Basic Algebra Rules

Exponential Rules:

$$a^m \cdot a^n = a^{m+n} \qquad (ab)^m = a^m b^m \qquad \frac{a^m}{a^n} = a^{m-n}; \quad a \neq 0$$

$$a^0 = 1 \quad a \neq 0 \qquad a^{1/m} = \sqrt[m]{a} \qquad \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}; \quad b \neq 0$$

$$(a^m)^n = a^{mn}$$

Distribution Law:

$$a(b + c) = ab + ac \qquad \frac{a + b}{c} = \frac{a}{c} + \frac{b}{c} \qquad \frac{a - b}{c} = \frac{a}{c} - \frac{b}{c}$$

Quadratic Factoring:

$$(a + b)^2 = a^2 + 2ab + b^2 \qquad (a - b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a - b)(a + b)$$

Properties of Logarithm:

$$\log_a(MN) = \log_a M + \log_a N \qquad \log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N \qquad \log_a(M)^t = t\log_a M$$

$$\log_a a = 1 \qquad \log_a 1 = 0$$

$$\log_a a^x = x \qquad a^{\log_a x} = x$$

Change of Base: $\log_a M = \dfrac{\log_b M}{\log_b a}$

$$\ln(MN) = \ln M + \ln N \qquad \ln\left(\frac{M}{N}\right) = \ln M - \ln N \qquad \ln(M)^t = t\ln M$$

$$\ln e = 1 \qquad \ln 1 = 0$$

$$\ln e^x = x \qquad e^{\ln x} = x$$

Math 10360 – Example Set 01A
Derivative and Integration Review

Basic Properties of Derivatives:

$[f(x) + g(x)]' \stackrel{?}{=}$

$[f(x) - g(x)]' \stackrel{?}{=}$

$[c \cdot f(x)]' \stackrel{?}{=}$

Product/Quotient/Chain Rule. Let $f(x)$ and $g(x)$ be differentiable functions. Derive formulas for the derivatives of $p(x) = f(x) \cdot g(x)$ and $q(x) = \dfrac{f(x)}{g(x)}$.

Product Rule:

$$\frac{d}{dx}(f(x)g(x)) = (f(x)g(x))' =$$

Chain Rule:

$$\frac{d}{dx}(f(g(x))) = [f(g(x))]' =$$

Quotient Rule: $\dfrac{d}{dx}\left(\dfrac{f(x)}{g(x)}\right) = \left(\dfrac{f(x)}{g(x)}\right)' =$

Some Common Derivatives. For any numbers k and n:

$\dfrac{d}{dx}(k) \stackrel{?}{=}$

$\dfrac{d}{dx}(x^n) \stackrel{?}{=}$ (Power Rule)

$\dfrac{d}{dx}(\sin(x)) \stackrel{?}{=}$

$\dfrac{d}{dx}(\cos(x)) \stackrel{?}{=}$

$\dfrac{d}{dx}(\tan(x)) \stackrel{?}{=}$

$\dfrac{d}{dx}(\csc(x)) \stackrel{?}{=}$

$\dfrac{d}{dx}(\sec(x)) \stackrel{?}{=}$

$\dfrac{d}{dx}(\cot(x)) \stackrel{?}{=}$

1. Find the following derivatives

a. $\dfrac{d}{dx}(x^3 \tan(x)) \stackrel{?}{=}$

b. $\dfrac{d}{dx}\left(\sqrt[3]{2x^2 - 5x + 3}\right) \stackrel{?}{=}$

2. Find the equation of the tangent line to the curve $x\cos(1 + 2y) = 2y^2 - 8$ at the point $(0, 2)$.

(Check $\frac{dy}{dx} = \frac{\cos(1+2y)}{4y+2x\sin(1+2y)}$)

Basic Integrals. For any numbers k and n:

$\int x^n \, dx \stackrel{?}{=}$ (Power Rule)

$\int \sin(x) \, dx \stackrel{?}{=}$

$\int \cos(x) \, dx \stackrel{?}{=}$

$\int \sec^2(x) \, dx \stackrel{?}{=}$

$\int \csc^2(x) \, dx \stackrel{?}{=}$

$\int \csc(x)\cot(x) \, dx \stackrel{?}{=}$

$\int \sec(x)\tan(x) \, dx \stackrel{?}{=}$

Method of Substitution

3. Find a formula for the function $f(x)$ if its slope is given by the $x\sin(x^2+1)$ and the graph of $f(x)$ passes through the point $(1, 2)$.

