.

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3.(6 pts.) Find the derivative of

$$y = x^{1/x} \ .$$

(a) $x^{\frac{1}{x}-2}(1-\ln x)$ (b) $-x^{\frac{1}{x}-3}$ (c) $x^{-1}\ln x(1-\ln x)$

(d) $x^{\frac{1}{x}}(\ln(\ln x)-x^{-2})$ (e) $x^{\frac{1}{x}-2}$

4.(6 pts.) Evaluate the following limit.

$$\lim_{x\to\infty} x^{1/x} \ .$$

Remark: Note the function is the same in both problems 3 and 4.

(a) 0 (b) e^{-1} (c) 1 (d) e (e) ∞

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5.(6 pts.) Find $f'(x)$ for

$$f(x) = \ln(2^x + x) + \arcsin(e^x)$$

(a) $\dfrac{2^x + 1}{2^x + x} + \dfrac{1}{\sqrt{1 - e^{2x}}}$

(b) $\dfrac{2^x \ln 2}{2^x + x} + \dfrac{e^x}{1 + e^{2x}}$

(c) $\dfrac{e^x \ln 2}{2^x + x} + \dfrac{e^x}{\sqrt{1 - 2e^x}}$

(d) $\dfrac{2^x \ln 2 + 1}{2^x + x} + \dfrac{e^x}{\sqrt{1 - e^{2x}}}$

(e) $\dfrac{e^x \ln 2 + 1}{2^x + x} + \dfrac{e^x}{\sqrt{e^{2x} - 1}}$

6.(6 pts.) Which line below is the tangent line to the parameterized curve $x = t - \cos t$, $y = t + \sin t$ when $t = 0$?

(a) $y = 2x + 2$

(b) $y = \dfrac{t + \sin t}{t - \cos t}\,(x + 1)$

(c) $x = -1$, a vertical tangent

(d) $y = \dfrac{\pi}{2}x + \dfrac{\pi}{2}$

(e) $y = \dfrac{1 + \cos t}{1 + \sin t}\,(x + 1)$

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7.(6 pts.) Evaluate the following definite integral.

$$\int_1^e \frac{\ln x}{x^3} dx \ .$$

(a) $\frac{1}{2}(1-2e^{-2})$ (b) $\frac{1}{4}(1-3e^{-2})$ (c) $\frac{1}{2}(e^{-2}-1)$

(d) $\frac{1}{4}(e^{-3}+2e^{-2}-1)$ (e) $\frac{1}{4}(1-e^{-3}+2e^{-2})$

8.(6 pts.) Evaluate

$$\int \frac{x^2}{\sqrt{9-x^2}}\, dx.$$

(a) $\frac{1}{2}x\sqrt{9-x^2}+C$ (b) $\frac{9}{2}\left[\arcsin(x/3)-\frac{x}{3}\right]+C$

(c) $\frac{9}{2}\left[\arcsin(x/3)-\frac{x\sqrt{9-x^2}}{9}\right]+C$ (d) $\frac{9}{2}\left[\arcsin(x/3)-\frac{x^2}{9}\right]+C$

(e) $9\arcsin(x/3)+C$

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9.(6 pts.) Evaluate

$$\int \frac{x+5}{x^2+x-2}\,dx.$$

(a) $\ln|x^2+x-2|+C$

(b) $\ln\left|\frac{2(x+2)}{x-1}\right|+C$

(c) $\ln(2|x-1|-|x+2|)+C$

(d) $\ln\left|\frac{(x-1)^2}{x+2}\right|+C$

(e) $\ln\left|\frac{(x+2)^2}{x-1}\right|+C$

10.(6 pts.) Find the Midpoint Rule approximation (using four intervals) of

$$\int_0^4 x^2\,dx\ .$$

(a) 22 (b) 64/3 (c) $\frac{119}{4}$ (d) 21 (e) $\frac{95}{4}$

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11.(6 pts.) Find the arclength of $y = 4 - 2x^{3/2}$, for $0 \leq x \leq 2$?

