1 Wind Regimes

- The proper design of a wind turbine for a site requires an accurate characterization of the wind at the site where it will operate.
- This requires an understanding of the sources of wind and of the turbulent atmospheric boundary layer.
- Wind speeds are characterized by their velocity distribution over time, $V(t)$.
- We will characterize this temporal variation through statistical analysis that will lead to statistical probability models.
- The wind is generated by pressure gradients resulting from non-uniform heating of the earth's surface by the sun. Approximately 2% of the total solar radiation reaching the earth's surface is converted to wind.

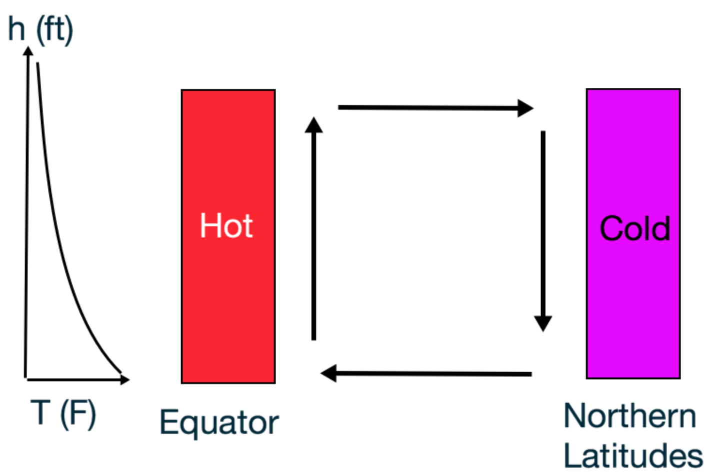

Figure 1: Mechanism of wind generation through global temperature gradients.

- The earth's rotation has an effect on the wind. In particular, it causes an acceleration of the air mass that results in a Coriolis force

$$f_c \sim [(\text{earth's angular velocity})\sin(\text{latitude})] \cdot \text{air velocity}. \quad (1)$$

- This results in a curving of the wind path as it flows from high pressure and low pressure regions (isobars).

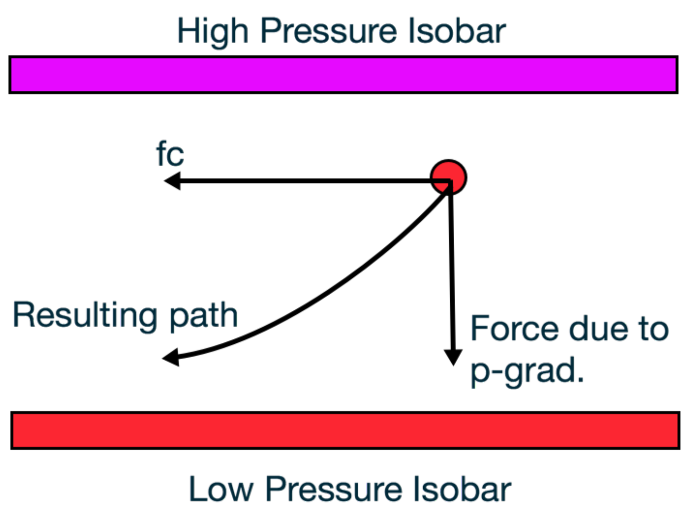

Figure 2: Effect of Coriolis force on the wind between pressure isobars.

- The Coriolis force balances the pressure gradient, leaving a resulting wind path that is parallel to the pressure isobars, referred to as the geostrophic wind.

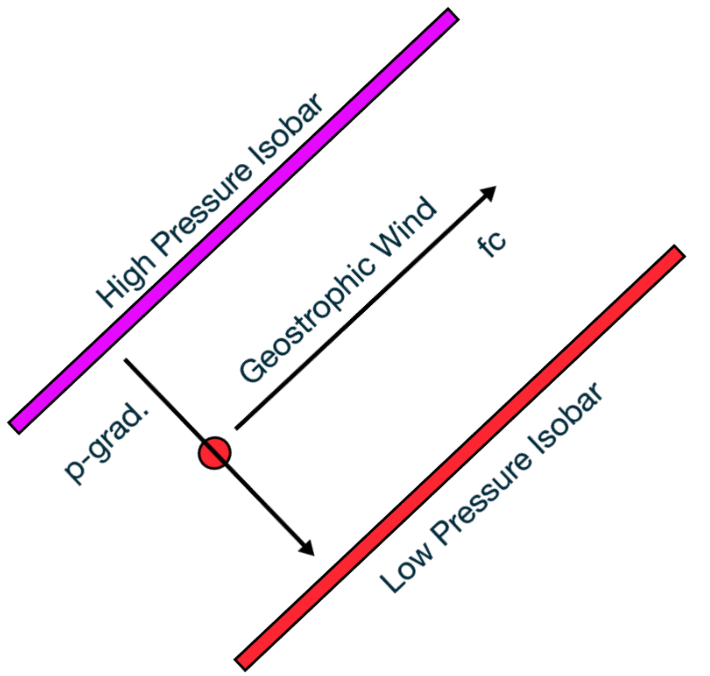

Figure 3: Schematic of geostropic wind in the Northern hemisphere that results from a steady state balance of Coriolis force and pressure isobars.

