Wind Turbine Control

- The control system on a wind turbine is designed to:
  1. seek the highest efficiency of operation that maximizes the coefficient of power, C_p,
  2. ensure safe operation under all wind conditions.
- Wind turbine control systems are typically divided into three functional elements:
  1. the control of groups of wind turbines in a wind farm,
  2. the supervising control of each individual wind turbine, and
  3. separate dedicated dynamic controllers for different wind turbine sub-systems.
- Generally, there exists an **optimum** tip-speed-ratio, λ that maximized C_p.
  - The exact λ depends on the individual wind turbine design ($6 \leq \lambda \leq 8$)

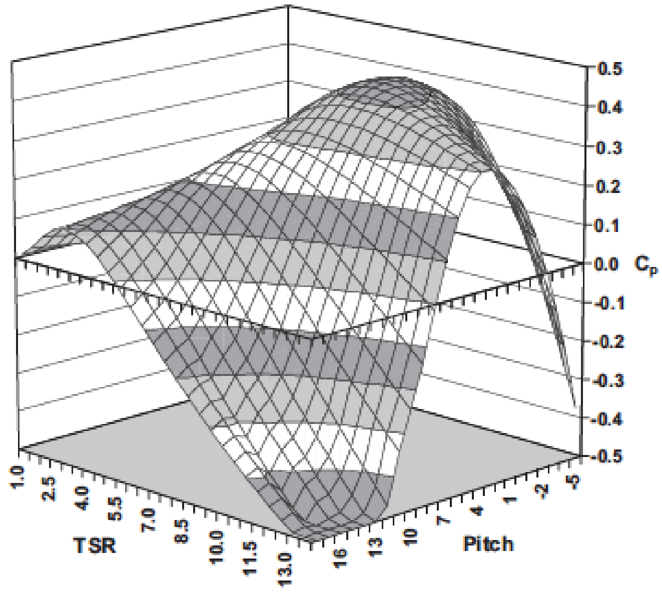

Figure 1: Example of the relation between the rotor tip-speed ratio and rotor pitch angle on the coefficient of power for a 600kW two-bladed horizontal wind turbine.

- The sensitivity of C_p to λ motivates closed-loop control focusing on the the rotation frequency

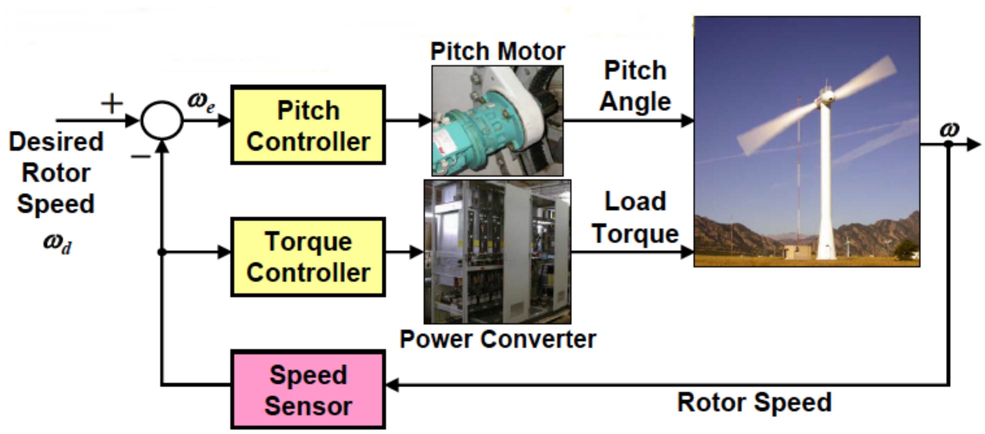

Figure 2: Schematic of a wind turbine closed-loop control system.

1 Axial Induction Control

Recall that the rotor blade tip speed ratio, λ is

$$\lambda = \frac{\Omega R}{U_\infty}. \quad (1)$$

The power generated from the wind is

$$P_{aero} = Q\Omega \quad (2)$$

where Q is the total torque generated by the rotor.

