Wind Farms

- **Wind farms** are a cluster of wind turbines that are located at a site to generate electricity.
- Wind farms are also sometimes referred to as a **“plant”**, **“array”** or a **“park”**.
- The first **onshore** wind farm was installed in 1980 in Southern New Hampshire.
  - It consisted of 20 wind turbines with rated power of 30 kW each, giving a combined capacity of 0.6 MW.
- The first **offshore** wind farm was build in 1991 off of the North coast of the Danish Island, Lolland.
  - It consisted of 11, 450 kW turbines that gave it a combined capacity of 4.95 MW.

- The trend is towards increased size and numbers of wind turbines that provide an overall larger power capacity.

Figure 1: Photographs of modern onshore and offshore wind farms.

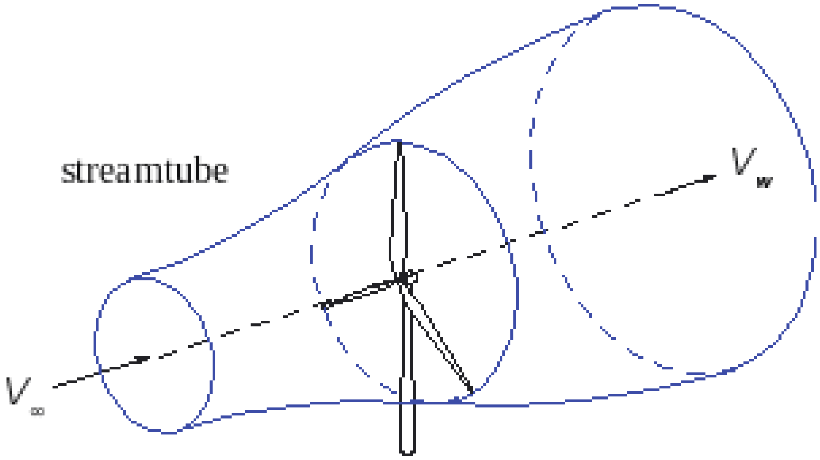
- The evolution towards larger size, smarter control and more advanced capabilities of wind turbines has resulted in a **more complex process of wind farm design**.

- Design objectives are often constrained by such aspects as
    1. economic factors,
    2. operation and maintenance,
    3. environmental impact,
    4. human factors.
- One of the most critical design factors is the arrangement of the wind turbines.
- The **goal** in this case is to determine the positions of the wind turbines within the wind farm to
    1. maximize the energy production,
    2. minimize the initial investment cost,
    3. minimize environmental factors, including land and acoustic noise.

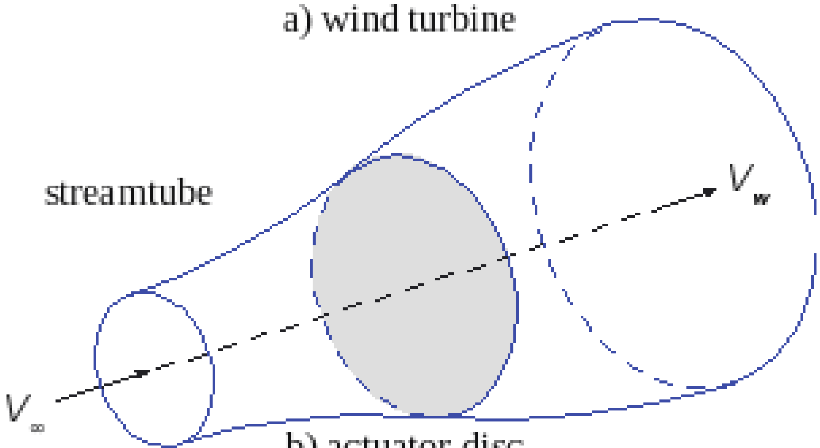
- Wind farm design optimization is a complex multi-objective problem that lacks an analytical formulation.
- Different approaches of wind farm design optimization have been proposed.
  1. Equally spaced turbines,
  2. Unequally spaced turbines,
  3. Staggered grid arrangements.
- More complex **random** arrangements based on Monte Carlo methods, and genetic algorithms.

Wind Turbine Wake Effects

- When a wind turbine extracts energy from the wind, it produces a cone-shaped wake of slower moving turbulent air.
- This was illustrated with respect to **Rotor Actuator Disk Theory**



a) wind turbine



b) actuator disc

Figure 2: Flowfield of a Wind Turbine and Actuator disc.

- Where the wake velocity, V_w, was expressed in terms a, and V_∞ as

$$V_w = V_\infty \left[1 - 2a \right]. \tag{1}$$

- A following is a remarkable photograph that illustrates the wakes produced by wind turbines in an offshore wind farm.

Figure 3: Photograph showing the wakes from wind turbines made visible by low level fog over an an offshore wind farm.

