Monetary Policy with a Touch of Basel

Ralph Chami and Thomas F. Cosimano
The typical portrait of monetary policy has the banks and the money supply being manipulated through changes in bank reserves. However, with only a small portion of bank deposits now subject to reserve requirements, an alternative explanation of how monetary policy influences banks is needed. Over the last decade, capital requirements have effectively replaced reserve requirements as the main constraint on the behavior of banks. This paper explores the implications of Basel capital requirements for monetary policy. In particular, we identify a “bank balance-sheet channel” of monetary policy, which operates through the impact on the money stock and the economy.

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I. INTRODUCTION

The typical portrait of monetary policy has the banks and the money supply being manipulated through changes in bank reserves. However, with only a small portion of bank deposits now subject to reserve requirements, an alternative explanation of how monetary policy influences banks is needed. Over the last decade capital requirements have effectively replaced reserve requirements as the main constraint on the behavior of banks. This paper explores the implications of risk-based capital requirements, à la Basel, for monetary policy. In particular, we analyze the “bank balance-sheet channel” of monetary policy, and highlight its effect on the bank capital, the money stock and the economy, when banks are subject to capital requirements similar to those adopted under the Basel Accord.

In the face of competition between commercial banks and other financial institutions, the Federal Reserve has progressively limited the scope of reserve requirements on bank deposits over the last decade. Starting in December 1990, the reserve requirements were removed from non-personal time accounts. In April 1992, the reserve requirement was reduced from 12% to 10% on transaction accounts. Moreover, by 1995, the commercial banks had significantly increased their use of deposit sweeping software. Under a sweep account the bank optimally removes funds from transaction accounts and places them in non-transaction accounts. This action effectively removes or diminishes the reserve requirement on these sweep accounts, since the majority of the time, funds are in accounts without reserve requirements. The impact of these changes may be seen by looking at reserve balances at Federal Reserve Banks and required reserves. Reserve balances, $33.3 Billion in November 1990, fell to only $7.1 Billion in November 2000, while required reserves fell from $61.1 Billion to $37.6 Billion over the same period. Thus, reserve requirements no longer apply to a significant portion of the money supply in the United States. Sellon and Weiner (1996) report a similar decline in reserve requirements in Germany, France, and Japan. In addition, there are no reserve requirements in Belgium, Canada, Denmark, New Zealand, Sweden, or the United Kingdom.

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2 See Rasche and Johannes (1987) for a comprehensive discussion of money multipliers.

3 See Cosimano and MacDonald (1998). Personal saving deposits already had a zero reserve requirement because of the Monetary Control Act, See Anderson and Rasche (2001).

4 See Federal Reserve (2000).


6 Anderson and Rasche (1996) discuss the increased reliance on vault cash to meet reserve requirements. Anderson and Rasche (2001, p. 71) find “...the willingness of regulators to permit use of deposit-sweeping software has made statutory reserve requirements a ‘voluntary constraint’ for most banks.”
Over the same time period, the Basel Accord, originally developed for the G-10 countries, was gradually adopted by a large percentage of countries in the world.\textsuperscript{7} A growing body of research has documented the implications of the new risk-based capital requirements for the banks’ behavior, in particular for asset portfolio allocation and lending behavior.\textsuperscript{8} While some of the implications were clearly intended by the designers of the Basel accord, there is now consensus on the unintended effects of the capital requirements on the banks’ behavior.\textsuperscript{9} This has given rise to a debate on the optimal design of bank regulations and supervision measures that rely on capital as a sufficient statistic, but which recognize the incentive effects of risk-based capital requirements on bank behavior.\textsuperscript{10} With the push towards global harmonization of bank regulations and the increased reliance on capital adequacy ratios to control bank behavior, the question now becomes, what are the implications for the conduct of monetary policy?

Bernanke and Lown (1991), in analyzing the reasons for 1990 credit crunch in the US, recognize that it is a “capital crunch,” and highlight its role in the subsequent recession and its implications for the conduct of monetary policy.\textsuperscript{11} Moreover, there are several papers that have analyzed the impact of monetary policy on banks with capital constrains, but with differing conclusions. Whether monetary policy affects bank lending or not seems to depend on the assumption that bank loans are financed by reservable deposits (See Bernanke and Blinder (1988)), or on the imperfect elasticity of the supply of nonreservable deposits (See Kashyap and Stein (1994)).\textsuperscript{12} For example, Labadie (1994), using an overlapping generations framework, shows that the addition of capital constraints on banks has no real effect. This result hinges on the assumption that banks can costlessly raise equity or external funds. On the other hand, Kopecky and VanHoose (1999), in a deterministic model, assume an increasing marginal cost of equity in a competitive banking industry. They distinguish between two cases, a short-term case where the bank equity is held constant and the capital

\textsuperscript{7} See Jackson \textit{et. al.} (1999) and Basel Capital Accord (1998) for complete details.

