Wind Regimes

Atmospheric Boundary Layer

Table 1: Classes of surface roughness for atmospheric boundary layers.

Category	Description	$\sim \delta$ (m)	z_0 (m)
1	Exposed sites in windy areas, exposed coast lines, deserts, etc.	270	0.005
2	Exposed sites in less windy areas, open inland country with hedges and buildings, less exposed coasts.	330	0.025-0.1
3	Well wooded inland country, built-up areas.	425	1-2

- A model for the atmospheric boundary layer wind velocity with elevation is

$$V(z) = V(10)\frac{\ln(z/z_0)}{\ln(10/z_0)} \tag{1}$$

  - where $z = 10\,\mathrm{m}$. is the reference height where the velocity measurement was taken,
  - and z_0 is the roughness height at the location where the velocity measurement was taken.

- If the roughness height at a proposed wind turbine site is different than that where the wind profile data was compiled then

$$V(z) = V(10)\frac{\ln(60/z_{0_1})\ln(z/z_{0_2})}{\ln(60/z_{0_2})\ln(10/z_{0_1})} \tag{2}$$

  - where z_{0_1} is the roughness height at the first location,
  - and z_{0_2} is the roughness height at the second location.

Temporal Statistics

- The lowest (first) order statistic is the time average (mean) that is defined as

$$V_m = \frac{1}{N}\sum_{i=1}^{N} V_i \quad \text{where} \quad V_i = V_1, V_2, V_3, \cdots, V_n \tag{3}$$

- Since the wind turbine power scales as V^3, the average power is

$$P_m \sim \frac{1}{N}\sum_{i=1}^{N} V_i^3 \neq V_m^3. \tag{4}$$

- Therefore, we use a *"power component"* time-averaged wind speed give as

$$V_{m_p} = \left[\frac{1}{N}\sum_{i=1}^{N} V_i^3\right]^{1/3}. \tag{5}$$

- Where $P \sim V_{m_p}^3$.

Wind Speed Probability

- Important wind speeds:
  V_{cut-in}
  V_{rated}
  $V_{cut-out}$

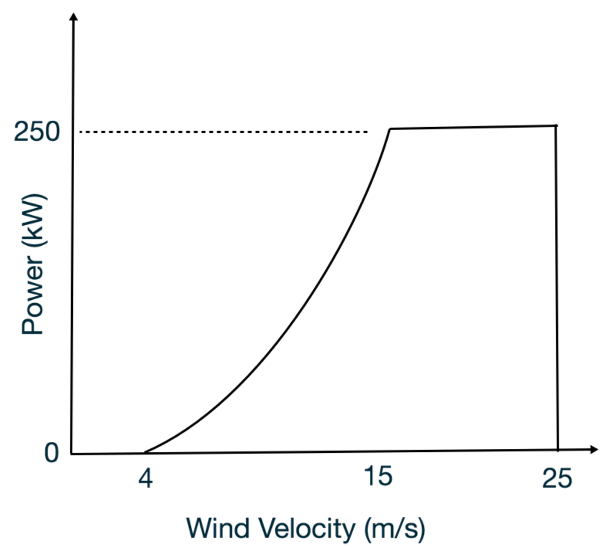

Figure 1: Hypothetical power curve for wind turbine with a rated power of 250 kW.

Statistical Models

- Weibull and Rayleigh (k=2) distributions can be used to describe wind variations with acceptable accuracy.
- In the Weibull distribution the probability of a wind speed, $V \geq V_p$, where V_p is an arbitrary wind speed is given as

$$p(V \geq V_p) = \exp\left[-(V_p/c)^k\right]. \tag{6}$$

- The number of hours in a year in which $V \geq V_p$

$$H(V \geq V_p) = (365)(24)\exp\left[-(V_p/c)^k\right]. \tag{7}$$

- In the Weibull distribution the probability of a wind speed being between two values, V_1 and (V_2)

$$\begin{aligned} \mathcal{P}(V_1 < V < V_2) &= p(V_2) - p(V_1) \qquad (8)\\ &= \exp\left[-(V_1/c)^k\right] - \exp\left[-(V_2/c)^k\right]. \qquad (9) \end{aligned}$$

- The statistical number of hours on a yearly basis that the wind speed will be between V and $(V + \Delta V)$ is then

$$H(V_1 < V < V_2) = (365)(24)\left(\exp\left[-(V_1/c)^k\right] - \exp\left[-(V_2/c)^k\right]\right). \tag{10}$$

- c and k are Weibull coefficients that depend on the elevation and location.

