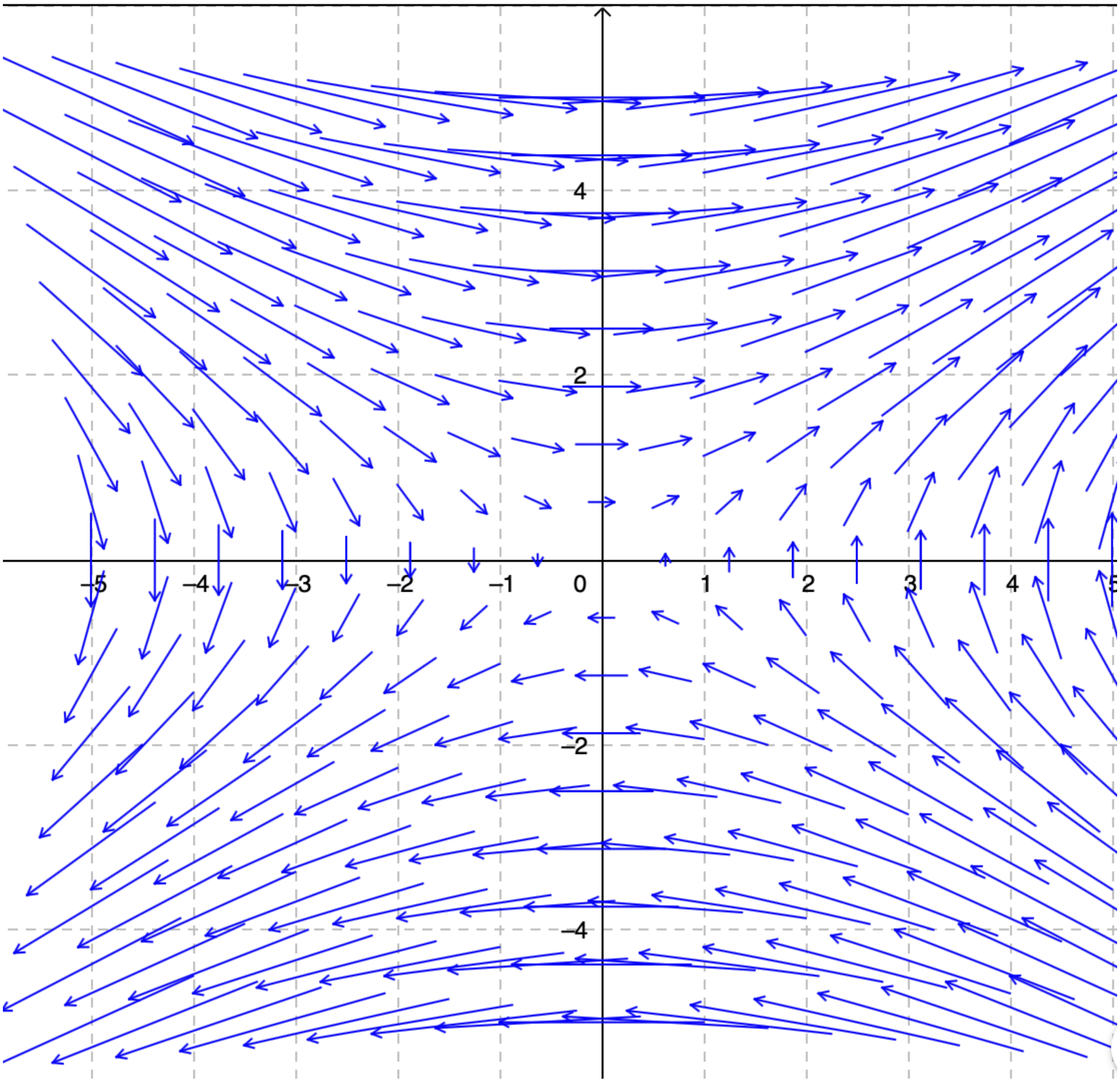
Goals for today

Vector fields

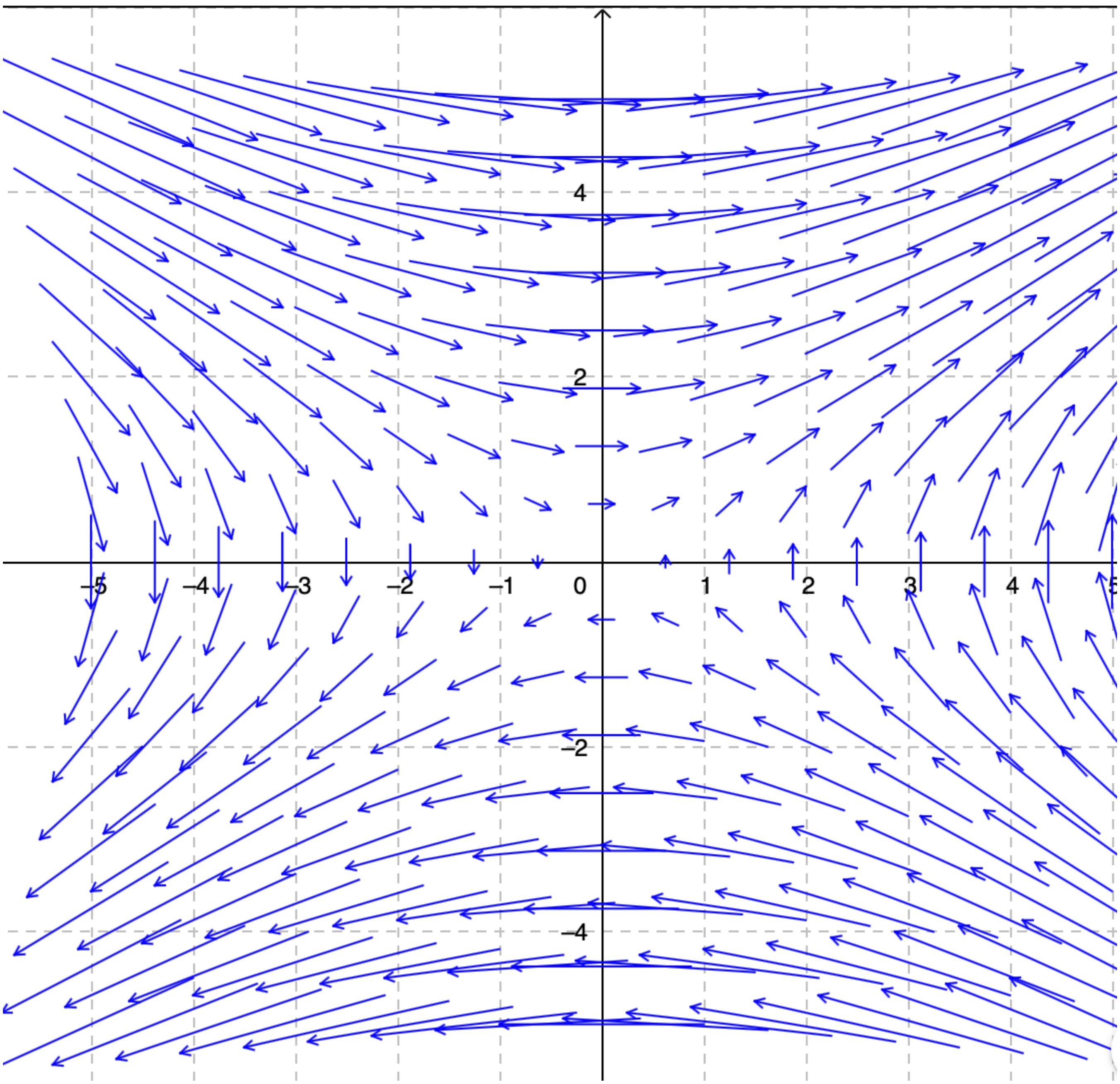
Line integrals

Vector field:

$$\vec{F}(x,y) = \langle P(x,y), Q(x,y) \rangle$$
$$\vec{F}(x,y,z) = \langle P(x,y,z), Q(x,y,z), R(x,y,z) \rangle$$



