1 Structural Design

• The structural design of the rotor and tower naturally follows from the aerodynamic design from which the aerodynamic loads are derived.

• As it often happens in the design of aerodynamic systems, their needs to be a compromise between the aerodynamic optimum and the structural optimum.

  – The **structural design** seeks to optimize strength, weight and cost.
  
  – Catastrophic failures of wind turbine structures are rare, but not impossible.

![Figure 1: Examples of rare structural failures of horizontal axis wind turbines.](image-url)
• Conditions leading to structural failures include
  1. extreme winds,
  2. an inadequate control system,
  3. cyclic-load fatigue that leads to cracks in the structure.

• Fatigue is a very important issue since wind turbines are designed
to operate for a minimum of 20 year over which the rotor
will rotate on the order of $10^9$ revolutions!

• Some of the loads repeat with every revolution of the rotor which
results in a cyclic straining of the structure that could lead to
strain hardening and brittle fracture.
• There are four primary sources of loads that are relevant to horizontal axis wind turbines. These are

1. aerodynamic loads,
2. gravitational loads,
3. dynamic loads, and
4. control loads.
• **Aerodynamic loads** include the lift, drag and pitch moment on the rotor such as can be determined by the BEM method.

![Figure 2: Force vectors based on BEM analysis (left) and illustration of 3-D lift and drag force distribution resulting in maximum shear forces and bending moments at the rotor root.](image_url)
• Structurally, the rotor is a cantilever beam with a fixed attachment at the rotor hub.

• The material stresses associated with these loads determines the structural design.

• The forces that act on the rotor can be transmitted through the rotor shaft to the gear box and tower.

  – Structural failure of the gear box continues to be an important issue.
• **Gravitational loads** are primarily associated with the weight of the rotor blades.

• This is a cyclic loading whose magnitude on a radial element is

\[ dF_g = \bar{g} dm \cos(\psi) \]  

(1)

• The cyclic gravitational loading on the rotor is converted into a **cyclic torque variation** on the rotor shaft that is then transmitted to the gear box.

Figure 3: Illustration of gravitational and centrifugal loads acting on a spinning wind turbine rotor.
• The gravitational loading generally acts through the rotor plane axis, except if the rotor bends out of plane, which is referred to as “flapping”.

![Diagram of rotor conditions]

Figure 4: Illustration of types of coned or “flapping” rotor conditions of the horizontal axis wind turbine.

• Out of plane or flapping angle is defined as $\beta$.
  
  – $\beta_0$ shows a rotor plane that is aligned with the wind direction. The loading on the blades is steady with respect to the rotor rotation angle, psi.

• $\beta_{1c}$, has the axis of the rotor aligned with the wind direction, but the coned rotor plane is canted upward
  
  – The rotor location that is tilted upwind (bottom portion) will have a larger effective angle of attack compared to the rotor that is tilted downwind.

  – This will produce a cyclic loading with a magnitude that varies as $\cos(\psi)$, where again $\psi = 0$ corresponds to the bottom of the rotation cycle.
• $\beta_{1s}$, has the axis of the coned rotor yawed with respect to the wind direction.
  
  – This produces a cyclic loading whereby the rotor that tilts upwind (right portion) will have an effectively larger angle of attack compared to the rotor that tilts downwind.
  
  – This will produce a cyclic loading with a magnitude that varies in this case, as $\sin(\psi)$.

• It is reasonable to sum the effects of the three coned rotor conditions to obtain an effective flapping angle, $\beta$ given as

$$\beta = \beta_0 + \beta_{1c} \cos(\psi) + \beta_{1s} \sin(\psi).$$

(2)

• In this case $\beta_0$ represents the collective or coned response, and $\beta_{1c}$ and $\beta_{1s}$ are the coefficients representing the respective cosine and sine cyclic responses.
• **Dynamic loading** is the result of changes in the motion of rotor.