\textsuperscript{8} In fact, most banks focus on capital as the binding constraint in deciding whether or not to issue a loan. A bank identifies the risk adjusted rate of return on a particular loan. This rate of return must exceed the return on capital for the loan to be funded.


\textsuperscript{10} See the proposal advanced by the U.S. Shadow Financial Regulatory Committee (2000), and Levonian (2000) for a critique of that proposal.

\textsuperscript{11} They conclude, however, that perhaps the balance sheet channel, more than the bank lending channel, was the major contributor to the ensuing recession.

\textsuperscript{12} See also Stein (1998).
constraint is binding, and a long-term case, where the bank has the ability to change equity in the long-term. Monetary policy in their framework will have real effects. Thakor (1996), using an asymmetric information model of bank lending, but maintaining the assumption of costly external funds, shows that monetary policy will impact bank lending. Recently, Van Der den Heuvel (2001), using a dynamic model of banking, analyzes the role of “bank capital channel” in the transmission of monetary policy. Banks in his model fail to fully hedge interest rate risk. He shows through simulations that the resulting interest rate mismatch implies that monetary policy will impact the supply of loans through its impact on the value of bank capital.

In this paper, we exploit the existence of an imperfectly competitive banking industry to show that monetary policy will influence a bank’s capital and, as a result, a bank’s lending behavior and the economy.\(^{13}\) We develop a dynamic model in which risk neutral banks are assumed to maximize the present value of all future profits, subject to a total capital constraint in an imperfectly-competitive banking industry. This industry structure implies that the optimal net interest margin on loans is usually above the marginal resource cost of deposits and loans. We show that in anticipation of the possibility that the total capital constraint binds, the bank will choose an optimal level of dividends and, hence, total capital this quarter to minimize the possibility of the total capital constraint binding next quarter. Thus, banks in this world find it optimal to hold capital above the regulatory level.

Monetary policy, in this model, impacts the supply of loans, by affecting the option value of holding bank capital, and the bank’s equity decision. For example, a tight monetary policy, which raises the federal funds rate, will induce the bank to raise its current and future deposit rate. In the presence of an imperfectly-competitive loan market, the bank will also reduce the net interest rate margin between loan and deposit rates. We show that the resulting impact on the loan supply is asymmetric, depending on whether the total capital constraint is binding or not. Moreover, the persistent increase in the deposit rate will raise the marginal cost of funding, which, all else being equal, reduces the supply of future loans and the probability that the capital constraint will bind during the next quarter. This reduces the option value of holding more capital. As a result, the bank will hold less equity this quarter and the constraint on loans issued next quarter becomes more restrictive. Thus, contractionary monetary policy, through the decline in the bank’s total capital, leads to a decline in loans in the next quarter. By affecting the bank capital, monetary policy impacts the capacity of banks to lend. This gives rise to a “bank capital financial accelerator” in our model, which is distinct from the well known financial accelerator discussed in the literature; the latter arises due to the impact of the monetary policy on the balance sheet of borrowers, and, consequently, on the demand.

\(^{13}\) The existence of market power in the banking sector accords with empirical evidence discussed later, and with the assumption in Bernanke and Blinder (1988), with the proponents of the “lending channel” view such as Kashyap and Stein (1994, 1997), and with Stein (1998), among others. The assumption made in these papers implies that borrowers are bank dependent.
for loans.  Nevertheless, the two effects, one arising from the supply side of loans and the other from the demand side for loans, together amplify the impact of monetary policy on the economy.

Capital regulations are shown to introduce asymmetries in the impact of monetary policy on the supply and cost of loans. For example, when the capital constraint binds and there is an easing of monetary policy, the resulting drop in the marginal cost of loans will not generate the desired increase in the quantity of loans or a drop in the loan rates. In fact, in this case, the lower marginal cost of lending only results in lower deposit rates and higher profits for the capital constrained banks. On the other hand, a rise in the marginal cost of lending, due to a tightening of monetary policy, will generate the desired quantity and price effects. This asymmetry is also present in the impact of demand for loan shocks on the initial supply of loans. When the capital constraint is binding, a sudden rise in the demand for loans (perhaps generated by a booming economy) would only translate into higher loan rates, as banks are prevented from extending more loans due to the presence of the capital constraint. On the other hand, a sudden drop in the demand for loans will result in lower rates and a lower supply of loans.

