Summery for curves:

$$s(t) = \int_a^t |\vec{r}'(t)|\, dt$$

$$\vec{\mathbf{T}} = \frac{\vec{r}'(t)}{|\vec{r}'(t)|} \qquad \frac{d\vec{\mathbf{T}}}{ds} = \kappa(s)\vec{\mathbf{N}} \qquad \vec{\mathbf{B}} = \vec{\mathbf{T}} \times \vec{\mathbf{N}}$$

$\vec{r}'(t) = \vec{\mathbf{v}}$ and $\vec{\mathbf{T}}$ point in the same direction.

$\vec{r}''(t) = \vec{\mathbf{a}}$ and $\vec{r}'(t) \times \vec{r}''(t) = \vec{\mathbf{v}} \times \vec{\mathbf{a}}$ point in same direction as $\vec{\mathbf{B}}$.

$\big(\vec{r}'(t) \times \vec{r}''(t)\big) \times \vec{r}'(t) = (\vec{\mathbf{v}} \times \vec{\mathbf{a}}) \times \vec{\mathbf{v}}$ point in the same direction as $\vec{\mathbf{N}}$.

The *normal plane* at a point on the curve is the plane perpendicular to $\vec{\mathbf{T}}$ at the point.

The *osculating plane* at a point on the curve is the plane perpendicular to $\vec{\mathbf{B}}$ at the point.

We never discussed the plane perpendicular to $\vec{\mathbf{N}}$.

$$\vec{\mathbf{a}} = a_T\vec{\mathbf{T}} + a_N\vec{\mathbf{N}}$$

$$a_T = \vec{\mathbf{a}} \bullet \vec{\mathbf{T}} = \frac{\vec{\mathbf{a}} \bullet \vec{\mathbf{v}}}{|\vec{\mathbf{v}}|} = \frac{\vec{r}'' \bullet \vec{r}'}{|\vec{r}'\,|}$$

$$a_N = |\vec{\mathbf{T}} \times \vec{\mathbf{a}}| = \frac{|\vec{\mathbf{v}} \times \vec{\mathbf{a}}|}{|\vec{\mathbf{v}}|} = \frac{|\vec{r}' \times \vec{r}''|}{|\vec{r}'|}$$