• One example is the centrifugal force generated by the rotation of the rotor.

\[ dF_c = rdm\Omega^2 \cos(\beta) \]  

– Again \( \beta \) is the effective flapping angle

• The centrifugal force can be considered as a point load that acts on the center of mass of the rotor blade, and is directed perpendicular to the axis of rotation.
• The moment produced by the centrifugal force acting on a differential element at radius $r$ is

$$dM_c = r \sin(\beta) \left[ rdm \Omega^2 \cos(\beta) \right].$$

(4)
• **Gyroscopic loads** are produced by yaw or flapping motions of the spinning rotor.

![Diagram of gyroscopic restoring moment](image)

Figure 6: Illustration of the gyroscopic restoring moment produced by the yawed motion of the rotor.

• Assuming that the rotor has a polar moment of inertia of $J$, and spins at a rate $\Omega$, it will have an angular momentum of $J\Omega$.

• Based on the theory of gyroscopes, if a body with angular momentum of $J\Omega$ is rotated about an axis that is perpendicular to the rotor $\Omega$ plane, it will generate a moment equal to the cross product, $\omega \times J\Omega$, where $\omega$ is the yawing rate.

• The generated bending moment acts on the bearing block

• These bending moments put stress on the rotor shaft and bearing block that could lead to structural failure.
• **Control loads** result from continuous changes in blade pitch and torque used to maintain the optimum tip-speed-ratio

• These control operations can produce intermittent loads on the rotor, shaft and gear box
2 Rotor Response to Loads

- The horizontal axis wind turbine rotor is designed to be stiff and light weight.

Figure 7: Section view of a HAWT rotor illustrating the internal structure.
• The rotor blade can be modeled as a cantilever beam.

• Like the BEM approach, the rotor blade is divided into small spanwise segments
  – The external loading of a rotor segment, \( p \, dx \) is known from the BEM analysis.
  – Loading results in shear forces, \( T \) and \( T + dT \), and bending moments, \( M \) and \( M + dM \) on each element.

Figure 8: Illustration of shear force and bending moment on a small spanwise element of the loaded rotor.
• A balance of forces and moments gives the following equations.

\[
\frac{dT_z}{dx} = -p_z(x) + m(x) \frac{d^2u_z(x)}{dt^2}
\]

(5)

\[
\frac{dT_y}{dx} = -p_y(x) + m(x) \frac{d^2u_y(x)}{dt^2}
\]

(6)

– Time derivative terms represent the inertia in the blade motion (BENDING).

• The bending moments are then found from

\[
\frac{dM_y}{dx} = T_z
\]

(8)

\[
\frac{dM_z}{dx} = -T_y
\]

(9)
• **Principle bending axis** is the point of bending elasticity where a normal force (out of the plane) does not produce bending of the beam.
  
  – If the airfoil section is symmetric (no camber) the first principle axis lies along the chord line, that is $\nu = 0$.
  
  – For normally twisted blades, $\theta_T \leq 0$, although $(\theta_T + \nu)$ is considered to be positive.

• The transformation of the bending moments due to the loads to those along the principle axes is

$$M_1 = M_y \cos(\theta_T + \nu) - M_z \sin(\theta_T + \nu) \quad (11)$$

and

$$M_2 = M_y \sin(\theta_T + \nu) - M_z \cos(\theta_T + \nu). \quad (12)$$
• From beam theory, the curvatures about the principle axes are

\[ \kappa_1 = \frac{M}{EI_1} \]  (13)

and

\[ \kappa_2 = \frac{M}{EI_2}. \]  (14)

• These curvatures are transformed back to the y and z axes by

\[ \kappa_z = -\kappa_1 \sin(\theta_T + \nu) + \kappa_2 \cos(\theta_T + \nu) \]  (15)

and

\[ \kappa_y = \kappa_1 \cos(\theta_T + \nu) + \kappa_2 \sin(\theta_T + \nu). \]  (16)
• The angular deformations are then calculated as

\[
\frac{d\theta_y}{dx} = \kappa_y \tag{17}
\]

and

\[
\frac{d\theta_z}{dx} = \kappa_z. \tag{18}
\]

• The deflections, \( u_z \) and \( u_y \) are found by integrating

\[
\frac{du_z}{dx} = -\theta_y \tag{19}
\]

and

\[
\frac{du_y}{dx} = -\theta_z. \tag{20}
\]
• **Numerical approach:** Consider a rotor blade divided into \( N \) spanwise elements, where the \( N^{th} \) element is at the rotor tip.

