Goals for today

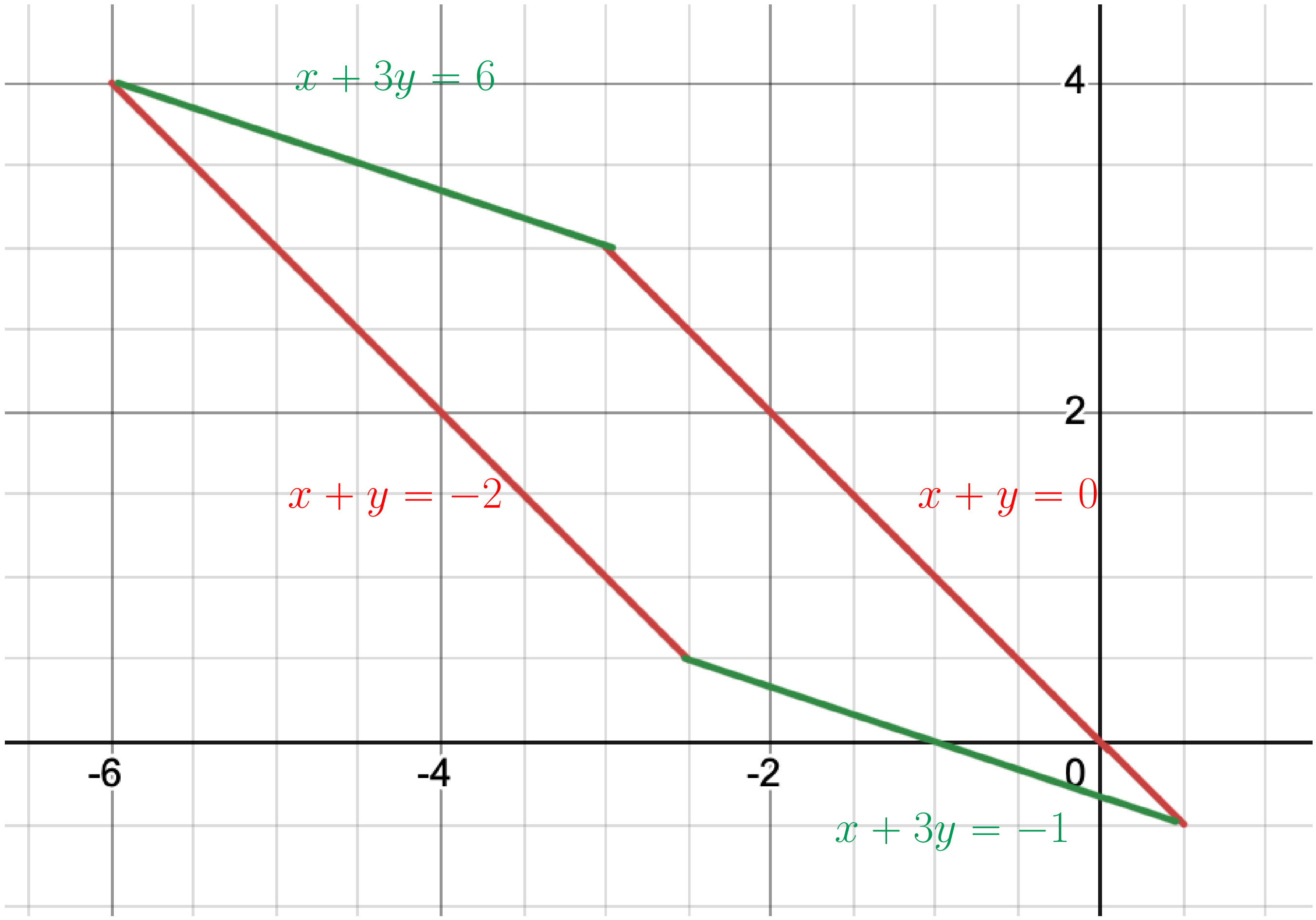
Change of variables in 2D

Transformation $T(u,w) = (x,y)\colon \mathbb{R}^2 \to \mathbb{R}^2$.