1.1 Atmospheric Boundary Layer

- The flow of air (a viscous fluid) over a surface is retarded by the frictional resistance with the surface.
- The result is a boundary layer in which the *minimum* velocity (ideally zero) is at the surface, and the *maximum* velocity (ideally $V_{geostropic}$) is at the edge of the boundary layer.
- The height or "thickness" of the boundary layer, δ, is affected by the "surface roughness".
- The surface roughness also affects the shape of the boundary layer which is defined by the change in velocity with height, $V(z)$.

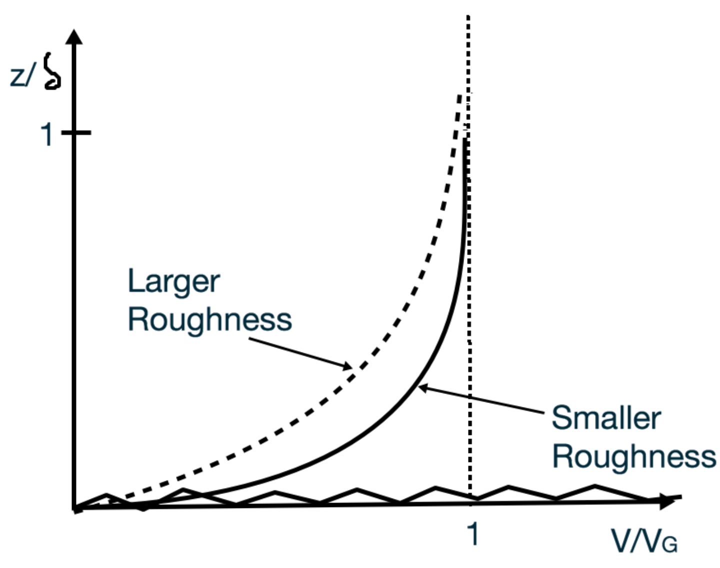

Figure 4: Schematic of atmospheric boundary layer profiles for small and large surface roughness .

- In atmospheric boundary layers, the surface roughness is represented by a category or "class".

Table 1: Classes of surface roughness for atmospheric boundary layers.

Category	Description	$\sim \delta$ (m)	z_0 (m)
1	Exposed sites in windy areas, exposed coast lines, deserts, etc.	270	0.005
2	Exposed sites in less windy areas, open inland country with hedges and buildings, less exposed coasts.	330	0.025-0.1
3	Well wooded inland country, built-up areas.	425	1-2

- Wind data is available at meteorological stations around the U.S. and the world. Most airports can also provide local wind data.
- Wind data is generally compiled at an elevation, z, of 10 meters. This is the recommendation of the World Meteorological Organization (WMO).
- A model for the atmospheric boundary layer wind velocity with elevation is

$$V(z) = V(10)\frac{\ln(z/z_0)}{\ln(10/z_0)} \tag{2}$$

where $z = 10\,\text{m.}$ is the reference height where the velocity measurement was taken, and z_0 is the roughness height at the location where the velocity measurement was taken.

- The impact of the wind speed variation with elevation on a wind turbine power generation is significant.
  - If at a site $V(10) = 7\,\text{m/s}$ and $V(40) = 9.1\,\text{m/s}$, the ratio of velocities is $V(40)/V(10) = 1.3$. However, the power generated by a wind turbine scales as V^3.
  - Therefore the ratio of power generated is $(V(40)/V(10))^3 = 2.2$!
- **In terms of sizing a wind turbine to produce a certain amount of power, knowing the wind speed at the site, at the elevation of the wind turbine rotor hub, is critically important.**

- Data may be available from a reference location at a certain elevation and roughness type that is different from the proposed wind turbine site.
- Therefore it is necessary to *project* the known wind speed conditions to those at the proposed site.
- To do this, it is assumed that there is a height in the atmospheric boundary layer above which the roughness height does not matter.
  - The literature suggests that this is above 60 m.
- Assuming the log profile of the atmospheric boundary layer, at a reference location where the wind speed and roughness height are known, the wind velocity at an elevation of 60 m. is given by

$$V(60) = V(10)\frac{\ln(60/z_{0_1})}{\ln(10/z_{0_1})}. \tag{3}$$

- At the second location, where you wish to project the wind speed at an elevation of 60 m. the velocity is

$$V(60) = V(z)\frac{\ln(60/z_{0_2})}{\ln(z/z_{0_2})}. \tag{4}$$

  where z_{0_2} is the roughness height at the second location.
- Dividing the two expressions, the velocity at any elevation at the second site, is

$$V(z) = V(10)\frac{\ln(60/z_{0_1})\ln(z/z_{0_2})}{\ln(60/z_{0_2})\ln(10/z_{0_1})} \tag{5}$$

