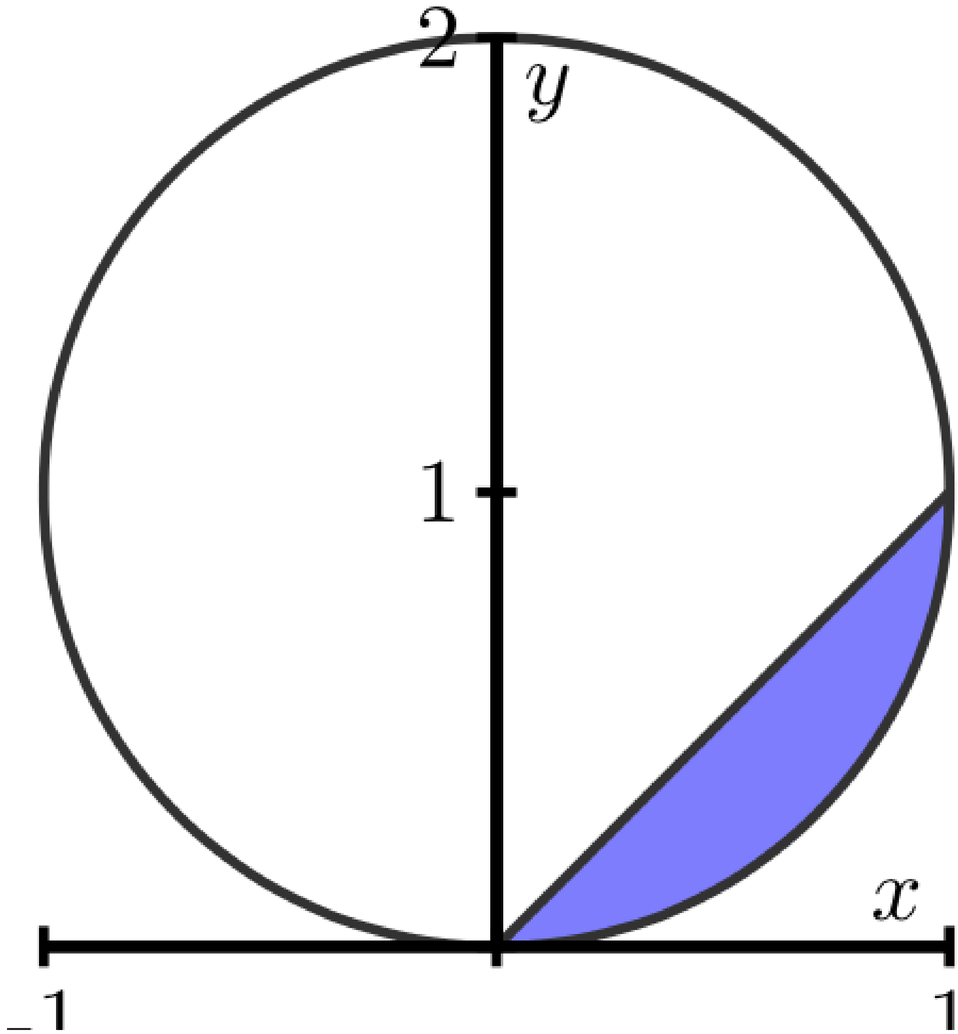
Goals for today

Double integrals in polar coordinates

$$x = r\cos(\theta)$$
$$y = r\sin(\theta)$$

$$r^2 = x^2 + y^2$$
$$\theta = \arctan\left(\frac{y}{x}\right)$$

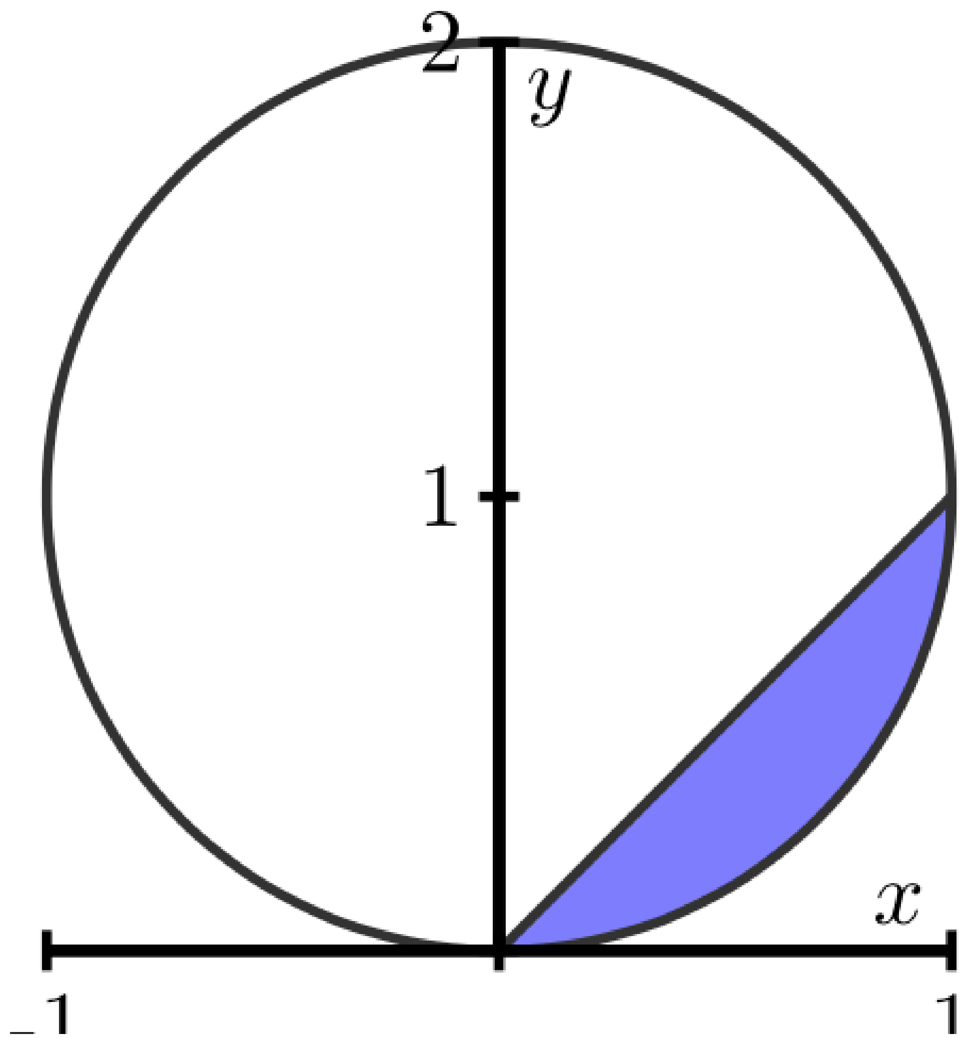
$$dA = r\,dr\,d\theta$$



$y = x$ and $x^2 + (y-1)^2 = 1$

$\theta = \dfrac{\pi}{4}$ and $r = 2\sin(\theta)$.

$$\iint\limits_R f\,dA = \int\int f(r,\theta)\,r\,dr\,d\theta$$



$y = x$ and $x^2 + (y-1)^2 = 1$

$\theta = \dfrac{\pi}{4}$ and $r = 2\sin(\theta)$.

$$\iint\limits_R f\,dA = \int_0^{\pi/4} \int_{r=0}^{r=2\sin(\theta)} f(r,\theta)\,r\,dr\,d\theta$$

$$\iint_D (x^2 + y^2)^{5/2} dA$$

where D is the disk $x^2 + y^2 \leqslant 4$.

Property of iterated integrals:

$$\int_a^b \int_c^d f(u) \cdot g(v) du dv = \left(\int_a^b f(u) du \right) \cdot \left(\int_c^d g(v) dv \right)$$

Convert the iterated integral

$$\int_0^{\sqrt{2}} \int_{-x}^{x} dy dx$$

into an iterated polar integral.

