Goals for today

Local Extrema

A point on a graph $z = f(x, y)$ has a *local maximum* at a point (a, b) provided that for all (x, y) near (a, b), $f(x, y) < f(a, b)$.

A point on a graph $z = f(x, y)$ has a *local minimum* at a point (a, b) provided that for all (x, y) near (a, b), $f(x, y) < f(a, b)$.

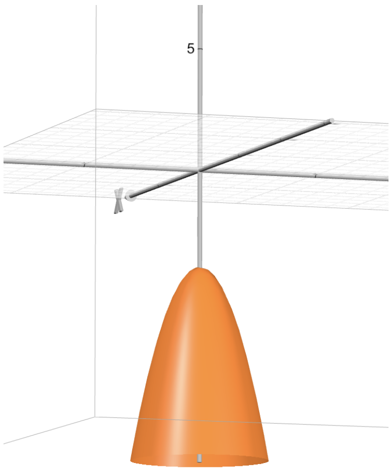
A *local extrema* is a point which is either a local max or a local min.

If f is differentiable at a point (a, b) which is a local extrema, then $\nabla f(a, b) = \langle 0, 0\rangle$.

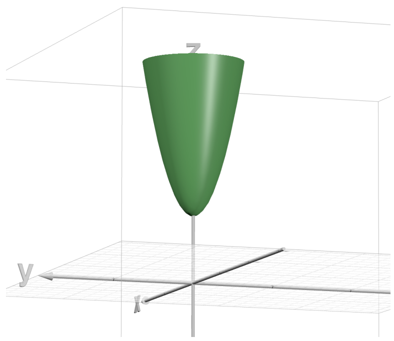
Points where $\nabla f = \langle 0, 0\rangle$ are called *critical points*.

In one variable a critical point is either a local max, a local min, or neither.

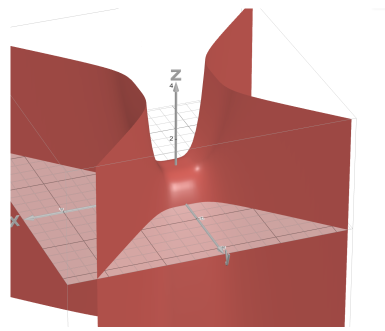
In multi-variables, there is a fourth option: a critical point is either a local max, a local min, a saddle point or none of these.



local max



local min



saddle

Second derivative test

If (a, b) is a critical point then compute

$$D(a,b) = \begin{vmatrix} f_{xx}(a,b) & f_{xy}(a,b) \\ f_{yx}(a,b) & f_{yy}(a,b) \end{vmatrix}$$

- If $D(a, b) > 0$ you have a local extrema:
  - if $f_{xx} > 0$ you have a local min
  - if $f_{xx} < 0$ you have a local max
- If $D(a, b) < 0$ you have a saddle point.
- If $D(a, b) = 0$ you have no idea.