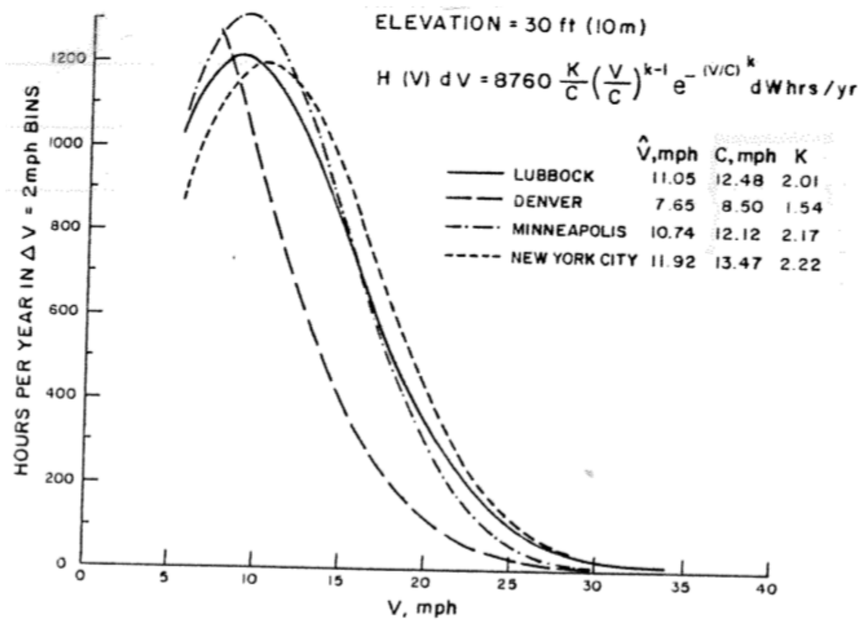

Figure 2: Sample Weibull distributions for atmospheric boundary layer data at different sites.

- Suggested corrections to Weibull coefficients k and c to account for different altitudes, z, are

$$k = k_{ref}\frac{[1 - 0.088\ln(z_{ref}/10)]}{[1 - 0.088\ln(z/10)]} \tag{11}$$

$$c = c_{ref}\left[\frac{z}{z_{ref}}\right]^n \tag{12}$$

$$n = \frac{[0.37 - 0.088\ln(c_{ref})]}{[1 - 0.088\ln(z_{ref}/10)]} \simeq 0.23 \tag{13}$$

- The *cumulative distribution* is the integral of the probability density function, namely
$$\mathcal{P}(V) = \int_0^\infty p(V)dV = 1 - \exp\left[-(V/c)^k\right] \tag{14}$$

- The average wind speed is then shown to be
$$V_m = c\Gamma(1 + \frac{1}{k}). \tag{15}$$

- The standard deviation of the wind speed, σ_v of the wind speeds is
$$\sigma_V = c\left[\Gamma\left(1 + \frac{2}{k}\right) - \Gamma^2\left(1 + \frac{1}{k}\right)\right]^{1/2} \tag{16}$$

- **Weibull Graphical Method.** The cumulative distribution probability is
$$\mathcal{P}(V) = 1 - \exp\left[-(V/c)^k\right] \tag{17}$$
or,
$$1 - \mathcal{P}(V) = \exp\left[-(V/c)^k\right] \tag{18}$$
so that taking the natural log of both sides of the equality,
$$\underbrace{\ln\left[-\ln[1 - \mathcal{P}(V)]\right]}_{y} = \underbrace{k\ln(V_i)}_{Ax} - \underbrace{k\ln(c)}_{B}. \tag{19}$$

- Plot $\ln\left[-\ln[1 - \mathcal{P}(V)]\right]$ versus $\ln(V_i)$ for the velocity samples $V_i,\ i = 1, N$

- the slope of the best fit straight line represents the Weibull coefficient, k, the y-intercept represents $-k\ln(c)$ from which c is found.