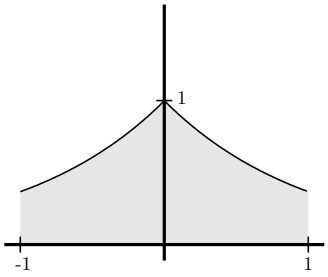
(a) $\frac{1}{9}[\sqrt{19} - 1]$ (b) $\frac{2}{3}[19\sqrt{19}]$ (c) $\frac{20}{3}\sqrt{10}$

(d) $\frac{2}{27}[10\sqrt{10} - 1]$ (e) $\frac{2}{27}[19\sqrt{19} - 1]$

12.(6 pts.) Find the center of mass of a plate with shape bounded below by the x–axis, on the left by $x = -1$, on the right by $x = 1$, and above by the curve

$$y = \begin{cases} e^x & -1 \leq x \leq 0 \\ e^{-x} & 0 \leq x \leq 1 \ . \end{cases}$$

The area is $2(1 - e^{-1})$.



(a) $\left(0, \frac{e^{-2} - 2}{8(1 - e^{-1})}\right)$ (b) $(0, 2(1 - e^{-1}))$ (c) $\left(0, \frac{2(2 - e^{-2})}{1 - e^{-1}}\right)$

(d) $\left(0, \frac{1 - e^{-2}}{4}\right)$ (e) $\left(0, \frac{1 - e^{-2}}{4(1 - e^{-1})}\right)$

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13.(6 pts.) Solve the initial value problem:

$$y' = (3 - 2x)(1 + y) \qquad y(1) = 0.$$

(a) $y(x) = e^{x^2-3x} - e^{-2}$ (b) $y(x) = e^{-2+3x-x^2} - 1$ (c) $y(x) = e^{3x-x^2} - e^2$

(d) $y(x) = e^{-2+3x-x^2}$ (e) $y(x) = e^{-1+3x-x^2} - e$

14.(6 pts.) If 100 grams of radioactive material with a half–life of two days are present at day zero, how many grams are left at day three?

(a) $\dfrac{100}{\sqrt{8}}$ (b) $\dfrac{100}{\sqrt{2}}$ (c) $\dfrac{100}{2^{1/3}}$ (d) $\dfrac{100}{4^{1/3}}$ (e) 50

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15.(6 pts.) If the function $y(x)$ satisfies the differential equation

$$xy' - 2y = x - 2$$

subject to the initial value $y(1) = 1$, what is $y(2)$?

(a) 5 (b) 4 (c) 3 (d) 2 (e) 1

16.(6 pts.) Which integral below computes the length of the parameterized curve $x(t) = t^3 + t$ and $y(t) = t^5 - t$ for $0 \leq t \leq 1$.

(a) $\displaystyle\int_0^1 (t^3 + t)\sqrt{2 + 6t^2 - t^4 + 25t^8}\ dt$ (b) $\displaystyle\int_0^1 (t^5 - t)\sqrt{t^3 + t^5}\ dt$

(c) $\displaystyle\int_0^1 \sqrt{t^3 + t^5}\ dt$ (d) $\displaystyle\int_0^1 \sqrt{2 + 6t^2 - t^4 + 25t^8}\ dt$

(e) $\displaystyle\int_0^1 (t^5 - t)(3t^2 + 1)\ dt$

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17.(6 pts.) Which integral below gives the area inside the polar curve $r = \sin(3\theta)$?

(a) $\frac{1}{2}\int_0^{\pi} \sin^2(3\theta)\ d\theta$

(b) $\frac{1}{2}\int_{\pi/6}^{\pi/3} \sin^2(3\theta)\ d\theta$

(c) $\frac{1}{2}\int_0^{\pi} \sqrt{\sin^2(3\theta) + 9\cos^2(3\theta)}\ d\theta$

(d) $\frac{1}{2}\int_0^{2\pi} \sin^2(3\theta)\ d\theta$

(e) $\frac{1}{2}\int_0^{2\pi} \sqrt{\sin^2(3\theta) + 9\cos^2(3\theta)}\ d\theta$

18.(6 pts.) Which statement below is true about the series $\sum_{n=1}^{\infty} \frac{2^n}{n^3 + 2^n}$

(a) $\lim_{n\to\infty} \frac{2^n}{n^3 + 2^n} = 0$ so the series converges.