The coefficient of power, C_p, is the ratio of the aerodynamic power extracted from the wind and the available aerodynamic power or,

$$C_p = P_{aero}/P_{available}. \quad (3)$$

The local axial and tangential induction factors are defined as

$$a = 1 - \frac{U_x}{U_\infty} \quad (4)$$

and

$$a' = \frac{U_y}{\Omega r} - 1 \quad (5)$$

where U_x and U_y are the respective axial and tangential velocities in the rotor plane.

The local flow angle at a given radial location on the rotor is then

$$\phi_r = \tan^{-1}\left(\frac{U_y}{U_x}\right) = \tan^{-1}\left(\frac{U_\infty(1-a)}{\Omega r(1+a')}\right) = \tan^{-1}\left(\frac{(1-a)}{(1+a')\lambda_r}\right) \quad (6)$$

where λ_r is the local tip speed ratio at the radial position, r.

The local effective rotor angle of attack at any radial location is then

$$\alpha_r = \phi_r - \psi_r - \theta \quad (7)$$

where ϕ_r is again the local flow angle, ψ_r is the local rotor twist angle, and θ is the global rotor pitch angle which is constant over the rotor radius.

The local lift and drag coefficients, $C_l(r)$ and $C_d(r)$, at a radial location on the rotor are then

$$C_l(r) = C_y \cos(\phi_r) - C_x \sin(\phi_r) \tag{8}$$

and

$$C_d(r) = C_y \sin(\phi_r) + C_x \cos(\phi_r) \tag{9}$$

where C_x and C_y are the force coefficients in the tangential and normal directions of the rotor section at the effective angle of attack, α_r.

The differential torque produced by radial segment of the rotor at radius, r, is

$$dQ = 4\pi\rho U_\infty(\Omega r)a'(1-a)r^2 dr - \frac{1}{2}\rho W^2 N c C_d \cos(\phi_r) r dr. \tag{10}$$

To simplify, the second term in Equation 10 is dropped (neglecting the drag on the rotor). The differential torque is then

$$dQ = 4\pi\rho U_\infty(\Omega r)a'(1-a)r^2 dr. \tag{11}$$

Substituting for a' in terms of a gives

$$dQ = 4\pi\rho U_\infty^2 \frac{a(1-a)^2 r^2}{\lambda} dr. \tag{12}$$

Assuming constant wind conditions (ρ and V_∞) and a fixed tip speed ratio, λ, then

$$dQ = C_1 a(1-a)^2 r^2 dr. \tag{13}$$

Assuming the axial induction factor is constant along the entire rotor span,

$$Q \propto a(1-a)^2. \tag{14}$$

In terms of the aerodynamic power,

$$P_{aero} = Q\Omega \tag{15}$$

or

$$P_{aero} \propto a(1-a)^2. \tag{16}$$

Wind Farms

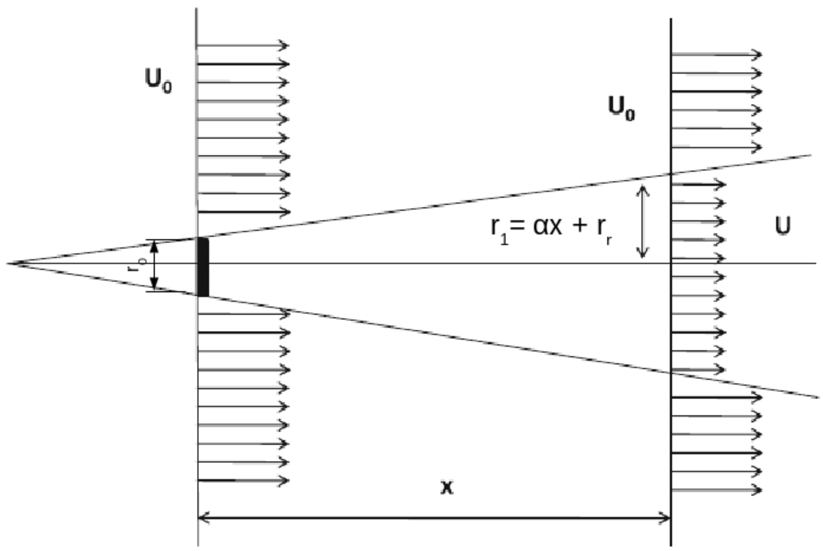

Figure 3: Schematic drawing of wind turbine wake model.