Rayleigh Distribution

- The Rayleigh distribution is a *special case* of the Weibull distribution in which $k = 2$. Then

$$V_m = c\Gamma\left(3/2\right) \tag{20}$$

or

$$c = 2\frac{V_m}{\sqrt{\pi}} \tag{21}$$

- In terms of the probability functions, substituting c into the Weibull expressions:

$$p(V) = \frac{\pi}{2}\frac{V}{V_m^2}\exp\left[-\frac{\pi}{4}\left(\frac{V}{V_m}\right)^2\right] \tag{22}$$

of which then

$$\mathcal{P}(V) = 1 - \exp\left[-\frac{\pi}{4}\left(\frac{V}{V_m}\right)^2\right] \tag{23}$$

so that

$$\mathcal{P}(V_1 < V < V_2) = \exp\left[-\frac{\pi}{4}\left(\frac{V_1}{V_m}\right)^2\right] - \exp\left[-\frac{\pi}{4}\left(\frac{V_2}{V_m}\right)^2\right] \tag{24}$$

and

$$\mathcal{P}(V > V_x) = 1 - \left[1 - \exp\left[-\frac{\pi}{4}\left(\frac{V_x}{V_m}\right)^2\right]\right] = \exp\left[-\frac{\pi}{4}\left(\frac{V_x}{V_m}\right)^2\right] \tag{25}$$

Energy Estimation of Wind Regimes

- The ultimate estimate to be made in selecting a site for a wind turbine or wind farm is the *energy* that is available in the wind at the site, namely wind energy density, E_D.
- Other parameters of interest are the most frequent wind velocity, $V_{F_{max}}$, and the wind velocity contributing the maximum energy, $V_{E_{max}}$, at the site.

Weibull-based Energy Estimation

- In terms of the Gamma function, the energy density is

$$E_D = \frac{\rho_a c^3}{2}\frac{3}{k}\Gamma\left(\frac{3}{k}\right). \tag{26}$$

- The energy that is available over a period of time, T (e.g. T=24 hrs)

$$E_T = E_D T = \frac{\rho_a c^3 T}{2}\frac{3}{k}\Gamma\left(\frac{3}{k}\right). \tag{27}$$

