Name: Date: 12/01/2016

**M20550 Calculus III Tutorial**
**Worksheet 10**

1. Determine whether or not the following vector fields are conservative:
   (a) $\mathbf{F} = (3 + 2xy)\,\mathbf{i} + (x^2 - 3y^2)\,\mathbf{j}$
   (b) $\mathbf{F} = \mathbf{i} + \sin z\,\mathbf{j} + y\cos z\,\mathbf{k}$

2. Evaluate $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F} = (y^2\cos(xy^2) + 3x^2)\,\mathbf{i} + (2xy\cos(xy^2) + 2y)\,\mathbf{j}$ is a conservative vector field and $C$ is any curve from the point $(-1, 0)$ to $(1, 0)$.

3. Use Green's Theorem to evaluate
$$\int_C \left(-\frac{y^3}{3} + \sin x\right)\,dx + \left(\frac{x^3}{3} + y\right)\,dy,$$
where $C$ is the circle of radius 1 centered at $(0, 0)$ oriented counterclockwise when viewed from above.

4. Write an equation of the tangent plane to the parametric surface
$$x = u^2 + 1, \quad y = v^3 + 1, \quad z = u + v,$$
at the point $(5, 2, 3)$.

5. Write the integral that computes the surface area of the surface $S$ parametrized by $\mathbf{r}(u, v) = \langle u^2\cos v, u^2\sin v, v\rangle$, where $0 \le u \le 1$ and $0 \le v \le \pi$.

6. Find the surface area of the part of the cylinder $x^2 + y^2 = 4$ that lies between the planes $z = 0$ and $z = 2$.

7. Find the area of the part of the paraboloid $z = x^2 + y^2$ which lies inside the cylinder $x^2 + y^2 = 1$.

8. (a) Compute div $\mathbf{F}$, where $\mathbf{F} = \langle e^y, zy, xy^2\rangle$.
   (b) Is there a vector field $\mathbf{G}$ on $\mathbb{R}^3$ such that curl $\mathbf{G} = \langle xyz, -y^2z, yz^2\rangle$? Why?