- The local downstream wake radius is r_1 given as

$$r_1 = \alpha x + r_r \tag{17}$$

  - r_0 is the physical radius of the upstream wind turbine rotor
  - α is the wake entrainment constant, also known as the wake decay constant, where

$$\alpha = \frac{0.5}{\ln\left(\frac{z}{z_0}\right)} \tag{18}$$

    where z is the wind turbine hub height, and z_0 is the surface roughness height at the site.
  - r_r is the effective radius of the upstream wind turbine rotor given as

$$r_r = r_0 \sqrt{\frac{1-a}{1-2a}}. \tag{19}$$

- If i is designated as the position of the wind turbine producing the wake, and j is the downstream position that is affected by the wake, then the wind speed at position j is

$$u_j = u_0(1 - u_{def_{ij}}) \tag{20}$$

  - where $u_{def_{ij}}$ is the *wake velocity deficit* induced on position j by an upstream wind turbine at position i.

- The **wake deficit** can be computed through the following relation

$$u_{def_{ij}} = \frac{2a}{1 + \alpha \left(\frac{x_{ij}}{r_r}\right)^2} \tag{21}$$

  - where a is the inflow induction factor that is related to the wind turbine thrust coefficient, C_T as

$$a = 0.5\left(1 - \sqrt{1 - C_T}\right) \tag{22}$$

  - x_{ij} is the downstream distance between positions i and j.

Wind Farm Design Optimization

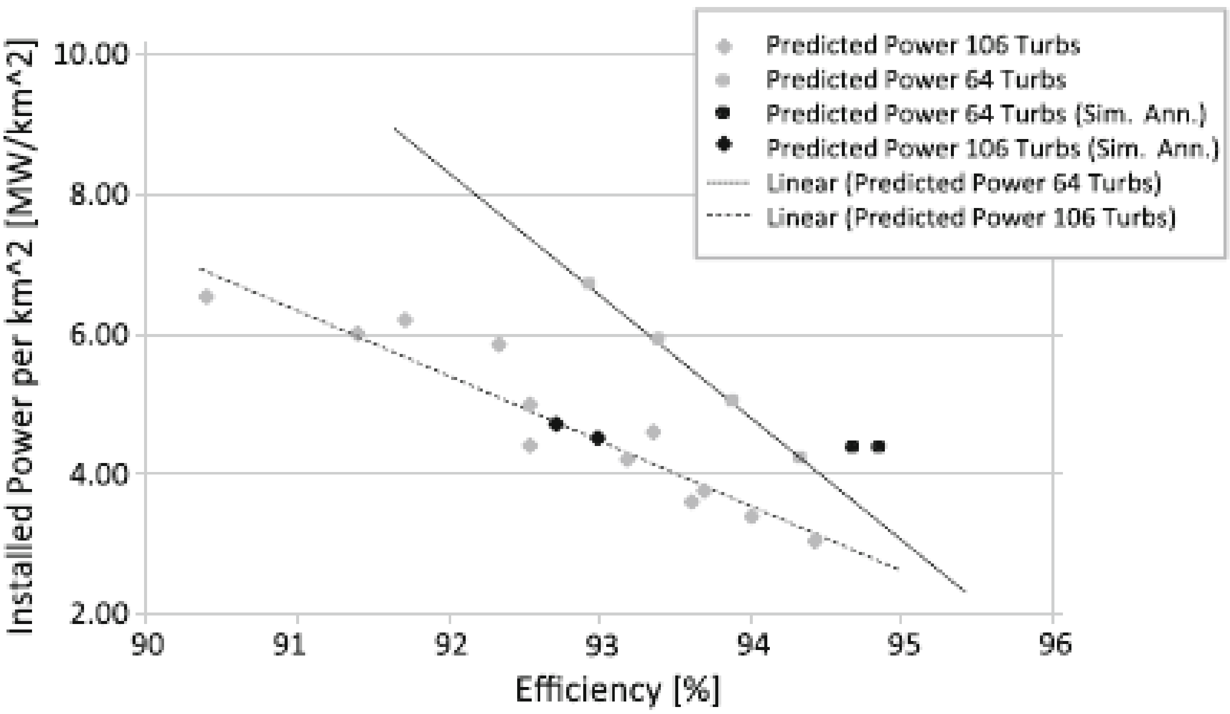

Figure 4: Impact of site area and number of wind turbines on wind farm efficiency.

