Synthesis and Control of Coherent Structures in Low Temperature Plasmas for Reconfigurable Electromagnetic Devices: Self-organized Plasma Lattice Metamaterial

Thomas Corke and Eric Matlis

Institute for Flow Physics and Control
Aerospace and Mechanical Engineering
University of Notre Dame
College of Engineering
Notre Dame, IN
tcorke@nd.edu
Research Objectives

- Experimentally generate spatially periodic plasma lattice structures in air
  - Exploit charge instability between electrodes separated by a double dielectric layer
- Demonstrate dynamic control of plasma lattice spacing, $a$
  - Document dependence on gas pressure ($P_s$), gap distance ($d$), and input power
- Experimentally determine EM wave transmittance under different plasma lattice conditions
Research Objectives

- Incorporate experimentally derived plasma lattice into EM wave simulation to determine predicted transmittance characteristics
- Compare EM transmittance obtained from experiments to EM wave simulation predictions
  - Independent variables: $P_s$ and $n_e$
- Use validated simulation to project EM transmittance to target 70Ghz probing frequency
Plasma frequency, $\omega_p$, is a function of electron density, which is controllable by the applied voltage.

Electron-neutrals collision frequency, $\nu$, is a function of the gas pressure.

The combination control the plasma permittivity.
Approach: Plasma Permittivity Regimes

\[
\varepsilon_p(\omega) = 1 - \left(\frac{\omega_p}{\omega}\right)^2 \frac{1}{1 - i\frac{\nu}{\omega}}
\]

\[\nu = (3.9 \times 10^9 \text{ s}^{-1} \text{Torr}^{-1}) P_s\]

Razier (1991)

Plasma Photonic Crystals
\[\omega \sim 10^{10}\]
\[\omega_p \sim 10^{11}\]
\[\nu \sim 10^{11}\]
Approach: Plasma Permittivity Control

\[ \epsilon_p(\omega) = 1 - \left( \frac{\omega_p}{\omega} \right)^2 \frac{1}{1 - i\nu/\omega} \]

\[ \omega/2\pi = 20 \text{GHz} \]

Air

Transmission

Absorption

Re\{\epsilon_p\} < 0

\[125 - 400 \text{ Torr}\]

\[10^{19} \text{ [cm}^{-3}\text{]}\]

\[10^{21} \text{ [cm}^{-3}\text{]}\]
Plasma charge instability* produces stationary, naturally spaced plasma lattice structure. (*Callegari et al., 2014)

Forms a periodic dielectric in 2-D.

Lattice spacing determines propagation characteristics: frequency cut-off, photonic band gap.

Sakai & Tachibana. 2012
Plasma Lattice Device

1. Circular Lower Electrode

2. Glass with Indium-Tin-Oxide layer on one surface

3. Complete Assembly

Assembly Schematic:
- Glass Pressure Plate
- ITO Electrode
- Dielectric (glass)
- Spacer Ring
- Dielectric Glass
- Circular Electrode
Plasma Lattice Structure

Effect of Voltage
Lattice Analysis: Node Locations, Radii and Spacing

Lattice Spacing

Lattice Interior Angles

150623_081316_rep01_acq02, 125.4 Torr, 6.48 kV, 4.48 W

150623_081316_rep01_acq02, 125.4 Torr, 6.48 kV, 4.48 W
Predictive Plasma Lattice Control

Follows Circle Packing Theory

Transmittance Experiment Conditions

- d=1.10mm
- d=0.63mm
- d=2.50mm

Number of nodes

Lattice spacing / Electrode Radius
EM Wave Experimental Setup

- **Viewing Mirror**
- **Waveguide Horn**
- **Plasma Lattice**
- **Aperture Plate**
- **S1**
- **S2**
- **B/W Camera**
- **Mirror View**
- **Aperture View**

Properties:
- **D=7cm (4.7\lambda_{20GHz})**
- **4.4 cm**
EM Wave Experimental Setup

- Experimental setup placed in cylindrical pressure vessel
- Vacuum-rated electrical pass-through connectors for plasma power, and S1 and S2 analyzer signals.

Rohde & Schwarz ZVN20 VNA
10MHz-20GHz
## Experimental Conditions

<table>
<thead>
<tr>
<th>d (mm)</th>
<th>Static Pressure, $P_s$ (Torr)</th>
<th>Power Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.50</td>
<td>125 200 250 300 400</td>
<td>22-56</td>
</tr>
<tr>
<td>1.10</td>
<td>- 200 - 300 400</td>
<td>35-80</td>
</tr>
<tr>
<td>0.62</td>
<td>- - - - 400</td>
<td>46-72</td>
</tr>
</tbody>
</table>

- Plasma AC frequency = 54kHz
- **Data Acquisition:**
  - Sequence of Plasma Lattice Off, On, Off
  - 1001pts. $S_{21}$: 17.5 - 20GHz
  - Plasma Lattice Image
Plasma Lattice Images: $P_s = 400\text{Torr}$
Image Analysis