1.2 Temporal Statistics

- The previous description of the atmospheric boundary layer was based on a *steady* (time averaged) viewpoint. Thus it refers to the mean wind and power.
- However the atmospheric boundary is turbulent so that the wind velocity and direction vary with time, $V(z,t)$.
- Time scales can be relatively short, $\mathcal{O}$1-5 seconds, diurnal (24 hour periods), or seasonal (12 month periods) which greatly affects power predictions.
- The temporal variation of the wind velocity naturally leads to the use of *statistical measures*.
- The lowest (first) order statistic is the time average (mean) that is defined as

$$V_m = \frac{1}{N} \sum_{i=1}^{N} V_i \quad \text{where} \quad V_i = V_1, V_2, V_3, \cdots, V_n \tag{6}$$

- Since the wind turbine power scales as V^3, the average power is

$$P_m \sim \frac{1}{N} \sum_{i=1}^{N} V_i^3 \neq V_m^3. \tag{7}$$

- Therefore, we use a *"power component"* time-averaged wind speed give as

$$V_{m_p} = \left[\frac{1}{N} \sum_{i=1}^{N} V_i^3 \right]^{1/3}. \tag{8}$$

Where now, $P \sim V_{m_p}^3$.

1.3 Wind Speed Probability

- Wind turbines at two different sites, with the same average wind speeds, may yield different energy output due to differences in the temporal velocity *distribution*.
  - At Site A, the wind speed is constant at 15 m/s for a 24 hour period.
  - At Site B, the wind speed 30 m/s for the first 12 hours, and 0 m/s for the last 12 hours.
- Consider a wind turbine with a rated power of 250 kW that has:
  $V_{cut-in} = 4\,\mathrm{m/s}$,
  $V_{rated} = 15\,\mathrm{m/s}$, and
  $V_{cut-out} = 25\,\mathrm{m/s}$.

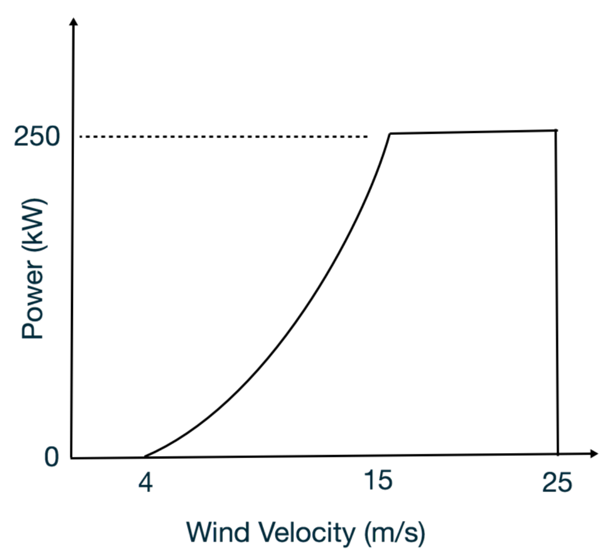

Figure 5: Hypothetical power curve for wind turbine with a rated power of 250 kW.

- Ans: At Site A, $24 \times 250 = 6000 kW - hr$. At Site B, 0 kW-hr (Why?)

- The impact that the wind speed variation can have on a wind turbine's power generation. Therefore it is important to **quantify the variation** that occurs in the wind speed over time.
- One such statistical measure is the "standard deviation" or second statistical moment which is defined as

$$\sigma_i = \left[\frac{1}{N} \sum_{i=1}^{N} (V_i - V_m)^2 \right]^{1/2} . \tag{9}$$

or

$$\sigma_i = \left[\frac{1}{N} \sum_{i=1}^{N} V_i^2 - (\frac{1}{N} \sum_{i=1}^{N} V_i)^2 \right]^{1/2} \tag{10}$$

- Wind data is most often grouped in the form of a frequency distribution

Table 2: Sample frequency distribution of monthly wind velocity

Velocity (m/s)	Hours/month	Cumulative Hours
0-1	13	13
1-2	37	50
2-3	50	100
3-4	62	162
4-5	78	240
5-6	87	327
6-7	90	417
7-8	78	495
8-9	65	560
9-10	54	614
10-11	40	654
11-12	30	684
12-13	22	706
13-14	14	720
14-15	9	729
15-16	6	735
16-17	5	740
17-18	4	744

- In the case of frequency data, the power-weighted time average is

$$V_{m_p} = \left[\frac{\Sigma_{i=1}^{N} f_i V_i^3}{\Sigma_{i=1}^{N} f_i}\right]^{1/3} \tag{11}$$

- The standard deviation is

$$\sigma_v = \left[\frac{\Sigma_{i=1}^{N} f_i \left(V_i - V_{m_p}\right)}{\Sigma_{i=1}^{N} f_i}\right]^{1/2}. \tag{12}$$

- For the frequency data in the table, $V_{m_p} = 8.34\,\mathrm{m/s}$ and $\sigma_v = 0.81\,\mathrm{m/s}$.