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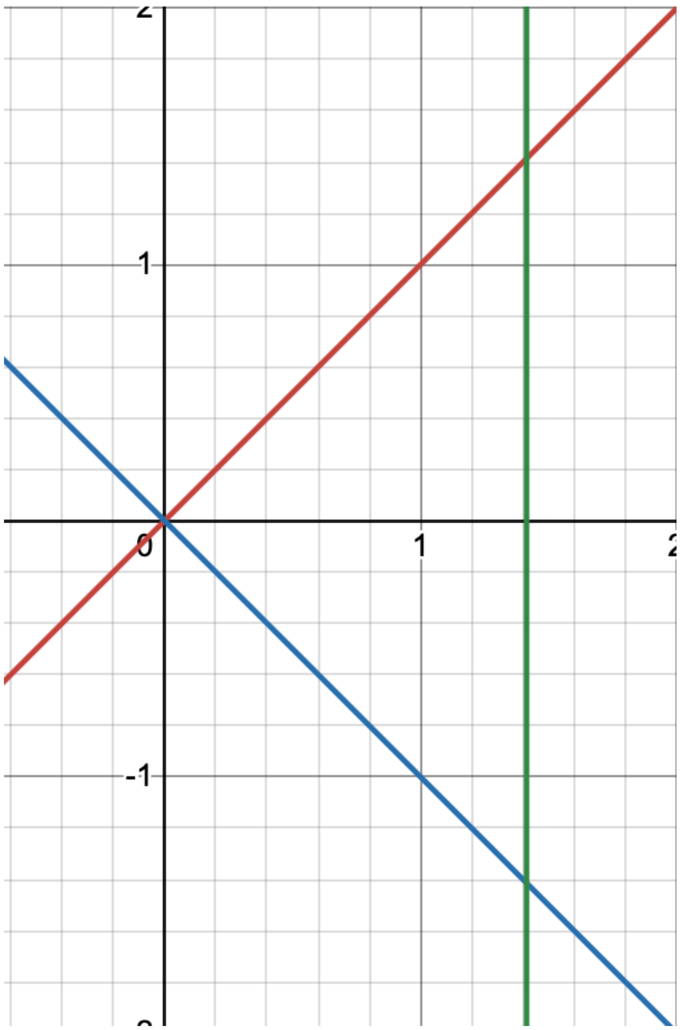

$$\int_{-\pi/4}^{\pi/4} \int_{r=0}^{r=\sqrt{2}\sec(\theta)} rdrd\theta$$

Evaluate the integral

$$\iint\limits_D r^2 \sin(\theta) dA$$

D is the region bounded by the polar axis and the upper half of the cardioid $r = 1 + \cos(\theta)$.