<table>
<thead>
<tr>
<th>Power Gain</th>
<th>d=2.50(mm)</th>
<th>d=1.11(mm)</th>
<th>d=0.63(mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a (mm)</td>
<td>λ/a</td>
<td>ωa/2πc</td>
<td>a (mm)</td>
</tr>
<tr>
<td>Min.</td>
<td>4.7</td>
<td>3.2</td>
<td>0.31</td>
</tr>
<tr>
<td>Mid.</td>
<td>4.2</td>
<td>3.6</td>
<td>0.28</td>
</tr>
<tr>
<td>Max.</td>
<td>4.6</td>
<td>3.3</td>
<td>0.31</td>
</tr>
</tbody>
</table>

- Metamaterial: \( \lambda/a \approx 10 \)
- Phototonic Crystal: \( \lambda/a \approx 1 \)
- Band Gap (Sakai et al., 2005): \( \omega a/2\pi c \approx 0.5 \)
Sample $S_{21}$ Transmittance Spectra

$S_{21}/S_{21}$ (Lattice off)

$d=2.5\text{mm}, P_s = 250\text{Torr}$

Average Baseline
**S\textsubscript{21} Transmittance Spectra**

\[ P_s = 400 \text{Torr}, \ d = 0.63 \text{mm}. \]

- **Area-averaged S\textsubscript{21} Ratio**
- **Freq (GHz)**

- Pressure = 400 Torr, Gap = 0.63 mm.
**S_{21} Transmittance Spectra**

\( P_s = 400 \text{Torr}, \ d = 0.63 \text{mm}. \)

<table>
<thead>
<tr>
<th>( a ) (mm)</th>
<th>( \lambda/a )</th>
<th>( \omega a/2\pi c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0</td>
<td>5.0</td>
<td>0.20</td>
</tr>
<tr>
<td>2.1</td>
<td>7.1</td>
<td>0.14</td>
</tr>
<tr>
<td>1.6</td>
<td>9.4</td>
<td>0.11</td>
</tr>
</tbody>
</table>
S\textsubscript{21} Transmittance Spectra

\begin{tabular}{|c|c|c|}
\hline
a (mm) & \(\lambda/a\) & \(\omega a/2\pi c\) \\
\hline
3.2 & 4.7 & 0.21 \\
2.4 & 6.2 & 0.16 \\
2.0 & 7.5 & 0.13 \\
\hline
\end{tabular}

\(P_s=400\text{Torr}, d=1.1\text{mm}.\)

\begin{tabular}{|c|c|c|}
\hline
a (mm) & \(\lambda/a\) & \(\omega a/2\pi c\) \\
\hline
4.7 & 3.2 & 0.31 \\
4.2 & 3.6 & 0.28 \\
4.6 & 3.3 & 0.31 \\
\hline
\end{tabular}

\(P_s=400\text{Torr}, d=2.50\text{mm}.\)
Effect of Power on Minimum $S_{21}$ Transmittance

$P_s = 300 \text{Torr}$, $d = 2.50 \text{mm.}$

$d = 2.50 \text{mm.}$, $125 \leq P_s \leq 400 \text{Torr}$

Normalized Power
Effect of Gap on $S_{21}$ Minimum Transmittance

$P_s = 400$ Torr, $0.63 \leq d \leq 2.50$ mm.
EM Wave Simulation

• Simulation of a 2-D plasma photonic crystal subject to planar electromagnetic wave fronts of a specific probing frequency.

• Utilized MIT open-source (Meep) software that solves Maxwell’s equations at each time step to realize the electromagnetic field at discrete spatial locations through an implementation of a finite-difference time-domain (FDTD) method.

• Dispersive materials are defined in Meep using a Lorentz-Drude model:

\[
\varepsilon(\omega) = \varepsilon_\infty + \sum_{m=1}^{N} \frac{\sigma_m \Omega_m^2}{\Omega_m^2 - \omega^2 + i\nu \omega}
\]

Where \( \varepsilon_\infty \) is the frequency-independent permittivity, \( N \) is the number of resonance frequencies, \( \Omega_m \) is a resonance frequency, \( \sigma_m \) is the strength associated with that frequency, and \( \nu \) is the electron elastic collision frequency.
Meep: Plasma Column Permittivity

• Permittivity, \( \varepsilon_p \), of each plasma column given by the dispersive relation

\[
\varepsilon_p(\omega) = 1 - \left( \frac{\omega_p}{\omega} \right)^2 \frac{1}{1 - i\frac{\nu}{\omega}}
\]

where \( \omega \) is the probing frequency, and \( \omega_p \) is the plasma frequency and \( \nu \) is the electron elastic collision frequency.