4. Evaluate $\displaystyle\int_0^1 \frac{x^2+2}{\sqrt{x^3+6x+5}}\,dx$.

Review of the Fundamental Theorem of Calculus

Given two numbers a and b, the phrase the *closed interval between a and b* means $[a, b]$ if $a \leqslant b$ or $[b, a]$ is $b \leqslant a$.

The Fundamental Theorem of Calculus states that if a function f is continuous on the closed interval between a and b then

- $\displaystyle\int_a^b f(x)dx$ and $\displaystyle\int_b^a f(x)dx$ are *numbers*.
- The two numbers are related by $\displaystyle\int_a^b f(x)dx = -\int_b^a f(x)dx$.
- Define a function $F(x) = \displaystyle\int_a^x f(t)dt$.
- The *domain* of $F(x)$ consists of all numbers x such that the function f is continuous on the closed interval between a and x.
- $F(a) = 0$.
- $F'(x) = f(x)$.
- $\displaystyle\int f(x)dx = F(x) + C$

Math 10360 – Example Set 01B
Derivative of Exponential & Logarithmic Functions: Section 3.9

1. Consider the area function $f(x) = \displaystyle\int_1^x \frac{1}{t}\,dt$ for $x > 0$. We call $f(x)$ the logarithm function and denote it by $f(x) = \ln x$.

a. $f'(x) = \dfrac{d}{dx}[\ln x] = \dfrac{d}{dx}\left[\displaystyle\int_1^x \frac{1}{t}\,dt\right] \stackrel{?}{=}$ ____________ $(x > 0)$

b. $\dfrac{d}{dx}[\ln|x|] \stackrel{?}{=}$ ____________ $(x \neq 0)$

c. What can you say about $\ln(1)$? Define the value of e using the definition of the natural logarithm.

d. Using the Fundamental Theorem of Calculus, show that $\ln(ax) = \ln(a) + \ln(x)$. Prove further that (i) $\ln(e^n) = n$ where n is an integer and (ii) $\ln(e^r) = r$ where r us any rational number.

Example A. Find the area under the graph of $y = \dfrac{-2}{4x-3}$ for $0 \le x \le 1/2$.

e. Give a sketch of the graph of $y = \ln x$. State clearly the domain and range of $\ln x$. What are the values of $\lim\limits_{x\to 0^+} \ln x$ and $\lim\limits_{x\to\infty} \ln x$?

f. The inverse $g(x)$ of $f(x) = \ln x$ exists. Why? Sketch the graph of $g(x) = \exp(x)$. Infer from (d) that we may write $\exp(x) = e^x$ for all real value x.

g. Explain why we may write: (i) $\ln(e^x) = x$ for all x, and $e^{\ln y} = y$ for $y > 0$.

h. Using the fact that $\dfrac{d}{dx}(e^x) = e^x$, the chain rule and the fact that $e^{\ln b} = b$ $(b > 0)$, show that $\dfrac{d}{dx}(b^x) = b^x \ln b$.

i. Using the change of base formula $\log_b x = \dfrac{\ln x}{\ln b}$, show that $\dfrac{d}{dx}(\log_b x) = \dfrac{1}{x \ln b}$.

Example B. Find the equation of the tangent line to the curve $y = 4 - 2e^x + \ln\left(\dfrac{1-x^2}{1+x^2}\right)$ at $x = 0$.

Review Exercise. Complete the following formulas:

Logarithmic Properties

$\ln(ab) \stackrel{?}{=}$

$\ln(a^n) \stackrel{?}{=}$

$\ln\left(\frac{a}{b}\right) \stackrel{?}{=}$

$\ln(e) \stackrel{?}{=}$

$\ln 1 \stackrel{?}{=}$

$\ln(e^x) \stackrel{?}{=}$

$e^{\ln x} \stackrel{?}{=}$

Exponential Rules

$a^n \cdot a^m \stackrel{?}{=}$

$\frac{a^n}{a^m} \stackrel{?}{=}$

$a^n \cdot b^n \stackrel{?}{=}$

$\frac{a^n}{b^n} \stackrel{?}{=}$

Derivative and Anti-derivative Rules

$\frac{d}{dx}(\ln x) \stackrel{?}{=}$

$\frac{d}{dx}(e^x) \stackrel{?}{=}$

$\frac{d}{dx}(\log_b x) \stackrel{?}{=}$

$\frac{d}{dx}(b^x) \stackrel{?}{=}$

$\int \frac{1}{x}\, dx \stackrel{?}{=}$

$\int e^x\, dx \stackrel{?}{=}$

$\int b^x\, dx \stackrel{?}{=}$