Goals for today

Line integrals

Evaluate $\int_C 4x^3 ds$ where

(1) C is the curve starting at $(-2,-1)$ and going straight to $(0,-1)$, then going up to $(1,0)$ along the graph of $y = x^3 - 1$ and then going straight up to $(1,2)$.

(2) C is the straight line from $(-2,-1)$ to $(1,2)$.

$$\int_C 4x^3 ds = \int_{C_1} 4x^3 ds + \int_{C_2} 4x^3 ds + \int_{C_3} 4x^3 ds$$

- C_1 is the line from $(-2,-1)$ to $(0,-1)$ or $\vec{r}(t) = \langle t, -1 \rangle$, $-2 \leqslant t \leqslant 0$.
- C_2 is the graph of $y = x^3 - 1$ from $(0,-1)$ to $(1,0)$ or $\vec{r}(t) = \langle t, t^3 - 1 \rangle$, $0 \leqslant t \leqslant 1$.
- C_3 is the line from $(1,0)$ to $(1,2)$ or $\vec{r}(t) = \langle 1, t \rangle$, $0 \leqslant t \leqslant 2$.

$$\int_C 4x^3 ds = \int_{C_1} 4x^3 ds + \int_{C_2} 4x^3 ds + \int_{C_3} 4x^3 ds$$

(1) C_1 is the line from $(-2,-1)$ to $(0,-1)$ or

$\vec{r}(t) = \langle t, -1 \rangle$, $-2 \leqslant t \leqslant 0$.

(2) C_2 is the graph of $y = x^3 - 1$ from $(0,-1)$ to $(1,0)$ or

$\vec{r}(t) = \langle t, t^3 - 1 \rangle$, $0 \leqslant t \leqslant 1$.

(3) C_3 is the line from $(1,0)$ to $(1,2)$ or

$\vec{r}(t) = \langle 1, t \rangle$, $0 \leqslant t \leqslant 2$.

(1) $\vec{r}'(t) = \langle 1, 0 \rangle$; $|\vec{r}'(t)| = 1$.

(2) $\vec{r}'(t) = \langle 1, 3t^2 \rangle$; $|\vec{r}'(t)| = \sqrt{1 + 9t^4}$.

(3) $\vec{r}'(t) = \langle 0, 1 \rangle$; $|\vec{r}'(t)| = 1$.

(1) $4x^3 = 4t^3$.

(2) $4x^3 = 4t^3$.

(3) $4x^3 = 4$.

$$\int_C 4x^3 ds = \int_{C_1} 4x^3 ds + \int_{C_2} 4x^3 ds + \int_{C_3} 4x^3 ds$$

(1) C_1 is the line from $(-2,-1)$ to $(0,-1)$ or

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(2) C_2 is the graph of $y = x^3 - 1$ from $(0,-1)$ to $(1,0)$ or

$\vec{r}(t) = \langle t, t^3 - 1 \rangle$, $0 \leqslant t \leqslant 1$.

(3) C_3 is the line from $(1,0)$ to $(1,2)$ or

$\vec{r}(t) = \langle 1, t \rangle$, $0 \leqslant t \leqslant 2$.

(1) $\displaystyle\int_{C_1} 4x^3 ds = \int_{-2}^{0} 4t^3 \cdot 1\, dt.$

(2) $\displaystyle\int_{C_2} 4x^3 ds = \int_{0}^{1} 4t^3 \cdot \sqrt{1 + 9t^2}\, dt.$

(3) $\displaystyle\int_{C_3} 4x^3 ds = \int_{0}^{2} 4 \cdot 1\, dt.$

Evaluate $\int_C 4x^3 ds$ where C is the straight line from $(-2,-1)$ to $(1,2)$.

$$\vec{r}(t) = (1-t)\langle -2,-1\rangle + t\langle 1,2\rangle = \langle -2+3t, -1+3t\rangle$$
$$0 \leqslant t \leqslant 1$$

Evaluate $\int_C 4x^3 ds$ where C is the straight line from $(-2,-1)$ to $(1,2)$.

$$\vec{r}(t) = (1-t)\langle -2,-1\rangle + t\langle 1,2\rangle = \langle -2+3t, -1+3t\rangle$$

$$0 \leqslant t \leqslant 1$$

$$\vec{r}'(t) = \langle 3,3\rangle \qquad |\vec{r}'(t)| = \sqrt{9+9} = 3\sqrt{2}$$

$$4x^3 = 4(-2+3t)^3$$

$$\int_C 4x^3 ds = \int_0^1 4(-2+3t)^3\, 3\sqrt{2} dt$$

Evaluate

$$\int_C \left\langle \sin(\pi y), yx^2 \right\rangle d\vec{r}$$

along the line segment from $(0,2)$ to $(1,4)$.

Equivalently evaluate

$$\int_C \sin(\pi y)dx + yx^2dy$$

along the same line segment.

Evaluate

$$\int_C \left\langle \sin(\pi y), yx^2 \right\rangle d\vec{r}$$

along the line segment from $(1,1)$ to $(0,2)$.

Equivalently evaluate

$$\int_C \sin(\pi y)dx + yx^2dy$$

along the same line segment.