- The photograph reveals downwind wind turbines that are completely engulfed in the wakes of the upwind turbines.
- How can we minimize the impact this has on the power generated by the downwind turbines?

- Jenson (1983) proposed an analytical **wake model** for a wind turbine.
- This considered that momentum is conserved within the wake, and that the wake region **expands linearly** in the downstream direction.

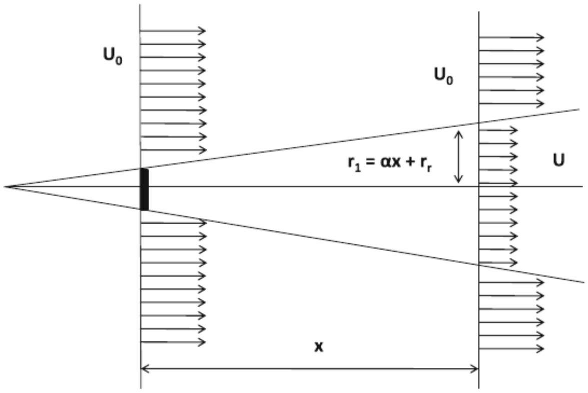


Figure 4: Schematic drawing of wind turbine wake model.

- Based on this, the local wake radius is r_1 given as

$$r_1 = \alpha x + r_r \tag{2}$$

  - r_r is the radius of the upstream wind turbine rotor
  - α is the wake entrainment constant, also known as the wake decay constant, where

$$\alpha = \frac{0.5}{\ln\left(\frac{z}{z_0}\right)} \tag{3}$$

  - z is the wind turbine hub height, and z_0 is the surface roughness height at the site.

- If i is designated as the position of the wind turbine producing the wake, and j is the downstream position that is affected by the wake, then the wind speed at position j is

$$u_j = u_0(1 - u_{def_{ij}}) \tag{4}$$

  - where $u_{def_{ij}}$ is the *wake velocity deficit* induced on position j by an upstream wind turbine at position i.

- The **wake deficit** can be computed through the following relation

$$u_{def_{ij}} = \frac{2a}{1 + \alpha\left(\frac{x_{ij}}{r_d}\right)^2} \tag{5}$$

  - where a is the inflow induction factor that is related to the wind turbine thrust coefficient, C_T as

$$a = 0.5\left(1 - \sqrt{1 - C_T}\right) \tag{6}$$

  - x_{ij} is the downstream distance between positions i and j.

- The term r_d in Equation 5 is called the **equivalent downstream rotor radius** and is given as

$$r_d = r_r\sqrt{\frac{1-a}{1-2a}}. \tag{7}$$

- How far downstream does it take for the wake of an **optimum upstream wind turbine** ($a = 1/3$) to dissipate?
- Answer: More than **40 rotor diameters** to recover the free-stream wind speed!
    - The standard streamwise spacing in wind farms is **5 diameters!**