1.4 Statistical Models

- Statistical models of the wind *velocity frequency of occurrence* are used to predict the power generated on a yearly basis.
- Weibull and Rayleigh (k=2) distributions can be used to describe wind variations with acceptable accuracy.
- The advantage of using well known analytic distributions like these is that the probability functions are already formulated.

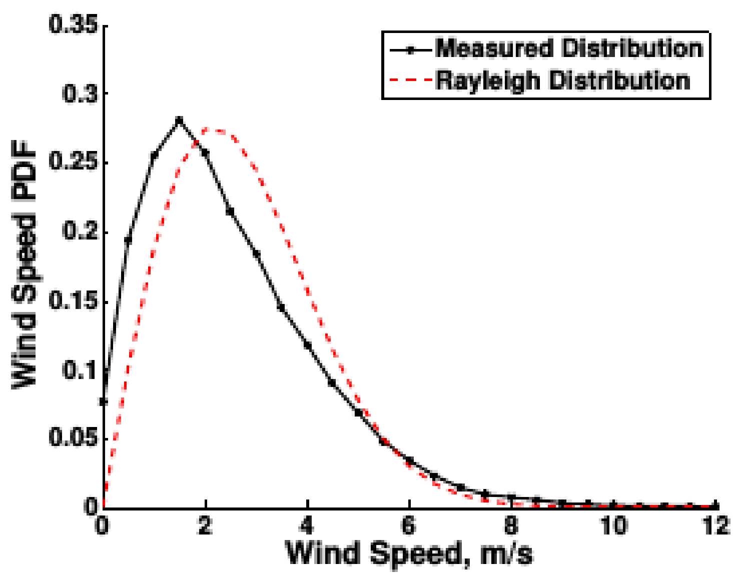

Figure 6: Probability distribution of wind speeds at the White Field wind turbine site, and a best-fit Rayleigh distribution.

1.4.1 Weibull Distribution

- In the Weibull distribution the probability in a years time of a wind speed, $V \geq V_p$, where V_p is an arbitrary wind speed is given as

$$p(V \geq V_p) = \exp\left[-(V_p/c)^k\right]. \tag{13}$$

- The number of hours in a year in which $V \geq V_p$

$$H(V \geq V_p) = (365)(24)\exp\left[-(V_p/c)^k\right]. \tag{14}$$

- The wind speed distribution density indicates the probability of the wind speed being between two values, V and $(V + \Delta V)$.

  This statistical probability is given as

$$p(V)\Delta V = \frac{k}{c}\left(\frac{V}{c}\right)^{k-1} \exp\left[-(V/c)^k\right]\Delta V. \tag{15}$$

- The statistical number of hours on a yearly basis that the wind speed will be between V and $(V + \Delta V)$ is then

$$H(V < V_p < V + \Delta V) = (365)(24)\frac{k}{c}\left(\frac{V_p}{c}\right)^{k-1} \exp\left[-(V_p/c)^k\right]\Delta V. \tag{16}$$

- c and k are Weibull coefficients that depend on the elevation and location.
- The wind frequency data can be accumulated at a particular site at the wind turbine hub-height elevation being considered and then fit to a Weibull distribution to find the best c and k.