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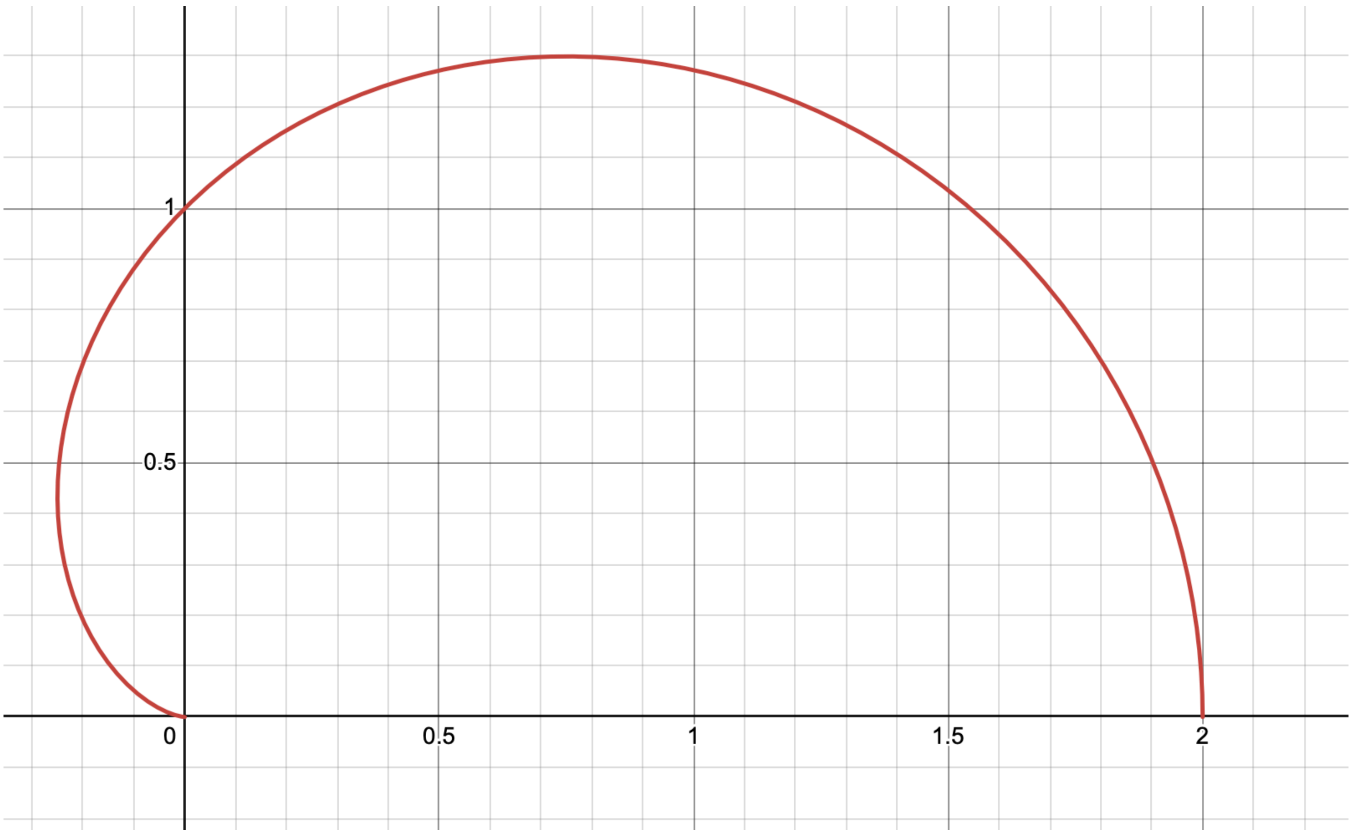

$$\int_0^{\pi} \int_{r=0}^{r=1+\cos(\theta)} r^2 \sin(\theta) r dr d\theta$$

$$\iint\limits_D e^{-(x^2+y^2)} dA$$

where D is the disk $x^2 + y^2 \leqslant R^2$.