2 Wind Turbine Acoustics

- The **sound pressure level** of a source in units of decibels (dB), is given as

$$L_P = 20 \log_{10} (P_{rms}/P_0) \tag{23}$$

  - P_{rms} is the root-mean-square of the pressure fluctuations,
  - P_0 is the reference threshold sound pressure level, $P_0 = 2 \times 10^{-5}$Pa.

2.1 Sound Pressure Measurement and Weighting

- **A-scale Weighting**, is the most common scale for assessing environmental and occupational noise. It approximates the response of the human ear to sounds of medium intensity.
- **B-scale Weighting**, approximates the response of the human ear for medium-loud sounds, around 70 dB. (not commonly used)
- **C-scale Weighting**, approximates the response of the human ear to loud sounds. (Can be used for low-frequency sound)
- **G-scale Weighting**, used for ultra-low frequency, infrasound.

2.2 dB Math

- The sum of two sound sources of 90 dB and 80 dB, in decibels, is

$$\begin{aligned}
90\text{dB} &= 20\log\left(\frac{P'_{90}}{2\times10^{-5}\text{Pa}}\right) &= 0.632\text{Pa} \\
80\text{dB} &= 20\log\left(\frac{P'_{80}}{2\times10^{-5}\text{Pa}}\right) &= 0.200\text{Pa} \\
\text{therefore} & & \\
(90+80)\text{dB} &= 20\log\left(\frac{0.832}{2\times10^{-5}\text{Pa}}\right) &= 92.38\text{dB}
\end{aligned} \tag{24}$$

2.3 Wind Turbine Sound Sources

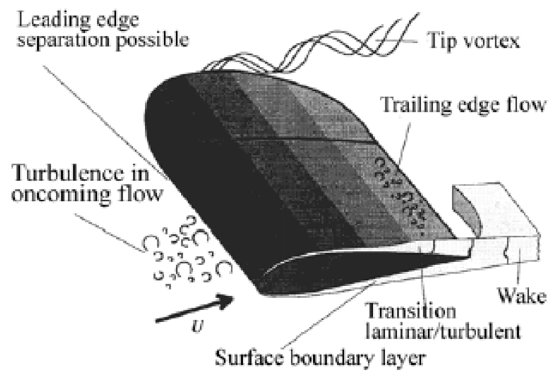

Figure 5: Mechanisms for sound generation due to the air flow over the turbine rotor.

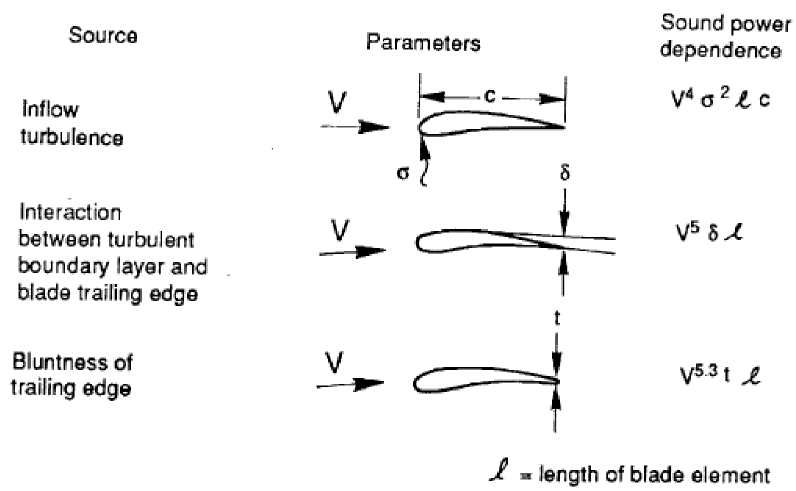

Figure 6: Sound level power scaling for different aerodynamic sound source mechanisms on the turbine rotor.