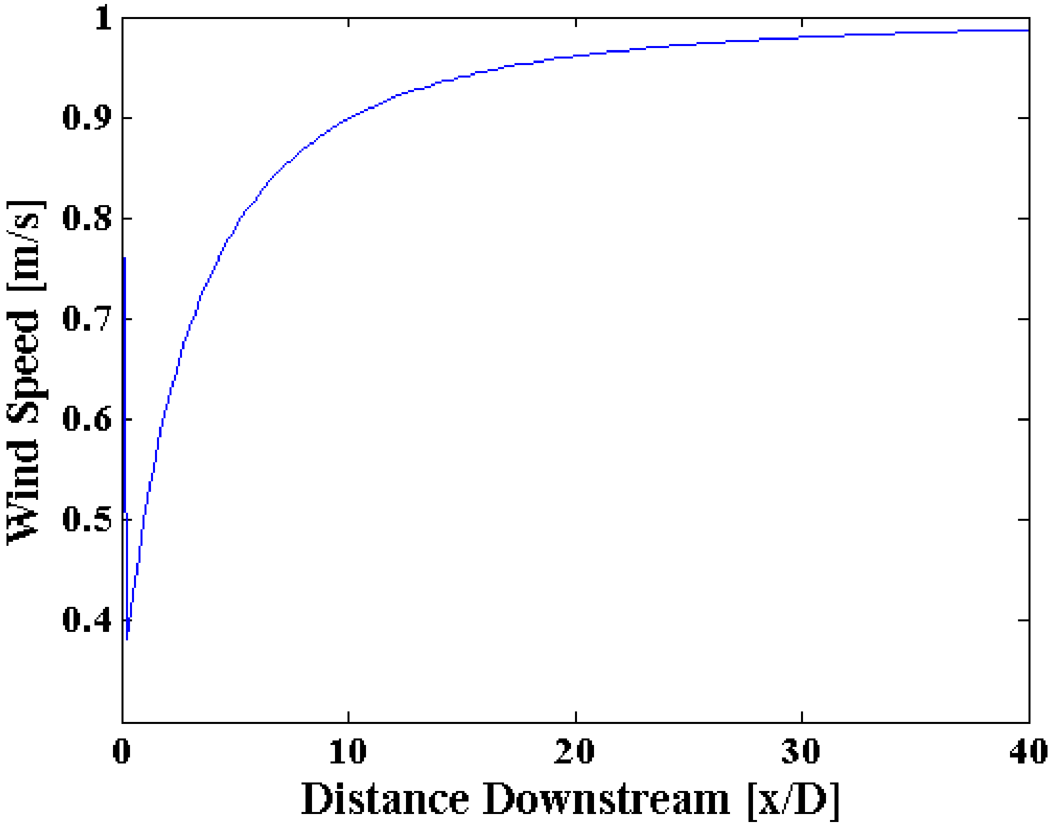


Figure 5: Velocity on the wake centerline of an upstream ideal, $a = 1/3$, wind turbine based on the wake model equations.

- To account for multiple wind turbines in which the wakes can intersect and affect a downstream turbine, **the velocity deficit is the sum of the deficits** produced by each wind turbine, namely

$$u_{def}(j) = \sqrt{\sum_{i \in W(j)} u^2_{def_{ij}}} \tag{8}$$

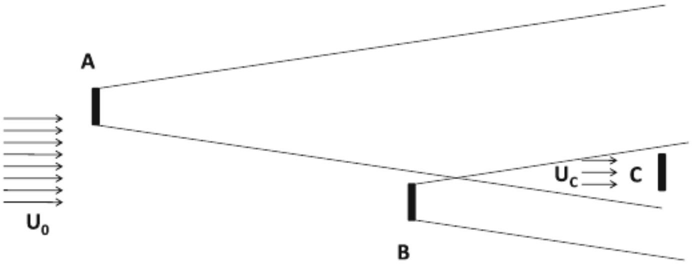
  - $W(j)$ is the set of upstream turbines affecting position j in the wake.

- The velocity deficit, $u_{def}(j)$ is then used in place of $u_{def_{ij}}$ so that

$$u_j = u_0(1 - u_{def_j}) \tag{9}$$

Example:

Consider the arrangement of three wind turbines in the following schematic in which wind turbine C is in the wakes of turbines A and B.



Given the following:

- $U_0 = 12\,\mathrm{m/s}$
- $x_{AC} = 500\,\mathrm{m}$
- $x_{BC} = 200\,\mathrm{m}$
- $z = 60\,\mathrm{m}$
- $z_0 = 0.3\,\mathrm{m}$
- $r_r = 20\,\mathrm{m}$
- $C_T = 0.88$

Compute the total velocity deficit, $u_{def}(C)$ and the velocity at wind turbine C, namely u_C.

Answer:

Based on the previous equations, $u_{def_{AC}} = 0.0208$ and $u_{def_{BC}} = 0.1116$. Then based on Equation 8, $u_{def}(C) = 0.1135$, that is the wind speed is reduced by 11.35% due to the wakes from A and B. The wind velocity approaching wind turbine C is then

$$U_C = U_0 \left(1 - u_{def}(C)\right) = 10.64\text{m/s}. \tag{10}$$

- This example highlights a very important property of multiple wake combinations, namely **the total velocity deficit depends most on the closest turbine that generates a wake**.

- The power generated by any one of the wind turbines is

$$P_j \propto a_j u_j^3 \tag{11}$$

  - a_j is the inflow induction for the wake-affected turbine
  - u_j is the wind velocity approaching the wake-affected turbine.

- The total power generated by all of the wind turbines is

$$P_{tot} \propto \sum_{i \in W(j)} a_j u_{ij}^3 \tag{12}$$

  - $W(j)$ is the set of turbines with inflow induction factors, a_j and approaching velocities u_{ij}.

- The wind farm efficiency is then defined as

$$\eta = \frac{P_{tot}}{N \cdot P_{iso}} \tag{13}$$

  - P_{iso} is the power produced **by an isolated wind turbine** under the same inflow velocity, U_0.

Wind Farm Design Optimization

- In an optimization of a wind farm one might seek to **maximize the power with respect to the initial cost** of the wind turbines purchased for the wind farm.
- Consider a cost model

$$\text{Cost}_{tot} = N_t \left(\frac{2}{3} + \frac{1}{3} e^{-0.00174 N_t^2} \right) \tag{14}$$

  - where N_t is the number of turbines installed,
  - and Cost_{tot} decreases as N_t increases, thus reflecting the **"economy of scale"**.