Evaluate

$$\int_C \left\langle \sin(\pi y), yx^2 \right\rangle d\vec{r}$$

along the line segment from $(0, 2)$ to $(1, 4)$.

Equivalently evaluate

$$\int_C \sin(\pi y)dx + yx^2dy$$

along the same line segment.

C is $\vec{r}_1(t) = (1-t)\langle 0, 1\rangle + t\langle 1, 4\rangle = \langle t, 1+3t\rangle$, $0 \leqslant t \leqslant 1$.

Evaluate

$$\int_C \left\langle \sin(\pi y), yx^2 \right\rangle d\vec{r}$$

along the line segment from $(0, 2)$ to $(1, 4)$.

C is $\vec{r}_1(t) = (1-t)\langle 0, 1\rangle + t\langle 1, 4\rangle = \langle t, 1+3t\rangle$, $0 \leqslant t \leqslant 1$.

$$d\vec{r} = \langle 1, 3\rangle\, dt$$

$$\vec{F}(t) = \left\langle \sin(\pi(1+3t)), (1+3t)x^2 \right\rangle$$

$$\int_C \left\langle \sin(\pi y), yx^2 \right\rangle d\vec{r} = \int_0^1 \left\langle \sin(\pi(1+3t)), (1+3t)x^2 \right\rangle \bullet \langle 1, 3\rangle\, dt$$

Evaluate

$$\int_C \left\langle \sin(\pi y), yx^2 \right\rangle d\vec{r}$$

along the line segment from $(1, 1)$ to $(0, 2)$.

Equivalently evaluate

$$\int_C \sin(\pi y)dx + yx^2 dy$$

along the same line segment.

C is $\vec{r}_2(t) = (1 - t)\langle 1, 4\rangle \langle 0, 1\rangle + t\langle 0, 1\rangle = \langle 1 - t, 4 - 3t\rangle$.

Note that the interval is $[a, b] = [0, 1]$ and

$$\vec{r}_2(t) = \vec{r}_1(a + b - t)$$

Evaluate

$$\int_C \langle y, x \rangle \bullet d\vec{r}$$

along the line segment from $(1,0)$ to $(0,1)$ to $(-1,0)$.

Evaluate

$$\int_C \langle y, x \rangle \bullet d\vec{r}$$

along the line segment from $(1,0)$ to $(-1,0)$.

Evaluate

$$\int_C \langle y, x\rangle \cdot d\vec{r}$$

along the line segment from $(1,0)$ to $(0,1)$ to $(-1,0)$.

$$\int_C \langle y, x\rangle\, d\vec{r} = \int_{C_1} \langle y, x\rangle\, d\vec{r} + \int_{C_2} \langle y, x\rangle\, d\vec{r}$$

- C_1 is $\vec{r}(t) = \langle 1-t, t\rangle$; $d\vec{r}(t) = \langle -1, 1\rangle\, dt$, $0 \leqslant t \leqslant 1$.
- C_2 is $\vec{r}(t) = \langle -t, 1-t\rangle$; $d\vec{r}(t) = \langle -1, -1\rangle\, dt$, $0 \leqslant t \leqslant 1$.

- $\displaystyle\int_0^1 \langle t, 1-t\rangle \cdot \langle -1, 1\rangle\, dt.$
- $\displaystyle\int_0^1 \langle 1-t, -t\rangle \cdot \langle -1, -1\rangle\, dt.$

Evaluate

$$\int_C \langle y, x\rangle \cdot d\vec{r}$$

along the line segment from $(1,0)$ to $(-1,0)$.

C is $\vec{r}(t) = \langle 1-2t, 0\rangle$; $d\vec{r}(t) = \langle -2, 0\rangle\, dt$, $0 \leqslant t \leqslant 1$.

$$\int_0^1 \langle 1-2t, 0\rangle \cdot \langle -2, 0\rangle\, dt$$

Evaluate

$$\int_C xdx + ydy + udu$$

along the curve $x = t^2$, $y = t^3$, $u = t$ for $-1 \leqslant t \leqslant 2$.

Evaluate

$$\int_C xdx + ydy + udu$$

along the curve $x = t^2$, $y = t^3$, $u = t$ for $-1 \leqslant t \leqslant 2$.

$$\int_C \langle x, y, z\rangle \bullet d\vec{r}$$

$$d\vec{r} = \left\langle 2t, 3t^2, 1\right\rangle dt$$

$$\int_C \langle x, y, z\rangle \bullet d\vec{r} = \int_{-1}^{2} \left\langle t^2, t^3, t\right\rangle \bullet \left\langle 2t, 3t^2, 1\right\rangle dt$$

Evaluate

$$\int_C xdx + ydy + udu$$

along the curve $x = t^2$, $y = t^3$, $u = t$ for $-1 \leqslant t \leqslant 2$.

- $dx = 2tdt$
- $dy = 3t^2dt$
- $du = dt$

$$\int_C xdx + ydy + udu = \int_{-1}^{2} t^2 \cdot 2tdt + \int_{-1}^{2} t^3 \cdot 3t^2dt + \int_{-1}^{2} t \cdot 1dt$$