Wind Turbine Control

• The control system on a wind turbine is designed to:
  1. seek the highest efficiency of operation that maximizes the coefficient of power, \( C_p \),
  2. ensure safe operation under all wind conditions.

• Wind turbine control systems are typically divided into three functional elements:
  1. the control of groups of wind turbines in a wind farm,
  2. the supervising control of each individual wind turbine, and
  3. separate dedicated dynamic controllers for different wind turbine sub-systems.

• Generally, there exists an optimum tip-speed-ratio, \( \lambda \) that maximized \( C_p \).
  – The exact \( \lambda \) depends on the individual wind turbine design \((6 \leq \lambda \leq 8)\)
Figure 1: Example of the relation between the rotor tip-speed ratio and rotor pitch angle on the coefficient of power for a 600kW two-bladed horizontal wind turbine.

- The sensitivity of $C_p$ to $\lambda$ motivates closed-loop control focusing on the rotation frequency.

Figure 2: Schematic of a wind turbine closed-loop control system.
1 Axial Induction Control

Recall that the rotor blade tip speed ratio, $\lambda$ is

$$\lambda = \frac{\Omega R}{U_\infty}. \quad (1)$$

The power generated from the wind is

$$P_{aero} = Q\Omega \quad (2)$$

where $Q$ is the total torque generated by the rotor.

The coefficient of power, $C_p$, is the ratio of the aerodynamic power extracted from the wind and the available aerodynamic power or,

$$C_p = \frac{P_{aero}}{P_{available}}. \quad (3)$$

The local axial and tangential induction factors are defined as

$$a = 1 - \frac{U_x}{U_\infty} \quad (4)$$

and

$$a' = \frac{U_y}{\Omega r} - 1 \quad (5)$$

where $U_x$ and $U_y$ are the respective axial and tangential velocities in the rotor plane.

The local flow angle at a given radial location on the rotor is then

$$\phi_r = \tan^{-1}\left(\frac{U_y}{U_x}\right) = \tan^{-1}\left(\frac{U_\infty(1 - a)}{\Omega r(1 + a')}\right) = \tan^{-1}\left(\frac{(1 - a)}{(1 + a')\lambda_r}\right) \quad (6)$$

where $\lambda_r$ is the local tip speed ratio at the radial position, $r$.

The local effective rotor angle of attack at any radial location is then

$$\alpha_r = \phi_r - \psi_r - \theta \quad (7)$$
where $\phi_r$ is again the local flow angle, $\psi_r$ is the local rotor twist angle, and $\theta$ is the global rotor pitch angle which is constant over the rotor radius.

The local lift and drag coefficients, $C_l(r)$ and $C_d(r)$, at a radial location on the rotor are then

$$C_l(r) = C_y \cos(\phi_r) - C_x \sin(\phi_r)$$

and

$$C_d(r) = C_y \sin(\phi_r) + C_x \cos(\phi_r)$$

where $C_x$ and $C_y$ are the force coefficients in the tangential and normal directions of the rotor section at the effective angle of attack, $\alpha_r$.

The differential torque produced by radial segment of the rotor at radius, $r$, is

$$dQ = 4\pi \rho U_\infty (\Omega r)a'(1-a)r^2dr - \frac{1}{2}\rho W^2 NcC_d \cos(\phi_r) rdr. \quad (10)$$

To simplify, the second term in Equation 10 is dropped (neglecting the drag on the rotor). The differential torque is then

$$dQ = 4\pi \rho U_\infty (\Omega r)a'(1-a)r^2dr. \quad (11)$$

Substituting for $a'$ in terms of $a$ gives

$$dQ = 4\pi \rho U_\infty^2 \frac{a(1-a)^2r^2}{\lambda} dr. \quad (12)$$

Assuming constant wind conditions ($\rho$ and $V_\infty$) and a fixed tip speed ratio, $\lambda$, then

$$dQ = C_1 a(1-a)^2r^2dr. \quad (13)$$

Assuming the axial induction factor is constant along the entire rotor span,

$$Q \propto a(1-a)^2. \quad (14)$$
In terms of the aerodynamic power,

\[ P_{aero} = Q\Omega \quad (15) \]

or

\[ P_{aero} \propto a(1 - a)^2. \quad (16) \]
Wind Farms

Figure 3: Schematic drawing of wind turbine wake model.