The rest of the paper is organized as follows: the next section provides a survey of the relevant literature. Section 3 demonstrates that under the Basel Accord, the total capital constraint is the relevant constraint for bank behavior. This section also shows that this constraint is related to the bank’s balance sheet and to the market value of its capital. Section 4 introduces the model of bank behavior. Section 5 provides the dynamics of the bank game, and demonstrates the existence of a trigger strategy equilibrium. Section 6 analyzes the impact of monetary policy under the Basel Accord, and Section 7 concludes.

II. Literature

In our framework, the presence of imperfect competition in the banking industry is important for the transmission of monetary policy through the “bank balance-sheet channel.” This assumption is also present in Van Den Heuvel (2001), but in contrast, his model relies on the inability or unwillingness of the banks to hedge interest rate risk, showing that monetary policy, through its impact on bank capital, affects the banks’ ability to supply loans. 15 The bank-dependence of some borrowers is also present in earlier work by Bernanke and Blinder

14 See Bernanke, Gertler and Gilchrist (1998), and Bernanke and Gertler (1995), among others, for a discussion of the impact of monetary policy on the creditworthiness of borrowers, and the resulting impact on loan demand.

15 Froot and Stein (1998), however, argue that it may be optimal for a bank to hedge such interest rate risk using swaps and other financial derivatives. Moreover, a large percentage of commercial loans are repriced within 30 days as a mark up over some base rate (see the Survey of Terms of Business Lending, Release E2, February 2001).
(1988) and Kashyap and Stein (1995, 1997), among others.\textsuperscript{16} Going beyond a two asset economy, these papers recognize that assets other than bonds and money are important in explaining the transmission of monetary policy. The presence of a demand for loans means that monetary policy impacts the spread between loan rates and the treasury rate, giving rise to an additional mechanism through which monetary policy can impact the economy. They refer to this mechanism as the “lending view” of monetary policy. However, in contrast to our work, the monetary transmission mechanism in these papers operates through bank reserves, rather than through the bank capital channel.

Bernanke and Blinder (1988, 1992) and Kashyap and Stein (1994, 1995) point out that another necessary condition for the lending view of monetary policy is an increasing marginal cost of raising external funds.\textsuperscript{17} This assumption, of imperfect elasticity of the supply of nonreservable deposits, however, is essential for generating a supply of loan effect when the banking industry is competitive. A constant marginal cost of loans in a competitive banking industry, in contrast, implies that there is no room for the demand for loans to influence the net interest margin between loans and deposits. In addition, there is no reason to think that monetary policy, in this case, would influence the marginal resource cost of loans and the net interest margin.\textsuperscript{18}

In this paper, increasing marginal cost of external financing concept is introduced into the banking model by using Freixas’s (1970) analysis of reserve management.\textsuperscript{19} This increasing marginal cost precludes a perfectly competitive market for external financing in which the spread between the treasury and deposit rate is equal to the constant marginal resource cost of deposits.\textsuperscript{20} In this case monetary policy can impact the spread between the treasury and

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\textsuperscript{17} Stein (1998) provides a banking model in which increasing cost of raising external funds arises due to the presence of asymmetric information. See also Romer and Romer (1990) for a critique.

\textsuperscript{18} Cosimano (1987, 1988) finds a short run impact of monetary policy in a competitive industry, due to the presence of adjustment costs.

\textsuperscript{19} See Freixas and Rochet (1997, pp. 228-229) for a recent exposition of this argument.

\textsuperscript{20} Berger (2000) surveys the literature dealing with economies of scale or scope in banking. He finds mild evidence of efficiency gains arising from revenue enhancements because of risk diversification across products and geography. Stiroh (2000) also finds some evidence of minor economies of scale for U.S. bank holding companies in the 1990s.
deposit rate, which results in banks substituting treasury securities for loans under a contractionary monetary policy.\(^{21}\)

It is still possible, however, to have a "bank-balance sheet channel" of monetary policy, even when the assumption of an increasing marginal cost of loans is relaxed. Market power in the banking industry—an assumption that we maintain in the paper—implies a markup of the loan rate over the marginal cost of loans. Then it is possible for the spread between the loan and deposit rates to fall when the treasury rate increases, even when the marginal cost of external financing is constant. Here the marginal cost of loans includes the marginal resource cost of both deposits and loans. This argument implies that the monetary policy would still impact bank capital, whenever there is anti-competitive pricing within the banking industry.