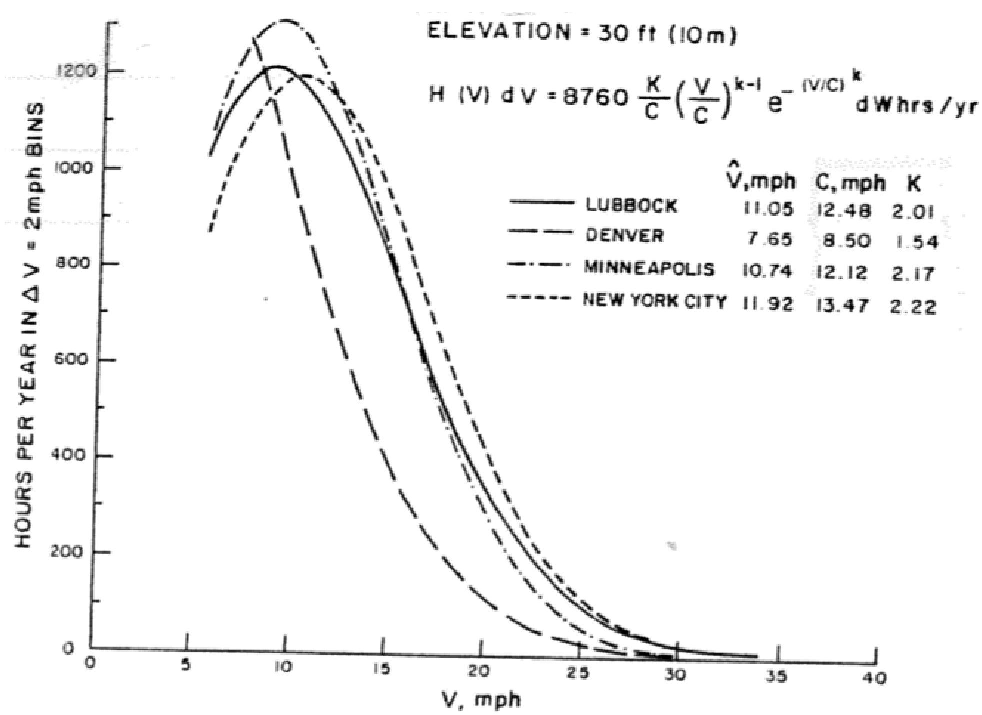

Figure 7: Sample Weibull distributions for atmospheric boundary layer data at different sites.

- Suggested corrections to Weibull coefficients k and c to account for different altitudes, z, are

$$k = k_{ref} \frac{[1 - 0.088 \ln(z_{ref}/10)]}{[1 - 0.088 \ln(z/10)]} \tag{17}$$

$$c = c_{ref} \left[\frac{z}{z_{ref}} \right]^n \tag{18}$$

$$n = \frac{[0.37 - 0.088 \ln(c_{ref})]}{[1 - 0.088 \ln(z_{ref}/10)]} \tag{19}$$

- The *cumulative distribution* is the integral of the probability density function, namely

$$\mathcal{P}(V) = \int_0^\infty p(V)dV = 1 - \exp\left[-(V/c)^k\right] \tag{20}$$

- The average wind speed is then

$$\begin{aligned} V_m &= \int_0^\infty V p(V) dV & (21)\\ &= \int_0^\infty V \frac{k}{c}\left(\frac{V}{c}\right)^{k-1} \exp\left[-(V/c)^k\right] dV & (22)\\ &= k \int_0^\infty \left(\frac{V}{c}\right)^{k} \exp\left[-(V/c)^k\right] dV. & (23)\end{aligned}$$

- Letting $x = (V/c)^k$ and $dV = (a/k)x^{(\frac{1}{k}-1)}dx$, and substituting into Equation 23,

$$V_m = c\int_0^\infty e^{-x}x^{1/k}dx \tag{24}$$

- Noting the similarity to the Gamma function

$$\Gamma_m = c\int_0^\infty e^{-x}x^{n-1}dx \tag{25}$$

- Then

$$V_m = c\Gamma(1 + \frac{1}{k}). \tag{26}$$

- Note that Gamma function calculators are readily available on the internet!

- The standard deviation of the wind speed, σ_v of the wind speeds can be written in terms of the Gamma function as well, namely

$$\sigma_V = c\left[\Gamma\left(1+\frac{2}{k}\right) - \Gamma^2\left(1+\frac{1}{k}\right)\right]^{1/2} \tag{27}$$

- Similarly, the cumulative distribution function, $\mathcal{P}(V)$, can be used to estimate the time over which the wind speed is between some interval, V_1 and V_2

$$\begin{aligned} \mathcal{P}(V_1 < V < V_2) &= p(V_2) - p(V_1) && (28) \\ &= \exp\left[-(V_1/c)^k\right] - \exp\left[-(V_2/c)^k\right]. && (29) \end{aligned}$$

- This can also be used to estimate the time over which the wind speed exceeds a value, namely

$$\begin{aligned} \mathcal{P}(V > V_x) &= 1 - \left[1 - \exp\left[-(V_x/c)^k\right]\right] && (30) \\ &= \exp\left[-(V_x/c)^k\right]. && (31) \end{aligned}$$

Example:

A wind turbine with a cut-in velocity of 4 m/s and a cut-out velocity of 25 m/s is installed at a site where the Weibull coefficients are $k = 2.4$ and $c = 9.8$ m/s. How many hours in a 24 hour period will the wind turbine generate power?