Example



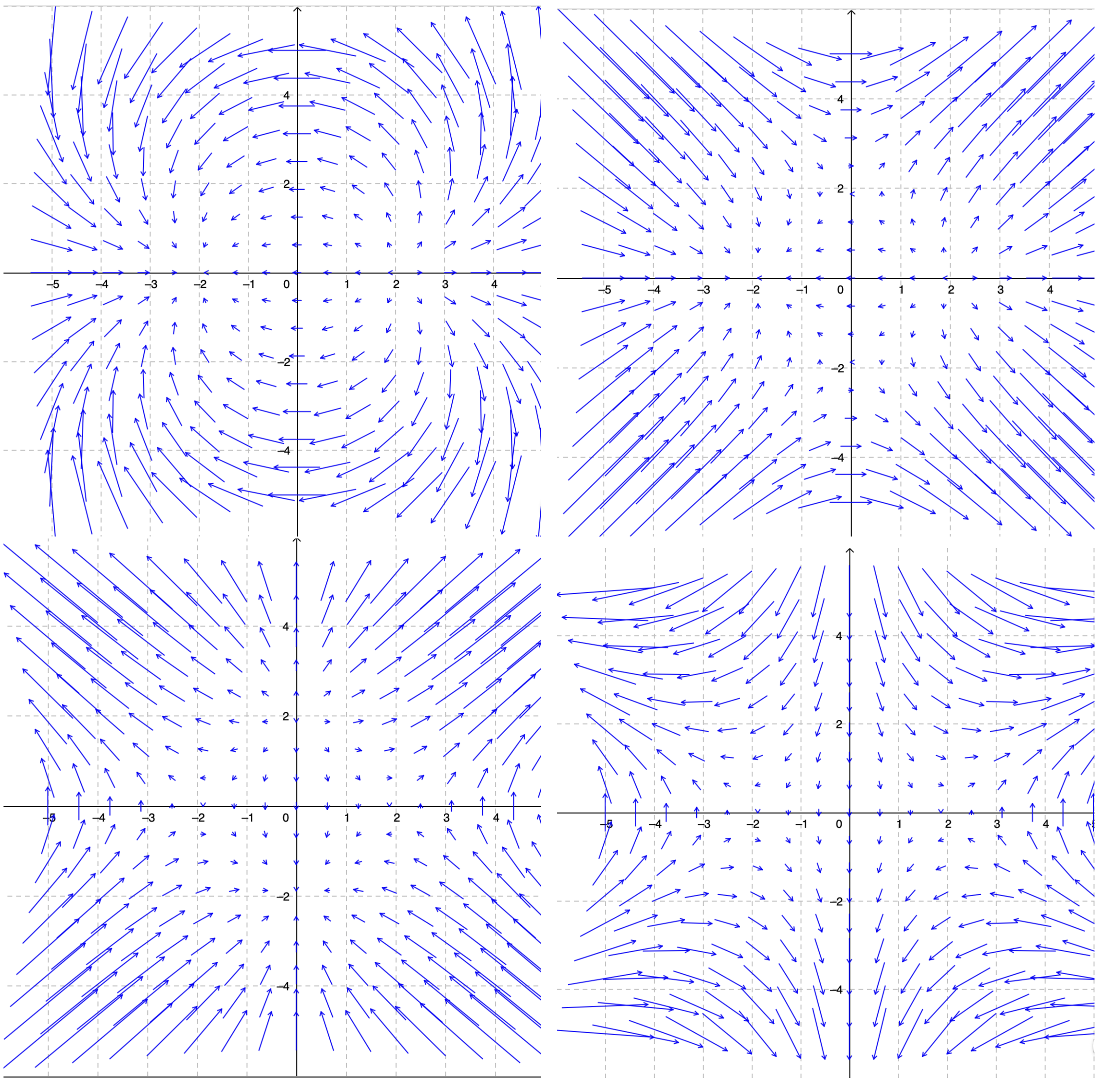
One of the equations below is the equation of this field. Which one is it?

(a)$\langle x^2, y^2 \rangle$

(b)$\langle \sin(2\pi y), \sin(2\pi x) \rangle$

(c)$\langle 2y, x \rangle$

(d)$\langle e^x, y \rangle$



Which field above is the field for

$$\left\langle x^2 - y^2 - 4, 2xy \right\rangle ?$$

One way to construct fields is to start with a function of 2 or 3 variables, say f. Then

$$\vec{F} = \nabla f$$

is a vector field called a *gradient field.* It may also be called a *conservative vector field.*

Any function f whose gradient is $\vec{F}$ is called a *potential function* for $\vec{F}$.

In 16.3 we will discuss how to determine if a field is a gradient field and how to find potential functions for it.

Line integrals: Given a function, say $f(x, y)$ and a curve $\vec{r}(t) = \langle x(t), y(t)\rangle$, $a \leqslant t \leqslant b$, define

$$\int_c^d f\big(x(s), y(s)\big)ds$$

just the way you think. Parametrize the curve by arc length s, starting at c and finishing at d, cut it up into little pieces, pick a point in each piece, evaluate f at the point, multiply by the length of the piece, add up all these values and take the limit.

Problem is you usually can't parametrize the curve via arc length. BUT $\dfrac{ds}{dt} = |\vec{r}\,'|$ so

$$\int_c^d f\big(x(s), y(s)\big)ds = \int_a^b f\big(x(t), y(t)\big)|\vec{r}\,'(t)|dt$$

An important source of examples comes from vector fields $\vec{F}$. We can get a function

$$f(x, y) = \vec{F} \bullet \frac{\vec{r}'}{|\vec{r}'|} = \vec{F} \bullet \vec{\mathbf{T}}$$

We sometimes write the resulting line integral as

$$\int_a^b \vec{F} \bullet d\vec{r}$$

or, if $\vec{F} = \langle P, Q \rangle$,

$$\int_a^b P dx + Q dy$$

with a similar formula in 3 space.