- The local downstream wake radius is $r_1$ given as

$$r_1 = \alpha x + r_r$$  \hspace{1cm} (17)

- $r_0$ is the physical radius of the upstream wind turbine rotor
- $\alpha$ is the wake entrainment constant, also known as the wake decay constant, where

$$\alpha = \frac{0.5}{\ln \left( \frac{z}{z_0} \right)}$$  \hspace{1cm} (18)

where $z$ is the wind turbine hub height, and $z_0$ is the surface roughness height at the site.

- $r_r$ is the effective radius of the upstream wind turbine rotor given as

$$r_r = r_0 \sqrt{\frac{1 - a}{1 - 2a}}.$$  \hspace{1cm} (19)
• If \( i \) is designated as the position of the wind turbine producing the wake, and \( j \) is the downstream position that is affected by the wake, then the wind speed at position \( j \) is

\[
  u_j = u_0 (1 - u_{def_{ij}})
\]

– where \( u_{def_{ij}} \) is the \textit{wake velocity deficit} induced on position \( j \) by an upstream wind turbine at position \( i \).

• The \textbf{wake deficit} can be computed through the following relation

\[
  u_{def_{ij}} = \frac{2a}{1 + \alpha \left( \frac{x_{ij}}{r_r} \right)^2}
\]

– where \( a \) is the inflow induction factor that is related to the wind turbine thrust coefficient, \( C_T \) as

\[
  a = 0.5 \left( 1 - \sqrt{1 - C_T} \right)
\]

– \( x_{ij} \) is the downstream distance between positions \( i \) and \( j \).
Wind Farm Design Optimization

Figure 4: Impact of site area and number of wind turbines on wind farm efficiency.
2 Wind Turbine Acoustics

• The sound pressure level of a source in units of decibels (dB), is given as

\[ L_P = 20 \log_{10} (P_{rms}/P_0) \]  \hspace{1cm} (23)

– \( P_{rms} \) is the root-mean-square of the pressure fluctuations,
– \( P_0 \) is the reference threshold sound pressure level, \( P_0 = 2 \times 10^{-5}\)Pa.

2.1 Sound Pressure Measurement and Weighting

• A-scale Weighting, is the most common scale for assessing environmental and occupational noise. It approximates the response of the human ear to sounds of medium intensity.

• B-scale Weighting, approximates the response of the human ear for medium-loud sounds, around 70 dB. (not commonly used)

• C-scale Weighting, approximates the response of the human ear to loud sounds. (Can be used for low-frequency sound)

• G-scale Weighting, used for ultra-low frequency, infrasound.
2.2 dB Math

- The sum of two sound sources of 90 dB and 80 dB, in decibels, is

\[
90\text{dB} = 20 \log \left( \frac{P'_{90}}{2 \times 10^{-5}\text{Pa}} \right) = 0.632\text{Pa} \quad (24)
\]

\[
80\text{dB} = 20 \log \left( \frac{P'_{80}}{2 \times 10^{-5}\text{Pa}} \right) = 0.200\text{Pa}
\]

therefore

\[
(90 + 80)\text{dB} = 20 \log \left( \frac{0.832}{2 \times 10^{-5}\text{Pa}} \right) = 92.38\text{dB}
\]
2.3 Wind Turbine Sound Sources

Figure 5: Mechanisms for sound generation due to the air flow over the turbine rotor.

Figure 6: Sound level power scaling for different aerodynamic sound source mechanisms on the turbine rotor.
Figure 7: Sound pressure level azimuthal radiation pattern for a wind turbine.
2.4 Sound Propagation

- **A simple model** based on the more conservative assumption of hemispherical sound propagation over a reflective surface, including air absorption is

\[
L_p = L_w - 10 \log_{10} (2\pi R^2) - \alpha R
\]  

(25)

- \(L_p\) is the sound pressure level (dB) a distance \(R\) from a sound source radiating at a power level, \(L_w\), (dB),
- \(\alpha = 0.005\) dB/m is the frequency-dependent sound absorption coefficient.