Answer:

$$\begin{aligned}
\mathcal{P}(V_4 < V < V_{25}) &= p(V_{25}) - p(V_4) && (32)\\
&= \exp\left[-(4/9.8)^{2.4}\right] - \exp\left[-(25/9.8)^{2.4}\right] && (33)\\
&= 0.890 - 7.75 \times 10^{-5} && (34)\\
&= 0.890 && (35)
\end{aligned}$$

Therefore the number of hours in a 24 hour period where the wind speed is between 4 and 25 m/s is: $H = (24)(0.89) = 21.36$ hrs.

1.4.2 Methods for Weibull model fits.

The methods for estimating the best k and c for a Weibull distribution include:

1. Graphical method,
2. Standard deviation method,
3. Moment method,
4. Maximum likelihood method, and
5. Energy pattern factor method.

Weibull Graphical Method. For a Weibull distribution, the cumulative distribution probability is

$$\mathcal{P}(V) = 1 - \exp\left[-(V/c)^k\right] \tag{36}$$

or,

$$1 - \mathcal{P}(V) = \exp\left[-(V/c)^k\right] \tag{37}$$

so that taking the natural log of both sides of the equality,

$$\underbrace{\ln\left[-\ln[1 - \mathcal{P}(V)]\right]}_{y} = \underbrace{k\ln(V_i)}_{Ax} - \underbrace{k\ln(c)}_{B}\,. \tag{38}$$

- Plot $\ln\left[-\ln[1 - \mathcal{P}(V)]\right]$ versus $\ln(V_i)$ for the velocity samples $V_i,\ i = 1, N$
- the slope of the best fit straight line represents the Weibull coefficient, k,
- the y-intercept represents $-k\ln(c)$, from which the Weibull scale factor, c can be found.
- Alternatively, one can perform a least-square curve fit of the linear function to find the slope and intercept.

- Sample set of wind velocity frequency data.
- First column: wind speeds (km/hr) at a site.
- Second column: frequency of occurrence (Hours/month)
- Third column: probability of occurrence of a given wind speed, $p(V)$.
  - equals the hours/month of a given wind speed (from column 2) divided by the total hours/month given by the sum of all the rows in column 2.
- Column four: cumulative probability, $\mathcal{P}(V)$, the running sum of $p(V)$.

Table 3: Sample wind velocity frequency distribution

V(km/h)	Hours/month	$p(V)$	$\mathcal{P}(V)$
0	1.44	0.002	0.002
2	3.60	0.005	0.007
4	5.76	0.008	0.015
6	10.08	0.014	0.029
8	18.00	0.025	0.054
10	26.64	0.037	0.091
12	34.56	0.048	0.139
14	36.72	0.051	0.190
16	41.04	0.057	0.247
18	36.72	0.051	0.298
20	49.68	0.069	0.367
22	50.40	0.07	0.437
24	52.56	0.073	0.510
26	53.28	0.074	0.584
28	51.84	0.072	0.656
30	47.52	0.066	0.722
32	41.76	0.058	0.780
34	38.88	0.054	0.834
36	29.52	0.041	0.875
38	23.76	0.033	0.908
40	20.16	0.028	0.936
42	15.12	0.021	0.957
44	12.24	0.017	0.974
46	7.92	0.011	0.985
48	5.76	0.008	0.993
50	2.88	0.004	0.997
52	1.44	0.002	0.999
54	0.72	0.001	1
56	0	0	1
58	0	0	1
60	0	0	1

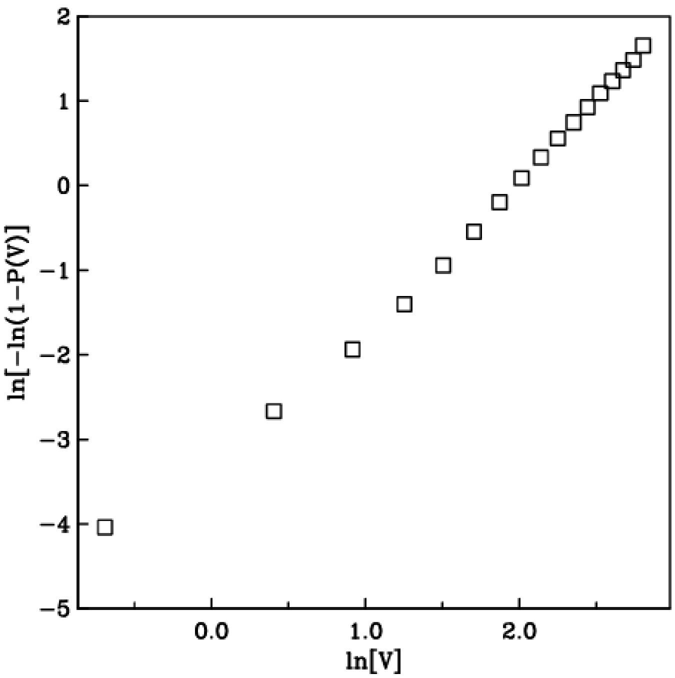

Figure 8: Weibull distributions fit for the data in Table 1.4.2. $k = 2.0$ and $c = 6.68\,\mathrm{m/s}$.