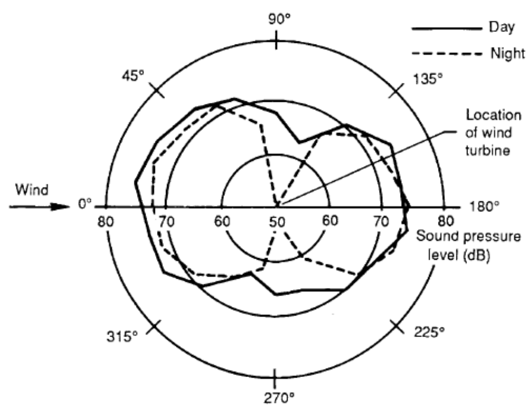

Figure 7: Sound pressure level azimuthal radiation pattern for a wind turbine.

2.4 Sound Propagation

- A **simple model** based on the more conservative assumption of hemispherical sound propagation over a reflective surface, including air absorption is

$$L_p = L_w - 10\log_{10}\left(2\pi R^2\right) - \alpha R \tag{25}$$

  - L_p is the sound pressure level (dB) a distance R from a sound source radiating at a power level, L_w, (dB),
  - $\alpha = 0.005$ dB/m is the frequency-dependent sound absorption coefficient.

2.5 Noise Standards

Table 1: ISO 1996-1971 Recommendations for Community Noise Limits

Location	Daytime - db(A) 7AM-7PM	Evening - db(A) 7PM-11PM	Night - dB(A) 11PM-7AM
Rural	35	30	25
Suburban	40	35	30
Urban Residential	45	40	35
Urban Mixed	50	45	40

3 Wind Turbine Energy Storage

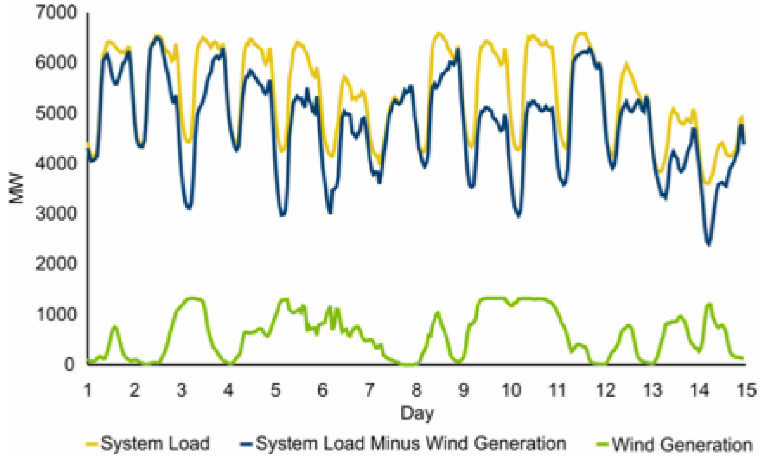

Figure 8: Example of a two week period of system loads, system loads minus wind generation, and wind generation.