There is ample evidence on the degree of concentration within the banking industry in various countries. Berger, Demsetz and Strahan (1999) find that the eight firm concentration ratio in the United States increased from 22% in 1988 to 35% in 1997.\(^{22}\) Dages, Goldberg, and Kinney (2000) find a 70% eight firm concentration ratio in Mexico and 80% for Argentina at the end of 1998.\(^{23}\) Allen and Gale (2000, chapter 3) survey the financial systems in developed nations in the 1990s. For example, they find that there are three major universal banks in Germany and the big four clearing banks in The United Kingdom. Concentration within the banking industry, however, does not necessarily mean that the loan rate is a markup over the marginal costs of loans.\(^{24}\) However, there is an abundance of evidence of anti-competitive pricing within the banking industry around the world. De Bandt and Davis (2000) use a test of contestability to find anti-competitive behavior in France, Germany, Italy and to a lesser extent in the United States over the period 1992-1996. Humphrey and Pulley (1997) find that large commercial banks in the United States set rates to maximize profits.

There are also several papers which provide indirect evidence of anti-competitive pricing in the banking industry. First, there is the literature documenting the slow adjustment of retail


\(^{22}\) Also see Stiroh (2000) and Stiroh and Poole (2000) for a discussion of bank holding company concentration in the 1990s.

\(^{23}\) Jaffee and Levonian (2000) examine the relation between developed and developing countries banking systems. Cetorelli and Gambera (2001) report that the 3 bank concentration ratio for 42 countries during 1989-1997 has a mean of 54% with a standard deviation of 18%.

\(^{24}\) See Tirole (1990, 221-223) for a discussion.
deposit rates. Second, Berger, Bonime, Covitz, and Hancock (2000) show that publicly traded banks tended to have persistent profits between 1970 and 1997 in the United States. This evidence is inconsistent with free entry into the industry, in that abnormal profits should lead to entry into the industry until excess profits disappears. Finally, Cosimano and McDonald (1998) find that publicly traded banks in the United States respond to an unanticipated decline in the marginal cost of loans with an increase in the market value of equity, which exceeds the magnitude of the present value of the decrease in cost. They show that this would occur when the loan rate is priced above the marginal cost of loans.

In this paper, we do not portend to provide a microfoundation for why banks exist; rather, we make use of some of the results that are well established in the literature. There is now a substantial body of literature, which has developed over the past twenty years, on why banks exist and how they operate. A bank, at a fundamental level, is an intermediary that matches up savers and investors due to some inefficiency within the economy. The banking literature has focused on the role of asymmetric information in giving rise to such an inefficiency. While some papers assume that borrowers possess private information or undertake some hidden action, others attribute that to the intermediary. For example, Diamond and Dybvig (1983) highlight the provision of liquidity to depositors, while Diamond (1984) and Williamson (1986) feature the delegation of monitoring of borrowers to the bank, by the information-constrained depositors. Gordon and Pennacchi (1990) stress the important role of a bank when one group of investors possesses private information. On the other hand, Calomiris and Kahn (1991) emphasize how demandable debt provides an incentive to control the bank’s behavior when banks possess private information. More recently, Stein (1998), Froot and Stein (1998), Kashyap, Rajan and Stein (1999) and Diamond and Rajan (1999, 2000) integrate these ideas to explain why liquid deposits, illiquid lending and bank capital all coexist within the banking organization. Finally, Allen and Gale (2000) and Boot and Thakor (1997a, 1997b, 2000) analyze the interaction between banks and other types of financial institutions such as investment banks.

While the role of asymmetric information is not explicitly modeled in this paper, however, some of the assumptions made accord with the presence of informational problems. For example, the presence of market power in the banking industry and the bank-dependence of some borrowers and projects—an assumption we maintain in the paper—can be attributed to

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26 Freixas and Rochet (1997) and Santos (2000a) provide surveys of the literature.


28 See Freixas and Rochet (1997) for a good review of this literature.
the private information that banks possess with regard to the quality of the loans. Moreover, costly equity and the fact that banks, in this paper, maintain an endogenously determined, positive level of equity, can also be linked to the role of internal equity as a signaling device, when informational asymmetries exist. Thus, the present framework can accommodate imperfect information assumptions, be it on the borrower’s side or the lender’s.

The incentive problems that arise in banking due to the presence of asymmetric information have also affected the recent approaches to prudential regulation and supervision of banks. The focus on bank capital as the solution to regulating bank behavior has been advocated by academics as well as policy makers, and is certainly at the core of the Basel Accord. The emphasis on capital as the sufficient statistic is meant to deal with as well as control bank behavior. These issues gain more urgency in the presence of deposit insurance, and in the absence of effective supervision by depositors. In this context, Dewatripont and Tirole (1993), using a corporate governance approach, argue that minimum capital requirements provide a trigger device to determine when the regulator must intervene to control the behavior of the bank. Koehn and Santomero (1980), on the other hand, highlight the incentive for banks to shift to riskier investments, due to the presence of costly capital requirements. More recently, Froot and Stein (1998) show that equity capital may induce banks to take on more risk, since it effectively reduces the banks’ degree of risk aversion.