1.4.3 Rayleigh Distribution

- The Rayleigh distribution is a *special case* of the Weibull distribution in which $k = 2$.

$$V_m = c\Gamma\left(3/2\right) \tag{39}$$

or

$$c = 2\frac{V_m}{\sqrt{\pi}} \tag{40}$$

- In terms of the probability functions, substituting c into the Weibull expressions:

$$p(V) = \frac{\pi}{2}\frac{V}{V_m^2}\exp\left[-\frac{\pi}{4}\left(\frac{V}{V_m}\right)^2\right] \tag{41}$$

of which then

$$\mathcal{P}(V) = 1 - \exp\left[-\frac{\pi}{4}\left(\frac{V}{V_m}\right)^2\right] \tag{42}$$

so that

$$\mathcal{P}(V_1 < V < V_2) = \exp\left[-\frac{\pi}{4}\left(\frac{V_1}{V_m}\right)^2\right] - \exp\left[-\frac{\pi}{4}\left(\frac{V_2}{V_m}\right)^2\right] \tag{43}$$

and

$$\mathcal{P}(V > V_x) = 1 - \left[1 - \exp\left[-\frac{\pi}{4}\left(\frac{V_x}{V_m}\right)^2\right]\right] = \exp\left[-\frac{\pi}{4}\left(\frac{V_x}{V_m}\right)^2\right] \tag{44}$$

1.5 Energy Estimation of Wind Regimes

- The ultimate estimate to be made in selecting a site for a wind turbine or wind farm is the *energy* that is available in the wind at the site.
- Involves calculating the wind energy density, E_D, for a wind turbine unit rotor area and unit time.
- Wind energy density is a function of the wind speed and temporal (frequency) distribution at the site.
- Other parameters of interest are the most frequent wind velocity, $V_{F_{max}}$, and the wind velocity contributing the maximum energy, $V_{E_{max}}$, at the site.
  - $V_{F_{max}}$, corresponds to the maximum of the probability distribution, $p(V)$.

- Horizontal wind turbines are usually designed to operate most efficiently at its *design power wind speed*, V_d.
- Therefore it is advantageous if V_d and $V_{E_{max}}$ at the site are made to be as close as possible.
- Once $V_{E_{max}}$ is estimated for a site, it is then possible to match the characteristics of the wind turbine to be most efficient at that condition.

1.5.1 Weibull-based Energy Estimation Approach

- The power that is available in a wind stream of velocity V over a unit rotor area is
$$P_V = \frac{1}{2}\rho_a V^3. \tag{45}$$

- The energy per unit time is $P_V p(V)$.
- The total energy for all possible wind velocities at a site is therefore
$$E_D = \int_0^\infty P_V p(V) dV. \tag{46}$$

- For a Weibull distribution, this is
$$E_D = \frac{\rho_a k}{2c^k} \int_0^\infty V^{(k+2)} \exp\left[-(V/c)\right]^k dV. \tag{47}$$

- In terms of the Gamma function, the energy density is
$$E_D = \frac{\rho_a c^3}{2}\frac{3}{k}\Gamma\left(\frac{3}{k}\right). \tag{48}$$

- The energy that is available over a period of time, T (e.g. T=24 hrs), is
$$E_T = E_D T = \frac{\rho_a c^3 T}{2}\frac{3}{k}\Gamma\left(\frac{3}{k}\right). \tag{49}$$

- To find the most frequent wind speed, V_F, we start with the Weibull probability distribution, $p(V)$,

$$p(V) = \frac{k}{c^k} V^{k-1} \exp\left[-(V/c)^k\right]. \tag{50}$$

- The most frequent wind speed is then the maximum of the probability function, found as

$$\frac{dp(V)}{dV} = 0 \tag{51}$$

or

$$\frac{k}{c^k} \exp\left[-(V/c)^k\right] \left[-\frac{k}{c^k} V^{2(k-1} + (k-1)V^{(k-2)}\right] = 0. \tag{52}$$

