Goals for today

Absolute Extrema

Review of the 1-variable case.

Let $f(x)$ be continuous on a closed, bounded interval $[a, b]$.

- The function $f$ has an *absolute maximum* at point $c \in [a, b]$ provided $f(c) \geqslant f(x)$ for all $x \in [a, b]$.
- The function $f$ has an *absolute minimum* at point $c \in [a, b]$ provided $f(c) \leqslant f(x)$ for all $x \in [a, b]$.

The value, $f(c)$ is an absolute max/min value.

There is a theorem which says that for $f(x)$ continuous on a closed, bounded interval $[a, b]$, there exists a least one point $x_{max}$ where $f$ has an absolute maximum, and at least one point $x_{min}$ where $f$ has an absolute minimum

To find these points, locate the critical points in $(a, b)$. Then compute the values of $f$ at the critical points and also $f(a)$ and $f(b)$. The absolute max value is the largest of this set of numbers and the absolute min value is the smallest.

There is no need for any second derivative test or anything else.

The 2-variable case has a similar outline.

First what is a closed, bounded region $R$ in the plane?

Bounded means that $R$ is contained in a disk with finite radius.

Closed is more complicated. A point $x \in \mathbb{R}^2$ is a *boundary point* provided every disk centered at $x$ contains points in $R$ and points in $\mathbb{R}^2 - R$. The set of all boundary points of $R$ is denoted $\partial R$.

$R - \partial R$ is the *interior of* $R$.

A set $R$ is *closed* if it contains all its boundary points.

Let $f$ be a function defined on $R$.

- The function $f$ has an *absolute maximum* at point $c \in R$ provided $f(c) \geqslant f(x)$ for all $x \in R$.
- The function $f$ has an *absolute minimum* at point $c \in R$ provided $f(c) \leqslant f(x)$ for all $x \in R$.

The value, $f(c)$ is an absolute max/min value.

There is a theorem which says that for $f(x)$ continuous on a closed, bounded interval $R$, there exists a least one point $x_{max}$ where $f$ has an absolute maximum, and at least one point $x_{min}$ where $f$ has an absolute minimum.

To find these points, locate the critical points in the interior of $R$. Then compute the values of $f$ at the critical points.

Then locate the critical points on $\partial R$. This is the subject of the next three lectures. Compute the values of $f$ at these points.

The absolute max value is the largest of this set of numbers and the absolute min value is the smallest.