We show that higher capital requirements, à la Basel, is likely to increase the option value of holding equity, and with it, loan supply. However, it may also increase the incentives for risky behavior on the part of the bank, since the value of the call option rises. Moreover, greater uncertainty regarding the demand for loans will raise the possibility that the capital

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29 See Sharpe (1990) and Rajan (1992), among others.

30 See, for example, Schneider (1998), Bernanke and Gertler (1987), Holmstrom and Tirole (1997) and Kashyap and Stein (2000).

31 An important concern also centers around the tax implications of bank bailouts, and the need for capital requirements to lessen the tax burden. On the role of bank capital regulation, see Berger, Herring, and Szego (1995) and Santos (2000b). See Jackson et. al. (1999) for related empirical literature.

32 Merton (1977) shows that deposit insurance provides a put option for the bank, in which the strike price is the deposits. See also Levonian (2000) for a more recent treatment using contingent-claim approach.

33 See also Hellman, Murdock, and Stiglitz (1998).

34 See also Jones (2000) for a good discussion of the dynamics of capital regulatory arbitrage.
constraint will bind in the future, which increases the option value of holding more capital for the bank, so as to avoid being constrained.

Before proceeding further with the analysis of bank behavior under the Basel capital requirements, we will first highlight the main features of the Basel capital requirements.

III. The Basel Accord and the Financial Market

The Basel Accord defines both capital and risk adjusted assets. Capital has two components: Tier 1, in principal, is the book value of bank capital defined as the difference between the accounting value of assets and liabilities;\(^\text{35}\) Tier 2 basically consists of preferred stock and subordinated debt. Total capital is the sum of Tier 1 and Tier 2 capital. In this paper Tier 1 capital is represented by the previous period’s market value of the bank equity, \(q_{t-1}s_t\), where \(q_{t-1}\) is the market price of the bank’s common stock and \(s_t\) is the number of shares. This assumption is meant to capture the notion of book value of equity, but it also recognizes the high correlation between book value and the market value of equity.\(^\text{36}\) One period bonds represent Tier 2 capital, \(b_t\), issued in the previous quarter.\(^\text{37}\) These bonds promise to pay an interest rate, \(r_t^b\). In order to maintain the book value feature of regulatory capital and its sensitivity to market valuation, we assume that last quarter’s market value of equity and bonds determines the capital constraint facing the bank this quarter.\(^\text{38}\)

Risk adjusted assets are defined by placing each balance sheet and off-balance sheet item into a risk category. The more risky assets are assigned a larger weight. Category 1 is mainly cash and treasury securities and has zero weight. Category 2 is more risky marketable securities and has a weight of 20%. Category 3 is essentially loans connected with home mortgages and has a weight of 50%. Category 4 is given a weight of 100% and consists of loans to private

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\(^\text{35}\) See Koch and MacDonald (2000 chapter 13) for complete specification.

\(^\text{36}\) Kane and Unal (1990) estimate the correlation between the book value and the market value of bank capital to be around 95%. Moreover, the recent push in the accounting profession toward the market valuation of bank assets and liabilities further erodes the perceived difference between the accounting and the economic value of bank capital.

\(^\text{37}\) See Evanoff and Wall (2000) for a discussion of the recent proposals, which argue for greater reliance on subordinated debt as part of regulatory capital. Usually subordinated debt has a longer term to maturity than one quarter; however, this does not materially affect the analysis, so the shorter term to maturity is used to keep the notation simple.

\(^\text{38}\) See Dewatripont and Tirole (1993, chapter 9) for a discussion of how historical cost accounting introduces a measurement error into the monitoring of banks.
agents. As a result, risk adjusted assets are a weighted average of the various assets of the banks. It is assumed that there are two assets in the banking model: loans, \( L_t \), and treasury securities, \( T_t \). Consequently, the risk adjusted assets in the model constitute the amount of loans issued by the bank.

The Basel Accord has three constraints on the capital required by a bank. The Tier 1 constraint requires that equity capital be no less than 4% of total risk adjusted assets. Let \( \theta_1 \) be the Tier 1 ratio of equity to risk adjusted assets. The Tier 1 capital constraint may be stated as

\[
\theta_1 L_t \leq q_{t-1} s_t. \tag{1}
\]

This constraint is represented by point A in Figure 1.