- Solving for V gives

$$V_{F_{max}} = c \left(\frac{k-1}{k}\right)^{1/k}. \tag{53}$$

- The wind velocity that results in the *maximum energy*, is shown in the text book to be

$$V_{E_{max}} = \frac{c(k+2)^{1/k}}{k^{1/k}} \tag{54}$$

1.5.2 Rayleigh-based Energy Estimation Approach

- For a Rayleigh wind speed distribution, the wind energy density is

$$E_D = \frac{3}{\pi}\rho_a V_m^3. \tag{55}$$

- The energy available for a unit rotor area over a period of time, T, is then

$$E_T = TE_D = \frac{3}{\pi}T\rho_a V_m^3. \tag{56}$$

- For a Rayleigh wind speed distribution, the most frequent wind speed is

$$V_{F_{max}} = \frac{1}{\sqrt{2K}} = \sqrt{\frac{2}{\pi}}V_m. \tag{57}$$

- For a Rayleigh wind speed distribution,the velocity that maximizes the energy is

$$V_{E_{max}} = \sqrt{\frac{2}{K}} = 2\sqrt{\frac{2}{\pi}}V_m. \tag{58}$$

Example:

The following monthly wind velocity data (m/s) at a location is given in the following table. From this, calculate the wind energy density, E_D, the monthly energy availability, E_T, the most frequent wind velocity, $V_{F_{max}}$, and the velocity corresponding to the maximum energy, V_E, based on a Rayleigh velocity distribution.

Table 4: Monthly average wind speed data.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
9.14	8.3	7.38	7.29	10.1	11.1	11.4	11.1	10.3	7.11	6.74	8.58

Answer:

Month	E_D (kW/m^2)	E_T (kW/m^2/month)	$V_{F_{max}}$ (m/s)	V_E (m/s)
Jan	0.90	666.95	7.29	14.58
Feb	0.67	451.11	6.62	13.24
Mar	0.47	351.09	5.89	11.77
Apr	0.46	327.49	5.82	11.63
May	1.20	889.30	8.03	16.05
Jun	1.59	1146.72	8.83	17.66
Jul	1.76	1307.78	9.13	18.25
Aug	1.59	1184.94	8.83	17.66
Sep	1.29	931.78	8.24	16.48
Oct	0.42	313.95	5.67	11.34
Nov	0.36	258.82	5.38	10.75
Dec	0.74	551.72	6.84	13.69

Note that the wind velocity where the energy is a maximum varies from month to month. This makes it difficult to design a wind turbine that is optimum for all wind conditions at a site.

2 Wind Condition Measurements

- **Cup Anemometer.** Invented in 1846 by John Thomas Romney Robinson.

  Independent of wind direction.

  Temporal Response?

Figure 9: Example of a cup anemometer and wind direction indicator.

- **Propeller Anemometer.** Provides wind speed and direction.

  Temporal Response?

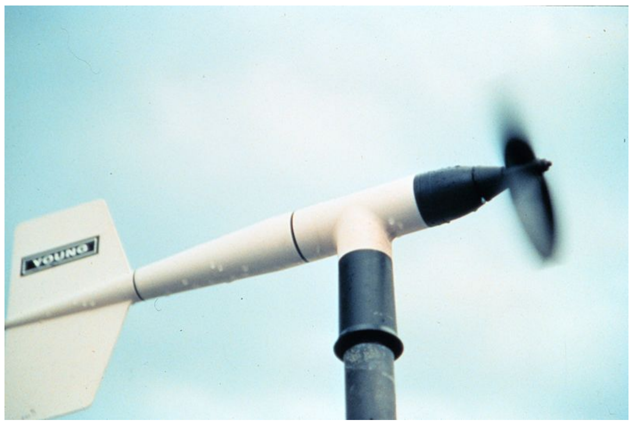

Figure 10: Example of a propeller anemometer that is designed to point into the wind.

- **Pitot-static Pressure Anemometers.** Invented by Henri Pitot in 1732 and modified to modern form in 1858 by Henry Darcy.

  No moving parts.

  Temporal response.

  Fouling and icing issues.

$$p_t = p_s + \left(\frac{\rho u^2}{2}\right) \tag{59}$$

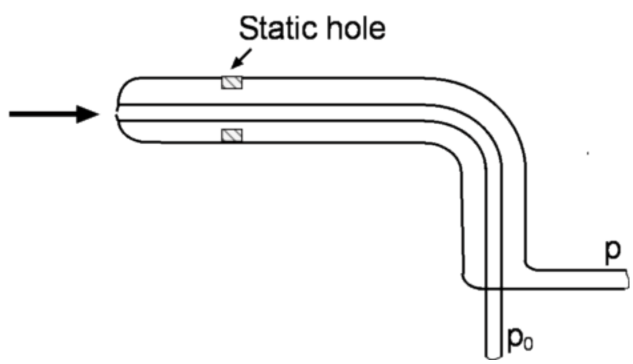

Figure 11: Schematic drawing of a Pitot-static probe anemometer.

- **Sonic Anemometers.** Use ultrasonic sound waves to measure wind velocity.

  First developed in the 1950s.

  Based on the time of flight of sonic pulses between pairs of transducers.

  Can be combined to provide multiple wind speed components.

Figure 12: Photograph of a three-component sonic anemometer.