$$\iint\limits_S e^{-(x^2+y^2)} dA$$

where S is the square $[-R, R] \times [-R, R]$

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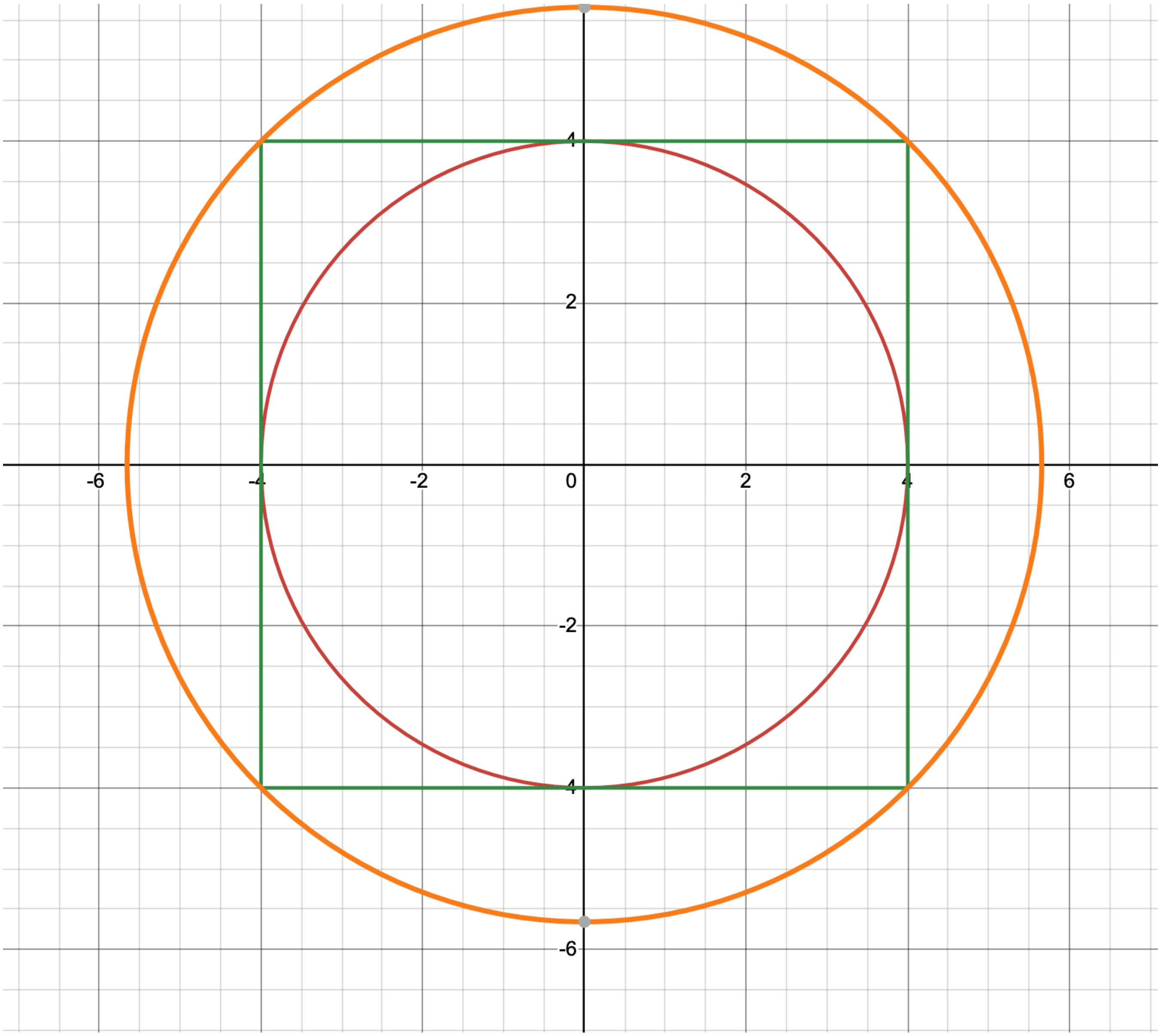
where D is the disk $x^2 + y^2 \leqslant R^2$.

$$\int_0^{2\pi} \int_{r=0}^{r=R} e^{-r^2} r dr d\theta = 2\pi \left(-\frac{1}{2}\right) e^{-r^2} \Big|_0^R = \pi - \pi e^{-R^2}$$

$$\iint_S e^{-(x^2+y^2)} dA$$

where S is the square $[-R, R] \times [-R, R]$

$$\int_{-R}^{R} \int_{y=-R}^{y=R} e^{-x^2} e^{-y^2} dy dx = \left(\int_{-R}^{R} e^{-t^2} dt \right)^2$$



$x^2 + y^2 = R^2$; $[-R, R] \times [-R, R]$; $x^2 + y^2 = 2R^2$

$$\pi - \pi e^{-R^2} < \left(\int_{-R}^{R} e^{-t^2} dt \right)^2 < \pi - \pi e^{-2R^2}$$

so passing to the limit,

$$\int_{-\infty}^{\infty} e^{-t^2} dt = \sqrt{\pi}$$