The total capital constraint requires that the sum of Tier 1 and Tier 2 capital be no less than 8% of risk adjusted assets. Let \( \theta_2 \) be the ratio of total capital to loans so that the total capital constraint is

\[
\theta_2 L_2 \leq q_{t-1} s_t + b_t. \tag{2}
\]

Point B portrays this constraint in Figure 1.

The final constraint is that Tier 1 capital must exceed Tier 2 capital.

\[ b_t \leq q_{t-1} s_t. \]

Combining this constraint with (2) implies that

\[
\theta_2 L_2 \leq 2q_{t-1} s_t.
\]

This constraint is given by point C in Figure 1.

It is clear from Figure 1 that the total capital constraint is the constraint that binds first. It turns out that the total capital constraint is binding as long as the total capital ratio is equal to twice the Tier 1 ratio. Examination of Figure 1 also reveals that bonds would have to

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39 The off balance sheet items are placed in similar types of categories. See Koch and MacDonald (2000, chapter 13).

40 The proof is in the appendix.
significantly exceed the amount of equity before the Tier 1 capital constraint becomes more binding than the total capital constraint.\footnote{Thus, only the total capital constraint (2) needs to be considered in the analysis.}

To establish the relation between bank profitability and the Basel Accord, a model of bank financing is introduced following Altuğ and Labadie (1994, pp. 165-168).\footnote{Labadie (1995) applies this analysis to a financial intermediary which receives a benefit from deposit insurance.} For now let profits of the bank, $\pi_t$, be given. The profits are used to meet the bank’s obligations. These obligations include dividend payments on equity, $d_t s_t$, and interest and principal payments on bonds, $(1 + r_t^b) b_t$. The retained earnings, after these payments are made, are given by the following condition

$$re_t = \pi_t - d_t s_t - (1 + r_t^b) b_t.$$  

The bank must invest in plant and equipment each period, the total plant and equipment of the bank being $k_t$. For simplicity the bank is assumed to maintain the same premises each period, so that the bank must finance only depreciation of its building and equipment, $\delta k_t$. These improvements in building and equipment are financed through retained earnings, new equity or new bonds, so that

$$\delta k_t = re_t + q_t[s_{t+1} - s_t] + b_{t+1}.$$  

If the expression for retained earnings is substituted into this condition, the net cash flow generated by the bank for an investor is given by

$$n_t = \pi_t - \delta k_t = d_t s_t + (1 + r_t^b)b_t - q_t[s_{t+1} - s_t] - b_{t+1}.$$  

The value of the bank to an investor is equal to the net cash flow plus the ex dividend value of the bank.

$$v_t = n_t + q_t s_{t+1} + b_{t+1}. \tag{3}$$  

The investor values the payments due in one quarter based on a stochastic discount factor, $m_{t+1}$, such that the value of the bank is
\[ v_t = n_t + E_t \left[ \sum_{j=1}^{\infty} m_{t,j} n_{t-j} \right], \]  

(4)

where \( m_{t,j} \) is the stochastic discount factor for payments due in \( j \) quarters.

The interplay between the Basel Accord and the valuation of the bank by the financial market can be seen by combining (3) and (4) to yield an expression, 
\[ q_t s_{t+1} + b_{t+1} = E_t \left[ \sum_{j=1}^{\infty} m_{t,j} n_{t+j} \right], \]

in which the value of the bank's total capital in this quarter is equal to the market's evaluation of the expected future cash flows of the bank. As a result, the total capital constraint on loans next quarter, (2) updated one period, is subject to the market's evaluation of the bank's expected future cash flows. Monetary policy can influence this valuation of the bank through its impact on the expected future profits of the bank. Thus, the monetary policy undertaken this quarter will influence the bank's total capital constraint next quarter and the amount of loans the bank can issue next quarter.\(^{43}\)

Before proceeding to the discussion of bank behavior in the next section, it is worth noting here that monetary policy is not the only factor that can impact the total capital constraint. There are two additional ways through which the total capital constraint next quarter can be affected. One way is that the bank could choose to issue more dividends. Following the definition of net cash flow, a decision to raise dividends this quarter would lead to lower total bank capital, for a given level of profits this quarter. A second way is that the market valuation of the bank would change when the public changes its stochastic discount factor for cash flows due in the future. For example, an increase in the risk-adjusted rate of return on investments would mean that investors would place a lower value on the expected future cash flows of the bank. As a result, the market's valuation of the bank would decline even though monetary policy did not change. In this paper, the bank will optimally choose dividends and with it a level of equity so that the first effect will be present. Since only the banking industry is modeled in this paper, we will take the consumer's discount factor as given, and leave the analysis of its role for future work.\(^{44}\)

In the next section we develop the structure of the banking industry with market power, describe the dynamics of the game, and derive results that highlight the implications of the capital adequacy requirements for bank behavior.