2.5 Noise Standards

Table 1: ISO 1996-1971 Recommendations for Community Noise Limits

<table>
<thead>
<tr>
<th>Location</th>
<th>Daytime - db(A) 7AM-7PM</th>
<th>Evening - db(A) 7PM-11PM</th>
<th>Night - dB(A) 11PM-7AM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rural</td>
<td>35</td>
<td>30</td>
<td>25</td>
</tr>
<tr>
<td>Suburban</td>
<td>40</td>
<td>35</td>
<td>30</td>
</tr>
<tr>
<td>Urban Residential</td>
<td>45</td>
<td>40</td>
<td>35</td>
</tr>
<tr>
<td>Urban Mixed</td>
<td>50</td>
<td>45</td>
<td>40</td>
</tr>
</tbody>
</table>
3 Wind Turbine Energy Storage

Figure 8: Example of a two week period of system loads, system loads minus wind generation, and wind generation.

Figure 9: Comparison of different electric power storage systems with regard to power rating and discharge rate.
3.1 Battery Case Study

\[ E_{\text{rated}} = C_{\text{rated}} V_{\text{nominal}} \ [W - h] \]  

- \( C_{\text{rated}} \) is the amp-hour capacity of the battery
- \( V_{\text{nominal}} \) is the nominal voltage of the battery
- General restriction on the “depth of discharge” (DOD) of 50% of capacity to ensure a long operating life

Example. The usable energy of a deep-cycle lead acid battery in which \( V_{\text{nominal}} = 60V \), and \( C_{\text{rated}} = 1200\text{A-hr} \) is

\[ E_{\text{usable}} = E_{\text{rated}} \cdot \text{DOD} \]
\[ = (1200)(60)(0.5) \]
\[ = 36[\text{kw-h}] \]

The efficiency for the battery “system” is

\[ \eta_{\text{battery/inverter}} = \eta_{\text{battery}} \eta_{\text{inverter}} \cdot \]

For an average voltage inverter efficiency of 85%, The overall efficiency of the battery-inverter combination is

\[ \eta_{\text{battery/inverter}} = (0.68)(0.85) = 0.578 (57.8\%) \]
3.2 Hydro-electric Storage Case Study

Figure 10: Schematic of a hydro-electric storage configuration.

- The energy generated is

\[ E_{\text{hydro}} = \rho ghVOL\eta \]  \hspace{1cm} (32)

where

\[ \begin{align*}
VOL &= \text{water volume stored} \ [\text{m}^3] \hspace{1cm} (33) \\
h &= \text{stored water elevation (pressure head)} \ [\text{m}] \hspace{1cm} (34) \\
\rho &= \text{water density} \ [1000 \text{ kg/m}^3] \hspace{1cm} (35) \\
g &= \text{gravitational constant} \ [9.8 \text{ m/s}^2] \hspace{1cm} (36) \\
\eta &= \eta_t\eta_{\text{pipe}} \hspace{1cm} (37) \\
\eta_t &= \text{turbine efficiency} \ (0.60) \hspace{1cm} (38) \\
\eta_{\text{pipe}} &= \text{pipe flow efficiency} \ (0.90). \hspace{1cm} (39)
\end{align*} \]

- Noting that \(1J = 1W\), the stored energy in units of [kW-h] is

\[ E = \frac{gVOLh\eta}{3600} \]  \hspace{1cm} (40)
• The required volume of water needed to supply a given amount of energy is

\[ VOL = \frac{3600E}{gh\eta} \]  

(Note that 3600 s/hr is a conversion between hours and seconds)
3.3 Buoyant Hydraulic Energy Storage Case Study

- The maximum amount of stored energy is

\[ E = m g \frac{h}{2} = \rho A \frac{h}{2} g \frac{h}{2} \eta_t = \rho A g \frac{h^2}{4} \eta_t \]

- \( A \) is the projected area of the floating structure
- \( A(h/2) \) is the volume of displaced water
- \( \eta_t \) is the efficiency of the turbine (\( \approx 60\% \))
4 Economics

Figure 12: Wind turbine rotor blade cost, labor cost, and baseline and advanced material cost correlations with rotor radius.

Baseline Rotor Cost = $3.1225R^{2.879}$ \hspace{1cm} (45)

$AEP = (P(V_{\text{rated}} - \text{cutout})(24)(365)(1500) = 4,312 \text{ MW-h.} \hspace{1cm} (46)$