Goals for today

Change of variables in 3D

Transformation $T(u, v, w) = (x, y, z)\colon \mathbb{R}^3 \to \mathbb{R}^3$.

$$\text{Jacobian of } T(u, v, w) = \begin{vmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v} & \dfrac{\partial x}{\partial w} \\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v} & \dfrac{\partial y}{\partial w} \\ \dfrac{\partial z}{\partial u} & \dfrac{\partial z}{\partial v} & \dfrac{\partial z}{\partial w} \end{vmatrix}$$

If S is a solid in the $u\,v\,w$ plane and E is a region in the $x\,y\,z$ plane, $T(S) = E$ provided every point $(x, y, z) \in E$ is the image of a unique point $(u, v, w) \in S$ except maybe along the boundary.

$$\iiint\limits_E f(x, y, z) dV_{x\,y\,z} = \iiint\limits_S f\big(T(u, v, w)\big) \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix} dV_{u\,v\,w}$$

The transformation to deal with the ellipse $\frac{x^2}{4} + \frac{y^2}{25} = 1$ is $x = \mathbf{2}r\cos(\theta)$, $y = 5r\sin(\theta)$ NOT $x = 4r\cos(\theta)$, $y = 5r\sin(\theta)$ as claimed in class.