\(^{43}\) Greenbaum and Thakor (1995, pp. 573-575) point out that monetary policy would have no impact on the bank's total capital constraint in this period.

\(^{44}\) This second effect would probably reinforce the effect of monetary policy discussed here. Using a standard asset pricing model, it is easy to see that an increase in the risk free rate, i.e. the treasury rate, would lead to an increase in the risk adjusted rate of return on investment.
IV. MODEL

The banking industry, in this model, is characterized by oligopolistic behavior, where banks enjoy market power. We assume that an individual bank precommits to a quantity of loans through its dividend policy this quarter, which is followed by loan rate competition in the next quarter. This assumption also captures the fact that the current level of regulatory capital determines future loans. Kreps and Scheinkman's (1983) present a model of an oligopoly subject to quantity constraints, in which a firm precommits to a quantity in the first period which is followed by price competition in the next period. Davidson and Deneckere (1986), however, show that the analysis is complicated by the sharing rule for demand. Thus, the banking industry here is characterized as a super game of oligopolistic behavior, following Abreu, Pearce, and Stacchetti (1986, 1990) and Rotemberg and Saloner (1986). The basic idea is to have each bank follow a coordinated policy for setting the loan rate as long as the other banks follow the same policy. If any bank undercuts this loan rate, then it is punished with Bertrand competition, in which the net interest margin between loans and deposits is set equal to its marginal resource cost.\(^{45}\)

An individual bank operates in an industry with N banks. The N banks follow the following strategy: each of the banks charges a loan rate consistent with monopoly power. This policy would yield profits \(\pi_i\) for the industry. For simplicity, assume a symmetric game in which costs are identical for each bank. As a result, each bank earns profits \(\frac{1}{N} \pi_i\) as long as it cooperates with the other banks. The value of the bank under cooperative behavior is \(\nu^C_i\). These profits are earned by producing \(\frac{1}{N}\) of the total loans in the industry.\(^{46}\) If a bank undercuts the loan rate by a small amount, it would capture a larger share of the market. This behavior would lead to current profits, \(\pi^{i'}\). However, the other banks respond to this action by following Bertrand competition indefinitely. Moreover, an important distinction here is that, in contrast to the case of an oligopolistic market with nondepository institutions, the cheating bank will not capture the entire market, since the individual bank is subject to a total

\(^{45}\) Market power in the banking industry can also be introduced using alternative frameworks. For example, VanHoose (1985) discusses monetary policy within a banking model with Cournot competition. Keeley (1990) looks at the impact of monopoly power on the Tobin Q for a bank. Cetorelli and Peretto (2001) examine capital accumulation by firms which are financed by banks. These banks are subject to Cournot competition. The focus of their work is on the optimal degree of competition which fosters the screening of loans. Pecchenino (1998) assumes risk averse bank managers choose loans subject to the optimal behavior of the other risk averse bank managers within a static context. Stein (1998) assumes that risk neutral banks choose the loan rates subject to an increasing marginal cost of loans in a two period framework. Bagliano, Dalmazzo, and Marini (2000) also follow Rotemberg and Saloner (1986), but they do not consider capital constraints.

\(^{46}\) Each of these terms will be derived shortly.
capital constraint. Under the Bertrand competition assumption, the net interest margin between the loan and deposit rate would be equal to the marginal resource cost of loans. As a result, the economic profits of the bank would be zero, so that the penalty imposed on the bank is $E \left[ m_{i,t} v_{i+1}^C \right]$. Following Rotemberg and Saloner (1986), the bank finds it optimal to cooperate as long as $\pi_i^0 \leq \pi_i + E \left[ m_{i,t} v_{i+1}^C \right]$. We will show, later, that in the presence of a total capital constraint, the probability of cooperating is higher since the bank’s profits from undercutting the loan rate will be lower.

To establish the equilibrium for this cooperative game, first consider the behavior of a bank operating as a monopolist. This optimal behavior will subsequently be included in the model of cooperative behavior among the N banks. While the loan market is assumed to be imperfectly competitive, the deposit and treasury markets are assumed to be perfectly competitive. The bank enters the current quarter with a predetermined level of capital and chooses the loan rate, deposit level, and dividend policy. Note that the dividend policy is intertemporal because current dividends determine the amount of total capital available next quarter, which, in turn, restricts the supply of loans next period.