(b) $\lim_{n\to\infty} \frac{2^n}{n^3 + 2^n} = 1$ so the series converges.

(c) $\lim_{n\to\infty} \frac{2^n}{n^3 + 2^n} = 1$ so the series diverges.

(d) $\lim_{n\to\infty} \frac{2^n}{n^3 + 2^n}$ does not exist so the series converges.

(e) $\lim_{n\to\infty} \frac{2^n}{n^3 + 2^n} = 0$ so the series diverges.

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19.(6 pts.) Sum the series $\sum_{n=1}^{\infty} \frac{3^n}{5^{2n}}$.

(a) $\frac{25}{22}$ (b) $\frac{3}{22}$ (c) $\frac{5}{3}$ (d) $\frac{96}{25}$ (e) $\frac{50}{11}$

20.(6 pts.) The interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(x+3)^n}{\sqrt{n}}$$

is

(a) $[2, 4]$ (b) $(-1, 1)$ (c) $(-4, -2)$

(d) $(2, 4)$ (e) $[-4, -2)$

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21.(6 pts.) Which series below absolutely converges?

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{n^3}$ (b) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n+1)}$ (c) $\sum_{n=1}^{\infty} \frac{\sqrt{n^3}}{n^2+1}$ (d) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}\pi^n}{3^n}$

(e) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3}$

22.(6 pts.) Which series below conditionally converges? **These are the same series as in problem 21**.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{n^3}$ (b) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n+1)}$ (c) $\sum_{n=1}^{\infty} \frac{\sqrt{n^3}}{n^2+1}$ (d) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}\pi^n}{3^n}$

(e) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3}$

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23.(6 pts.) Find a power series for the function

$$f(x) = -\frac{3}{(x+2)^2}.$$

(a) $\frac{3}{2}\sum_{n=1}^{\infty}(-1)^n n\frac{x^{n-1}}{2^n}$ (b) $\frac{3}{2}\sum_{n=0}^{\infty}\frac{(-1)^n}{(n+1)}\frac{x^{n+1}}{2^n}$ (c) $\frac{3}{2}\sum_{n=1}^{\infty}n\frac{x^{n-1}}{2^n}$

(d) $\frac{3}{2}\sum_{n=0}^{\infty}(-1)^n\frac{x^n}{2^n}$ (e) $\frac{3}{2}\sum_{n=1}^{\infty}(-1)^n n\frac{x^n}{2^n}$

24.(6 pts.) Which series below represents $\frac{\sin x}{x}$?

(a) $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}$ (b) $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{n!}$ (c) $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n+1)!}$

(d) $\sum_{n=0}^{\infty}(-1)^n\binom{1/2}{n}x^{2n}$ (e) $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n)!}$

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25.(6 pts.) Which series below gives the arc length of the curve $y = \dfrac{x^2}{2}$ from $x = 0$ to $x = 1$?

(a) $\displaystyle\sum_{n=0}^{\infty} \binom{1/2}{n} \frac{1}{2n+2}$ (b) $\displaystyle\sum_{n=0}^{\infty} \binom{1/3}{n} \frac{1}{2n+1}$ (c) $\displaystyle\sum_{n=0}^{\infty} \binom{1/3}{n} \frac{1}{2n+2}$

(d) $\displaystyle\sum_{n=0}^{\infty} \binom{1/2}{n} \frac{(-1)^n}{2n+1}$ (e) $\displaystyle\sum_{n=0}^{\infty} \binom{1/2}{n} \frac{1}{2n+1}$

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Math 126, Final Exam Instructor: ANSWERS
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