Problems for today

Find the plane containing $\langle 1, 1, 2\rangle$, $\langle 3, 0, 1\rangle$, and $\langle 2, 1, 0\rangle$.

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Two vectors in the plane:

$\vec{v}_1 = \langle 3,0,1\rangle - \langle 1,1,2\rangle = \langle 2,-1,-1\rangle$

and

$\vec{v}_2 = \langle 3,0,1\rangle - \langle 2,1,0\rangle = \langle 1,-1,1\rangle$

$\vec{v}_1 \times \vec{v}_2 = \langle -2, -(3), -1\rangle = \langle -2,-3,-1\rangle$.

The normal vector can be any multiple of $\langle -2,-3,-1\rangle$ so let $\vec{n} = \langle 2,3,1\rangle$.

An equation for the plane is

$\langle 2,3,1\rangle \cdot \langle x,y,z\rangle = \langle 2,3,1\rangle \cdot \langle 1,1,2\rangle = 7.$

Equivalently

$$2x + 3y + z = 7$$

Find the point on all three planes
$x + y - z = 6$, $2x - y - 5z = 6$ and $x + 2y + 7z = 1$

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$x = 6 - y + z$ from first so plugging into second:

$2(6 - y + z) - y - 5z = 6$, $12 - 3y - 3z = 6$, $3y + 3z = 6$, $y + z = 2$.

Hence $y = 2 - z$ so $x = 6 - (2 - z) + z = 4 + 2z$.

Plugging into third:

$(4 + 2z) + 2(2 - z) + 7z = 1$, $8 + 7z = 1$, $z = -1$, $y = 3$, $x = 2$.

$\boxed{(2, 3, -1).}$

Find an equation of the line which is the intersection of $2x - y + 5z = 10$ and $3x + y - 7z = 10$.

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A vector $\vec{v}$ lies in a plane with normal vector $\vec{n}$ if and only of $\vec{v}$ and $\vec{n}$ are orthogonal.

If the line has equation $\vec{r} = \vec{p} + \vec{m}t$ then $\vec{m}$ is orthogonal to the two normal vectors and hence we may use any non-zero multiple of

$$\begin{matrix} 2 & -1 & 5 \\ & \times & \\ 3 & 1 & -7 \end{matrix}$$

$\langle 2, -(-29), 5\rangle$ so take $\vec{m} = \langle 2, 29, 5\rangle$.

To find a point solve $2x - y + 5z = 10$ and $3x + y - 7z = 6$.

$x = \dfrac{10 + y - 5z}{2}$ so

$3\left(\dfrac{10 + y - 5z}{2}\right) + y - 7z = 10$, $30 + 3y - 15z + 2y - 14z = 20$, $10 + 5y - 29z = 0$.

One solution is $y = -2$, $z = 0$ and therefore

$x = \dfrac{10 + (-2) + 0}{2} = 4$ or $\vec{p} = \langle 4, -2, 0\rangle$ and

$$\boxed{\vec{r} = \langle 4, -2, 0\rangle + \langle 2, 29, 5\rangle t.}$$

Find the plane containing

$\vec{r}_1 = \langle 1, 3, 5 \rangle + \langle 1, 1, 1 \rangle t$ and
$\vec{r}_2 = \langle 5, 7, 9 \rangle + \langle 1, -1, 1 \rangle t$

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The line $\vec{r} = \vec{p} + \vec{m}t$ lies in the plane $\vec{n} \bullet \langle x, y, z \rangle = d$ if and only if $\vec{n} \bullet \vec{m} = 0$ and $\vec{n} \bullet \vec{p} = d$.

$\langle 1, 1, 1 \rangle \times \langle 1, -1, 1 \rangle = \langle 2, 0, -2 \rangle$ so if our plane exists it has $\vec{n} = \langle 1, 0, -1 \rangle$ for a normal vector.

If the first line lies in the plane $\vec{n} \bullet \langle x, y, z \rangle = d$, $d = \langle 1, 0, -1 \rangle \bullet \langle 1, 3, 5 \rangle = -4$.

Since $\langle 1, 0, -1 \rangle \bullet \langle 5, 7, 9 \rangle = -4$ both lines lie in
$\boxed{\langle 1, 0, -1 \rangle \bullet \langle x, y, z \rangle = -4.}$

Are $\vec{r}_1 = \langle 1, 4, 5 \rangle + \langle 1, 1, 1 \rangle t$ and

$\vec{r}_2 = \langle 5, -7, 9 \rangle + \langle 1, -1, 1 \rangle t$ skew?

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$\vec{r}_2 = \langle 5,-7,9\rangle + \langle 1,-1,1\rangle t$ skew?

Need to solve $\langle 1,4,5\rangle + \langle 1,1,1\rangle t = \langle 5,-7,9\rangle + \langle 1,-1,1\rangle s$.

Take cross product of both sides with $\langle 1,1,1\rangle$ and remember $\vec{m}\times\vec{m} = \vec{0}$:

$$\langle 1,1,1\rangle \times \langle 1,4,5\rangle = \langle 1,1,1\rangle \times \langle 5,-7,9\rangle + \langle 1,1,1\rangle \times \langle 1,-1,1\rangle s.$$

$$\langle 1,-4,3\rangle = \langle 16,-4,-12\rangle + \langle 2,0,-2\rangle s$$

$$\langle -15,0,15\rangle = \langle 2,0,-2\rangle s;\ s = -15/2.$$

If they intersect, they intersect at $\langle 5,-7,9\rangle + \langle 1,-1,1\rangle(-15/2) = \frac{1}{2}\Big(\langle 10,-14,18\rangle + \langle -15,15,-15\rangle\Big) = \frac{1}{2}\Big(\langle -5,1,3\rangle\Big)$.

Solve $\langle 1,4,5\rangle + \langle 1,1,1\rangle t = \frac{1}{2}\Big(\langle -5,1,3\rangle\Big)$ or

$\langle 1,1,1\rangle t = \frac{1}{2}\Big(\langle -5,1,3\rangle\Big) - \frac{1}{2}\Big(\langle 2,8,10\rangle\Big) = \frac{1}{2}\langle -7,7,-7\rangle$ so $t = -\frac{7}{2}$. Hence the lines intersect.

Are $\vec{r}_1 = \langle 2, 4, 5\rangle + \langle 1, 1, 1\rangle t$ and

$\vec{r}_2 = \langle 5, -7, 9\rangle + \langle 1, -1, 1\rangle t$ skew?

Need to solve $\langle 2, 4, 5\rangle + \langle 1, 1, 1\rangle t = \langle 5, -7, 9\rangle + \langle 1, -1, 1\rangle s$.

Take cross product of both sides with $\langle 1, 1, 1\rangle$ and remember $\vec{m} \times \vec{m} = \vec{0}$:

$\langle 1, 1, 1\rangle \times \langle 2, 4, 5\rangle = \langle 1, 1, 1\rangle \times \langle 5, -7, 9\rangle + \langle 1, 1, 1\rangle \times \langle 1, -1, 1\rangle s$.

$\langle 1, -3, 2\rangle = \langle 16, -4, -12\rangle + \langle 2, 0, -2\rangle s$

$\langle -15, -7, 14\rangle = \langle 2, 0, -2\rangle s$; no solution. Hence the lines are skew.

If they had been parallel, $\langle 1, 1, 1\rangle \times \langle 1, -1, 1\rangle$ would have been $\vec{0} = \langle 0, 0, 0\rangle$.