- The objective function for the optimization process could then be

$$\text{Obj} = \frac{1}{P_{tot}} w_1 + \frac{Cost_{tot}}{P_{tot}} w_2 \tag{15}$$

  - where w_1 and w_2 are weighting coefficients where $w_1 + w_2 = 1$.

- Consider a wind turbine patterns where the wind direction is from the bottom to the top.

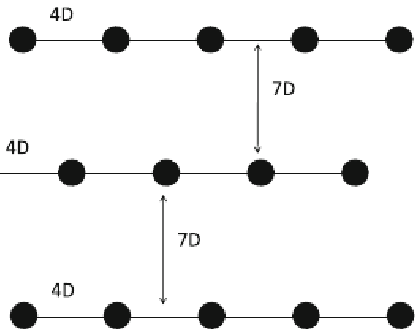


Figure 6: Rule of thumb pattern of wind turbines in a wind farm. The predominant wind direction is from bottom to top.

- An optimization study was conducted to examine the potential of optimized patterns of wind turbines.
  - This considers the impact of **site area** and **number of wind turbines** on wind farm efficiency.
  - It considers either **64, 5 MW** turbines or **106, 3 MW** turbines.
  - The total power installed is similar for the two cases.
  - A predefined site area is imposed, which **defines a power density**.

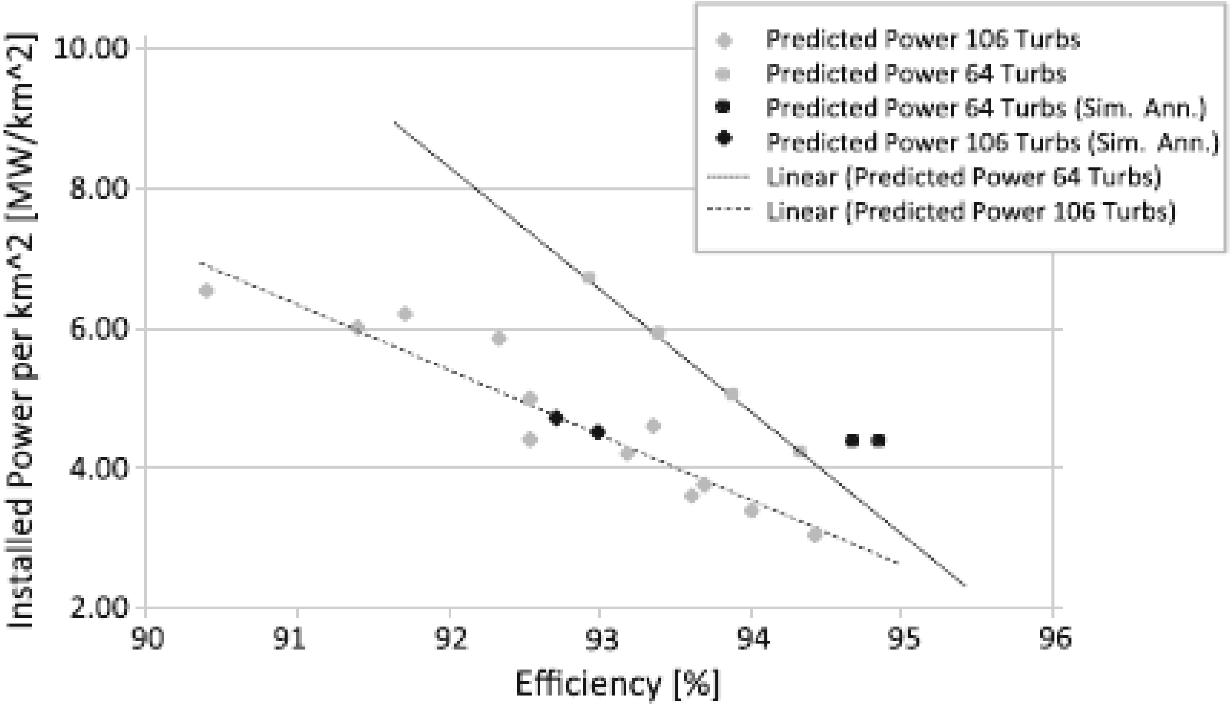


Figure 7: Impact of site area and number of wind turbines on wind farm efficiency.

- The light dots represent the results obtained by the rule of thumb pattern.
    - For this, as the power density decreases (site area increases), the efficiency of the wind farm increases. **Why?**
    - Ans: Placing wind turbines further apart reduced wake effect.
- The dark dots in the figure represent the results obtained by seeking an optimum pattern.
    - The optimization process **improved the efficiency** for the case with the **smaller number**, 64, 5 MW turbines (black-filled circles).

  - There was **no improvement** with the **larger number** of turbines (106, 3 MW).

- Thus the potential improvement over the rule of thumb pattern is more evident if the turbines are fewer and larger.
- This may be a product of the optimization method which clearly is more complex as the number of wind turbines increases.