• The plasma frequency is defined as

\[
\omega_p = \sqrt{\frac{n_e q^2}{m \varepsilon_0}}
\]

where \( n_e \) is the electron density, \( q \) is the electron charge, \( m \) is electron mass, and \( \varepsilon_0 \) is the free-space permittivity. Based on Razier (1991), \( \nu = (3.9 \times 10^9 \text{ s}^{-1} \text{Torr}^{-1})P_s \)

• For the Meep parameters, we equate two forms of the permittivity, namely

\[
\varepsilon(\omega) = \varepsilon_\infty + \sum_{m=1}^{N} \frac{\sigma_m \Omega_m^2}{\Omega_m^2 - \omega^2 + i\omega \nu} = 1 + \frac{\omega_p^2}{-\omega^2 + i\omega \nu}
\]

then \( \varepsilon_\infty = 1 \) and \( N = 1 \).

• To achieve the best representation of the plasma material, \( \Omega_m \ll 1 \) which requires that \( \sigma_m = (\omega_p)^2/(\Omega_m)^2 \). Used \( \Omega_m = 0.0001 \).
EM Wave Simulation

- Computational domain consists of rectangular region \(12r_{\text{lattice}}\) by \(6r_{\text{lattice}}\)
- A non-reflecting PML perimeter of width \(4\lambda_{\text{Probe}}\) surrounds the domain
- Plasma grating patterns from the experiments are inset in computational domain

![Diagram showing computational domain with PML (Perfectly Matched Layer) and plasma grating patterns.](diagram)
Simulation: 400 Torr, $a=4\text{mm}$, $n_e=1\times10^{21}\text{m}^{-3}$
Simulation: 400 Torr, $a=4\text{mm}$, $n_e=1\times10^{21}\text{m}^{-3}$

$100\times(\text{Plasma} - \text{No Crystal})/\text{No Crystal}$ at $\eta_e=1\times10^{21}$, 4MM, 20GHz
Simulation Results

\[ P_s = 400 \text{Torr}, \ 0.63 \leq d \leq 2.50 \text{mm}. \]

Experiment:
- 17.75–20.00GHz
- \( d = 2.50 \text{mm} \)
- \( d = 1.10 \text{mm} \)
- \( d = 0.63 \text{mm} \)

Simulation: 400 Torr
- 20GHz, \( a = 4 \text{mm}, 1 \times 10^{-20} \text{m}^{-3} \)
- 19GHz, \( a = 2 \text{mm}, 1 \times 10^{-20} \text{m}^{-3} \)
- 20GHz, \( a = 2 \text{mm}, 1 \times 10^{-21} \text{m}^{-3} \)
- 20GHz, \( a = 2 \text{mm}, 1 \times 10^{-20} \text{m}^{-3} \)
Simulation Projection

<table>
<thead>
<tr>
<th>$a$ (mm)</th>
<th>$\lambda/a$</th>
<th>$\omega a/2\pi c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2.15</td>
<td>0.47</td>
</tr>
<tr>
<td>8</td>
<td>0.54</td>
<td>1.87</td>
</tr>
</tbody>
</table>

Sakaguchi et al., 2007
17 and 30 discharge rows

![Graph showing transmittance vs electron density](graph.png)
Summary

- Demonstrated dynamic control of plasma lattice structures with different lattice spacings, \( a \).
- Performed EM wave transmittance experiments for probing frequencies up to 20GHz.
- Experiments demonstrated effects of \( P_s \), \( d \), and power on S21 transmittance.
- Results indicated a narrow frequency band S21 attenuation.
- Largest S21 attenuation occurred with highest \( P_s \) were \( \lambda/a=10 \).
- The largest attenuation was comparable to other non-configurable plasma lattices in the literature, e.g. Sakaguchi et al., 2007.
- EM wave simulations performed on experiment-based plasma lattice configurations produced transmittance values that were in good agreement with the experiments.
  - Indicates validity of the simulation to predict effect at higher probing frequencies.
Way Forward

- Perform EM wave transmittance experiments at higher probing frequencies (~70GHz).
  - Acquire a 70GHz Vector Network Analyzer and matched components.
- Repeat transmittance measurements at the higher probing frequencies that include off-axis locations.
  - Seek wave energy changes in the $\Gamma - M$ direction that is a characteristic of a photonic crystal.
- Perform further comparisons to the EM wave simulations.
- Generate band diagrams for a range of lattice spacing and electron densities derived from the experiments. Correlate these to the experimental observations.
- Investigate charge instability lattice control approaches that include:
  - Silicon dielectric with surface layer doping patterns
  - Real-time UV patterns projected on the glass dielectric

Wave Guides and Conduits
Way Forward

- Even at the current probing frequencies, with our control over the plasma, we are in a position to look at photonic crystals where the columns can have positive, negative or 0 permittivity.

- This is quite unique and we think, not realizable with non-configurable plasma lattices in the literature such as that of Sakaguchi et al.