- The most frequent wind speed

$$V_{F_{max}} = c\left(\frac{k-1}{k}\right)^{1/k}. \tag{28}$$

- The wind speed that maximizes the energy

$$V_{E_{max}} = \frac{c(k+2)^{1/k}}{k^{1/k}} \tag{29}$$

Rayleigh-based Energy Estimation

- Energy density

$$E_D = \frac{3}{\pi}\rho_a V_m^3. \tag{30}$$

- The energy over a period of time, T,

$$E_T = TE_D = \frac{3}{\pi}T\rho_a V_m^3. \tag{31}$$

- The most frequent wind speed

$$V_{F_{max}} = \frac{1}{\sqrt{2K}} = \sqrt{\frac{2}{\pi}}V_m. \tag{32}$$

- The wind speed that maximizes the energy

$$V_{E_{max}} = \sqrt{\frac{2}{K}} = 2\sqrt{\frac{2}{\pi}}V_m. \tag{33}$$

Aerodynamic Performance

Actuator Disk Momentum Theory

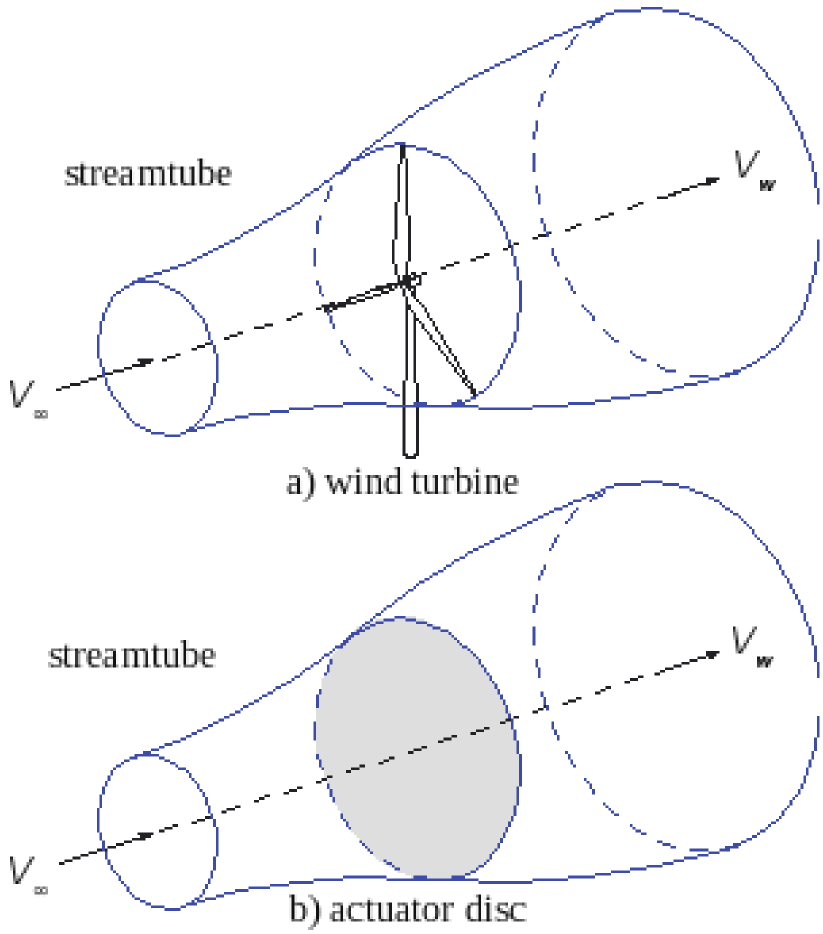

Figure 3: Flowfield of a Wind Turbine and Actuator disc.

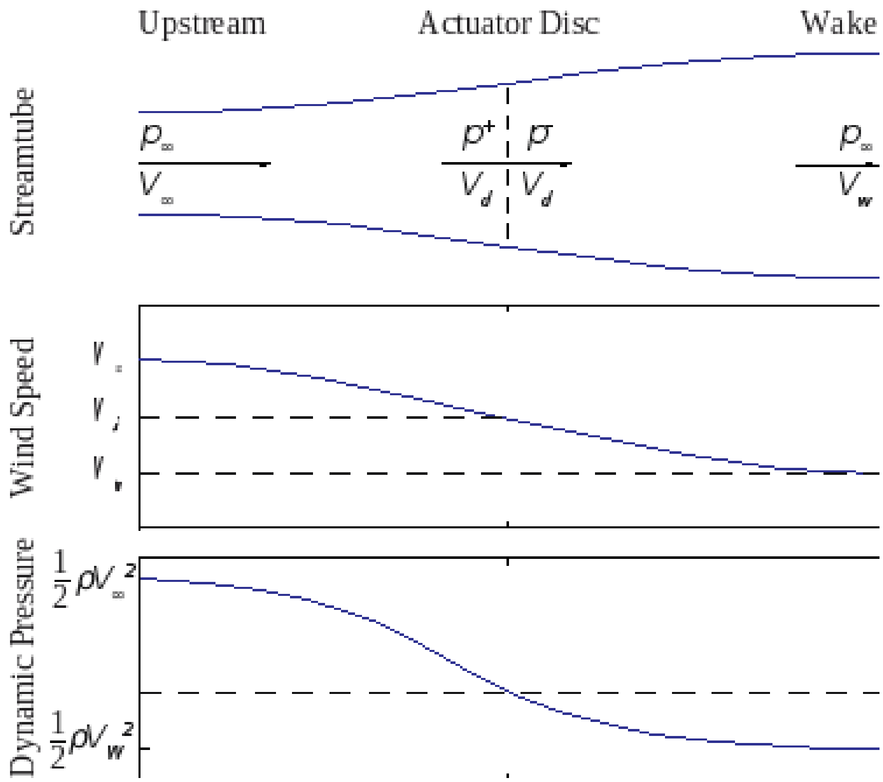

Figure 4: Variation of the velocity and dynamic pressure through the stream-tube.

$$(AV)_\infty = (AV)_d = (AV)_w \tag{34}$$

- Inflow (axial) induction factor, a,

$$a = \frac{V_\infty - V_d}{V_\infty} \tag{35}$$

- The velocity at the actuator disc, V_d

$$V_d = V_\infty [1 - a] . \tag{36}$$

- The wake velocity, V_w

$$V_w = V_\infty [1 - 2a] . \tag{37}$$

- The the thrust on the rotor

$$T = 2\rho A_d V_\infty^2 a \left[1 - a\right] \tag{38}$$

- The thrust coefficient

$$C_T = T/\left[\frac{1}{2}\rho A_d V_\infty^2\right] \;=\; 4a\left[1 - a\right]. \tag{39}$$

- The power extracted from the wind by the actuator disc

$$P = TV_d \;=\; 2\rho A_d V_\infty^3 a\left[1 - a\right]^2. \tag{40}$$

- The power coefficient, C_p, is defined as the ratio of the power extracted from the wind, P, and the available power of wind, or

$$C_P = P/\left[\frac{1}{2}\rho A_d V_\infty^3\right] \;=\; 4a\left[1 - a\right]^2. \tag{41}$$

- The maximum theoretical power coefficient, $C_{P_{max}} = 0.593$, for which $a = 1/3$. Called the Betz limit.