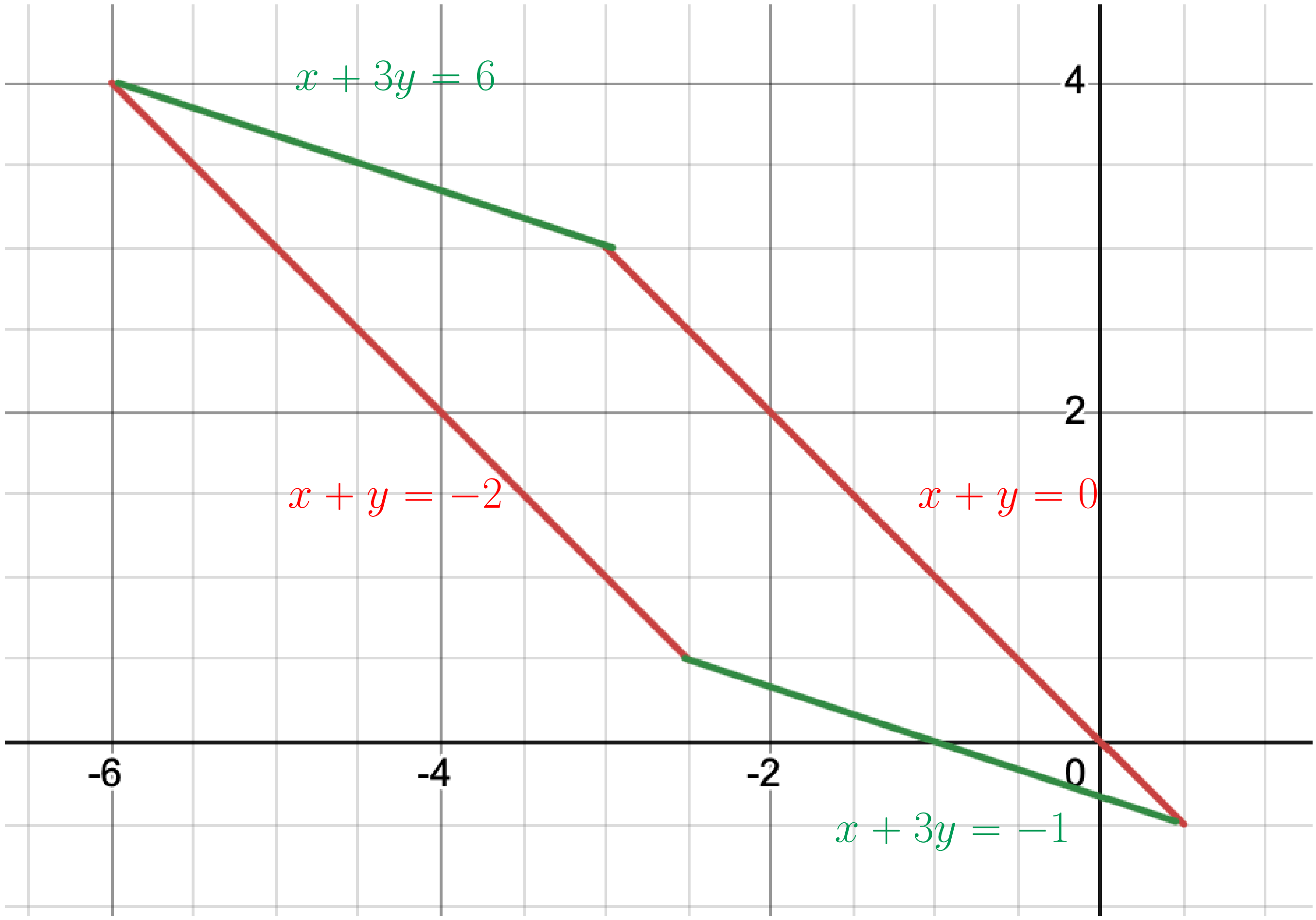
$$\text{Jacobian of } T(u,w) = \begin{vmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial w} \\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial y} \end{vmatrix}$$

If R is a region in the $u\,w$ plane and D is a region in the $x\,y$ plane, $T(R) = D$ provided every point $(x,y) \in D$ is the image of a unique point $(u,w) \in R$ except maybe along the boundary.

$$\iint\limits_D f(x,y)dA_{x\,y} = \iint\limits_R f\big(T(u,w)\big)\begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial y} \end{vmatrix} dA_{u\,w}$$



D



$$\iint_D dA = ?$$

$$\iint_D x dA = ?$$

$$\iint_D y dA = ?$$

Evaluate

$$\iint\limits_D xy^3 dA$$

where D is the region bounded by $xy = 1$, $xy = 3$, $y = 2$ and $y = 6$ using the transform $T(u, w) = \left(\frac{w}{6u}, 2u\right)$.

Derive a transformation that will convert the triangle with vertices $(1, 0)$, $(6, 0)$ and $(3, 8)$ into a right triangle with sides parallel to the axes.