Name: Date: 11/03/2016

M20550 Calculus III Tutorial
Worksheet 8

1. Evaluate the given integral.
$$\iint_R \arctan\left(\frac{y}{x}\right)\, dA$$
where $R = \{(x, y) : 1 \leq x^2 + y^2 \leq 4, 0 \leq y \leq x\}$.

2. (a) Let E_1 be the solid lies under the plane $z = 1$ and above the region in the xy-plane bounded by $x = 0$, $y = 0$, and $2x + y = 2$. Write the triple integral $\iiint_{E_1} xz\, dV$ but do not evaluate it.

   (b) Let E_2 be the solid region in the first octant that lies under the paraboloid $z = 2 - x^2 - y^2$. Write the triple integral $\iiint_{E_2} xz\, dV$ in cylindrical coordinates (you don't need to evaluate it).

   (c) Let E_3 be the solid region that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the plane $z = 2$. Write the triple integral $\iiint_{E_3} xz\, dV$ in spherical coordinates (you don't need to evaluate it).

3. Write the integral that computes the volume of the part of the solid cylinder $x^2 + y^2 \leq 1$ that lies between the planes $z = 0$ and $z = 2 - y$.

4. Set up, but do not solve, the integral that gives the volume of the solid region bounded by the paraboloid $z = 3y^2 + 3x^2$ and the cone $z = 4 - \sqrt{x^2 + y^2}$.

5. Find the mass of the solid between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$ whose density is $\rho(x, y, z) = x^2 + y^2 + z^2$.

6. Find the center of mass of the solid S bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 1$ if S has constant density 1 and total mass $\frac{\pi}{2}$. (Hint: $\bar{x}$ and $\bar{y}$ can be found by symmetry).