- The maximum theoretical power coefficient, $C_{P_{max}} = 0.593$, for which $a = 1/3$ also holds when **wake rotation is included**.
- The tangential flow is represented through an angular induction factor, a', where
$$a' = \frac{\omega}{2\Omega} \tag{42}$$
- Define λ_r as the local speed ratio
$$\lambda_r = \frac{\Omega r}{V_\infty}. \tag{43}$$
- Define λ is the tip speed ratio
$$\lambda = \frac{\Omega R}{V_\infty}. \tag{44}$$
- A useful relation
$$a(1-a) = a'\lambda_r^2. \tag{45}$$

Blade Element (BEM) Theory

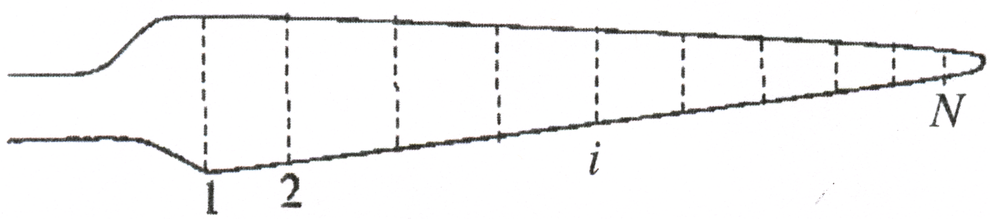

Figure 5: Example of a wind turbine blade divided into 10 sections for BEM analysis.

- The resultant velocity, V_R, is made up of the vector sum of the wind speed and the rotational speed of the blade section

$$V_R = \sqrt{[V_\infty(1-a)]^2 + [\Omega r(1+a')]^2} \tag{46}$$

- The angle that the resultant velocity makes with respect to the plane of rotation is the angle

$$\phi = \tan^{-1}\left[\frac{V_\infty(1-a)}{\Omega r(1+a')}\right]. \tag{47}$$

- The local angle of attack at any radial location on the rotor is

$$\alpha(r) = \phi(r) - [\theta_T(r) + \theta_{cp}]. \tag{48}$$

- Defining

$$C_n = C_L \cos\phi + C_D \sin\phi \tag{49}$$

and

$$C_t = C_L \sin\phi - C_D \cos\phi \tag{50}$$

- Then

$$dF_n = B\frac{1}{2}\rho V_R^2 C_n c dr \tag{51}$$

and

$$dF_t = B\frac{1}{2}\rho V_R^2 C_t c dr. \tag{52}$$

- The differential torque and power are

$$dQ \;=\; r dF_t \;=\; B\frac{1}{2}\rho V_R^2 C_t c r dr \tag{53}$$

and

$$dP \;=\; \Omega dQ \;=\; B\Omega\frac{1}{2}\rho V_R^2 C_t c r dr. \tag{54}$$

- Defining a new parameter

$$\sigma_r = \frac{Bc}{2\pi r} \tag{55}$$

then

$$a = \frac{1}{\frac{4\sin^2\phi}{\sigma_r C_n} + 1}. \tag{56}$$

and

$$a' = \frac{1}{\frac{4\sin\phi\cos\phi}{\sigma_r C_t} - 1}. \tag{57}$$

BEM Theory Tip Loss

- Tip loss factor

$$F = \frac{2}{\pi}\cos^{-1}\left(e^{-f}\right) \tag{58}$$

where

$$f = \frac{B}{2}\frac{R-r}{r\sin\phi} \tag{59}$$

- The tip loss factor is introduced into the differential thrust as

$$dT = 2F\rho V_{\infty}^{2}a(1-a)2\pi r dr. \tag{60}$$

and

$$dQ = 2Fa'(1-a)\rho V_{\infty}\Omega r^{2}(2\pi r dr). \tag{61}$$