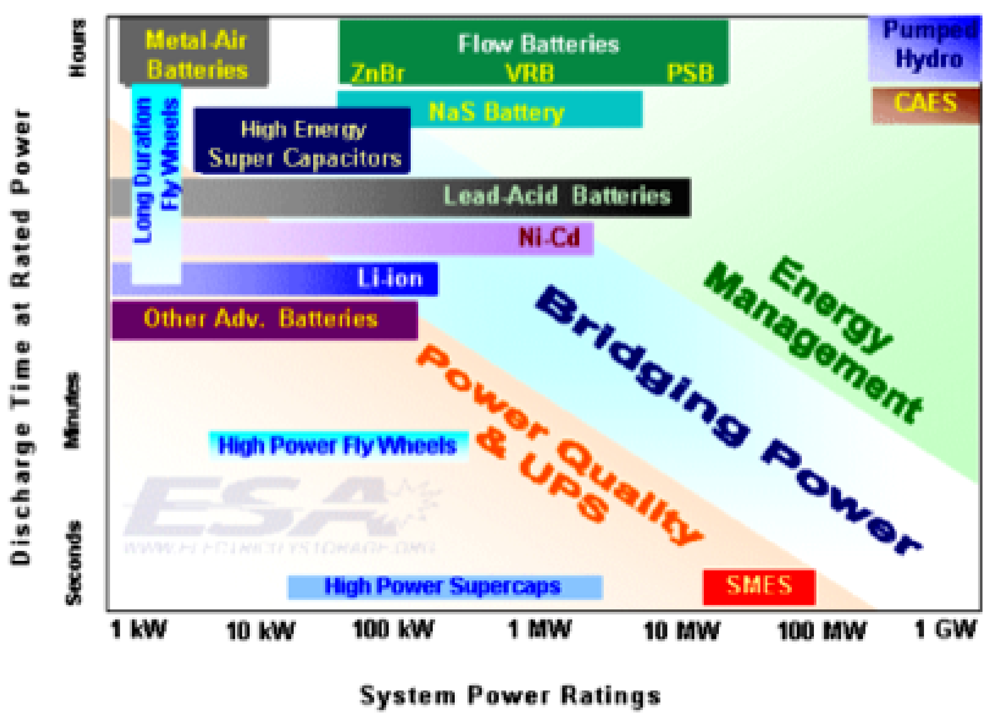

Figure 9: Comparison of different electric power storage systems with regard to power rating and discharge rate.

3.1 Battery Case Study

–
$$E_{rated} = C_{rated} V_{nominal} \ \ [W - h] \tag{26}$$

– C_{rated} is the amp-hour capacity of the battery

– $V_{nominal}$ is the nominal voltage of the battery

– General restriction on the "depth of discharge" (DOD) of **50% of capacity** to ensure a long operating life

– Example. The usable energy of a deep-cycle lead acid battery in which $V_{nominal} = 60$V, and $C_{rated} = 1200$A-hr is

$$E_{usable} = E_{rated} \cdot \text{DOD} \tag{27}$$
$$= (1200)(60)(0.5) \tag{28}$$
$$= 36[\text{kw-h}] \tag{29}$$

The efficiency for the battery "system" is

$$\eta_{battery/inverter} = \eta_{battery} \eta_{inverter}. \tag{30}$$

For an average voltage inverter efficiency of 85%, The overall efficiency of the battery-inverter combination is

$$\eta_{battery/inverter} \ = \ (0.68)(0.85) \ = \ 0.578 \ (57.8\%) \tag{31}$$

3.2 Hydro-electric Storage Case Study

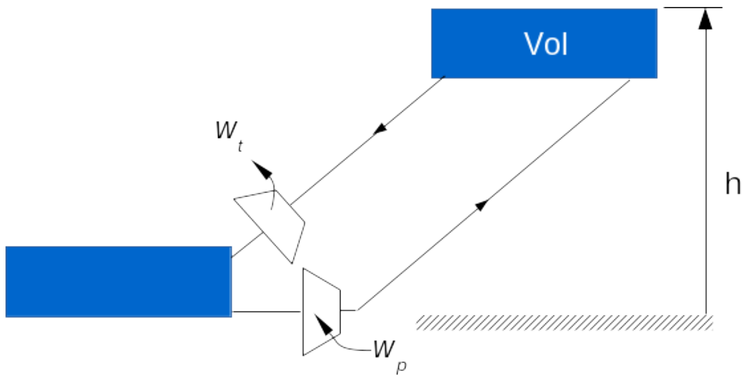

Figure 10: Schematic of a hydro-electric storage configuration.