• **Shear force:**

\[
T_{y}^{i-1} = T_{y}^{i} + \frac{1}{2} \left( p_{y}^{i-1} + p_{y}^{i} \right) \left( x^{i} - x^{i-1} \right) ; \ i = N, N - 1, \ldots 2
\]

and

\[
T_{z}^{i-1} = T_{z}^{i} + \frac{1}{2} \left( p_{z}^{i-1} + p_{z}^{i} \right) \left( x^{i} - x^{i-1} \right) ; \ i = N, N - 1, \ldots 2.
\]

• **Bending Moments:**

\[
M_{y}^{i-1} = M_{y}^{i} - T_{z}^{i} \left( x^{i} - x^{i-1} \right) - \left( \frac{1}{6} p_{z}^{i-1} + \frac{1}{3} p_{z}^{i} \right) \left( x^{i} - x^{i-1} \right)^{2} ; \ i = N, N - 1, \ldots
\]

and

\[
M_{z}^{i-1} = M_{z}^{i} - T_{y}^{i} \left( x^{i} - x^{i-1} \right) - \left( \frac{1}{6} p_{y}^{i-1} + \frac{1}{3} p_{y}^{i} \right) \left( x^{i} - x^{i-1} \right)^{2} ; \ i = N, N - 1, \ldots
\]

• **Rotor Deflections:**

\[
u_{y}^{i+1} = u_{y}^{i} + \theta_{z}^{i} \left( x^{i+1} - x^{i} \right) + \left( \frac{1}{6} \kappa_{z}^{i+1} + \frac{1}{3} \kappa_{z}^{i} \right) \left( x^{i+1} - x^{i} \right)^{2} ; \ i = 1, 2, \ldots N - 1
\]

and

\[
u_{z}^{i+1} = u_{z}^{i} + \theta_{z}^{i} \left( x^{i+1} - x^{i} \right) + \left( \frac{1}{6} \kappa_{y}^{i+1} + \frac{1}{3} \kappa_{y}^{i} \right) \left( x^{i+1} - x^{i} \right)^{2} ; \ i = 1, 2, \ldots N - 1
\]
where

$$
\theta_{y}^{i+1} = \theta_{y}^{i} + \frac{1}{2} \left( \kappa_{y}^{i+1} + \kappa_{y}^{i} \right) (x_{i+1}^{y} - x_{i}^{y}) ; \ i = 1, 2, \cdots N-1 \quad (27)
$$

and

$$
\theta_{z}^{i+1} = \theta_{z}^{i} + \frac{1}{2} \left( \kappa_{z}^{i+1} + \kappa_{z}^{i} \right) (x_{i+1}^{z} - x_{i}^{z}) ; \ i = 1, 2, \cdots N-1 \quad (28)
$$
- Boundary conditions on the shear force are

\[
T_y^N = 0 \quad (29)
\]
\[
T_z^N = 0 \quad (30)
\]
\[
T_y^1 = \sum_i^N (R^i) \quad (31)
\]
\[
T_z^1 = \sum_i^N (L^i). \quad (32)
\]

- The boundary conditions on the moments are

\[
M_y^N = 0 \quad (34)
\]
\[
M_z^N = 0 \quad (35)
\]
\[
M_y^1 = \sum_i^N (L^i)(x^i) \quad (36)
\]
\[
M_z^1 = \sum_i^N (R^i)(x^i). \quad (37)
\]

- Assuming a rigid rotor support, the boundary conditions on the displacements are

\[
u_y^1 = 0 \quad (39)
\]
\[
u_z^1 = 0. \quad (40)
\]