- The energy generated is

$$E_{hydro} = \rho g h VOL \eta \tag{32}$$

where

$$VOL = \text{water volume stored [m}^3\text{]} \tag{33}$$
$$h = \text{stored water elevation (pressure head) [m]} \tag{34}$$
$$\rho = \text{water density [1000 kg/m}^3\text{]} \tag{35}$$
$$g = \text{gravitational constant [9.8 m/s}^2\text{]} \tag{36}$$
$$\eta = \eta_t \eta_{pipe} \tag{37}$$
$$\eta_t = \text{turbine efficiency (0.60)} \tag{38}$$
$$\eta_{pipe} = \text{pipe flow efficiency (0.90).} \tag{39}$$

- Noting that $1J = 1W$, the stored energy in units of [kW-h] is

$$E = \frac{gVOLh\eta}{3600} \tag{40}$$

- The required volume of water needed to supply a given amount of energy is

$$VOL = \frac{3600E}{gh\eta} \tag{41}$$

  - Note that 3600 s/hr is a conversion between hours and seconds

3.3 Buoyant Hydraulic Energy Storage Case Study

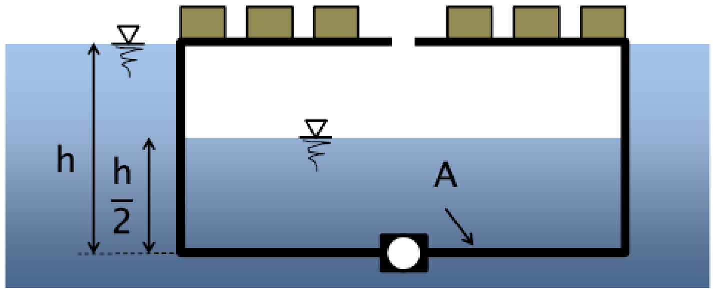

Figure 11: Schematic representation of the buoyant energy storage.

- The maximum amount of stored energy is

$$E = mg\frac{h}{2} \tag{42}$$
$$= \rho A\frac{h}{2}g\frac{h}{2}\eta_t \tag{43}$$
$$= \rho Ag\frac{h^2}{4}\eta_t \tag{44}$$

  - A is the projected area of the floating structure
  - $A(h/2)$ is the volume of displaced water
  - η_t is the efficiency of the turbine (≃60%)

4 Economics

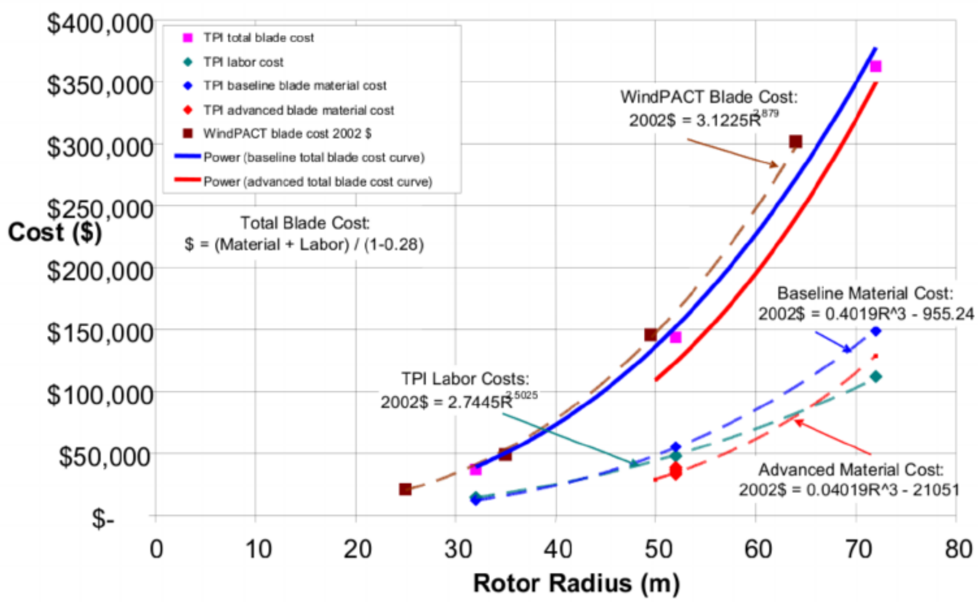

Figure 12: Wind turbine rotor blade cost, labor cost, and baseline and advanced material cost correlations with rotor radius.

$$\text{Baseline Rotor Cost} = 3.1225R^{2.879} \tag{45}$$

$$AEP = (P(V_{rated-cutout})(24)(365)(1500) = 4,312\,\text{MW-h}. \tag{46}$$