The bank attracts deposits, $D_i$, at a fixed marginal cost, $c_i$. This marginal resource cost represents the cost of check clearing and bookkeeping for these deposits. Deposits serve as money in the economy excluding currency, i.e., they are components of $M2$. It is assumed that the supply of deposits by individuals is still positive, even when the deposit rate, $r_i^D$, is less than the treasury rate, $r_i^T$. Because these deposits serve as money there is always the possibility of unanticipated deposit withdrawals. To guard against these withdrawals, the bank holds liquid treasury securities in addition to any required reserves, $\alpha D_i$, where $\alpha$ is the required reserve ratio. Following Frost (1970), assume that the unanticipated deposit withdrawals, $x$, comes from a probability density function, $f(x)$, which is assumed to be independent of time. If unanticipated deposit withdrawals exceed the amount of marketable treasury securities, $T_i$, then the bank has to liquidate assets at some penalty rate, $r_i^p$. The cost of deposit withdrawals is

47 This be can rationalized due to either the transaction services provided by bank deposits or due to the existence of deposit insurance. Mulligan (1997) provides estimates for the supply of deposits by firms. Mulligan and Sala-i-Martin (2000) do the same for individuals. They find that the key issue is whether or not individuals decide to use banking services. Once they do, the key variable is the product of the spread between treasury rates and deposit rates times the amount of financial assets.

48 Note that since the channel through which monetary policy in this paper works falls outside the reserve channel, the reserve requirement can be set to zero without affecting the results. The reduction or elimination of reserve requirements would mean that $\alpha$ would approach zero.
\[ C(T_i) = r_i^p \int_0^\infty [x - T_i] f(x)dx = \frac{r_i^p}{2D} [\overline{D} - T_i]^2. \]

For simplicity, assume that the unanticipated deposit withdrawals have a uniform distribution with support \([\underline{D}, \overline{D}]\) so that the cost of liquidation or additional external funding is a quadratic function of treasury securities.\(^49\)

Any funds left over after providing liquidity are lent out by the bank as one period loans, \(L_t\), at the loan rate, \(r_i^L\).\(^50\) These loans have a constant marginal cost, \(c_i\), which represents the cost of monitoring and screening loans. The bank faces a demand for loans given by

\[ L_t = l_0 - l_1 r_i^L + l_2 M_t + \varepsilon_{L,t}. \]

Here, \(M_t\) is a generic variable that represents the level of economic activity in the bank’s loan market and \(\varepsilon_{L,t}\) is a random shock to the demand for loans that has support \([L, \overline{L}]\) and is independent of the state vector \(x_t\). \(F(\cdot)\) is the cumulative distribution of the shock to loans. The elasticity of demand is \(\frac{l_1 r_i^L}{l_0}\), which would tend to \(\infty\) in a perfectly competitive environment.

The bank’s problem is to maximize the value of the bank, (4), by choosing the loan rate, deposits and treasury securities

\[ v(q_{t-1}, s_t, b_t, x_t) = \text{Max} \{ \pi_t + \lambda_t [q_{t-1} s_t + b_t - \theta_t L_t] \]

\[ - \tau_t [q_t s_{t+1} + b_{t+1}] + E_t [m_{t+1} v(q_{t+1}, s_{t+1} + b_{t+1}, x_{t+1})] \}

subject to

\[ \pi_t = r_i^L L_t + r_i^T T_t - r_i^D D_t - c_i L_t - c_D D_t - \frac{\rho^D}{2D} [\overline{D} - T_i]^2, \]

\[ L_t = l_0 - l_1 r_i^L + l_2 M_t + \varepsilon_{L,t}, \]

\[ q_t s_{t+1} + b_{t+1} = (d_t + q_t) s_t + (1 + r_i^b) b_t - \pi_t, \]

\(^49\) As in Froot and Stein (1998), Stein (1998), and Kashyap and Stein (1995), the marginal cost of external funding, \(T_i\), here, is increasing.

\(^50\) Cosimano (1987) allows for two period loans. This complication introduces a new state variable, but does not influence the qualitative results.
and

\[ T_i = (1 - \alpha) D_i - L_i + q_{t-1} s_i + b_i. \]

Here, \( \lambda_i \) is the Lagrange multiplier for the total capital constraint, \( x_i \) is a vector of exogenous stochastic variables to be defined below, and \( E_i [z] \) is the expectation of \( z \) conditional on the bank's information at time \( t \). Following Froot and Stein (1998), \( \tau_i \) is the deadweight cost of total capital (that is, equity and debt).\(^{51}\) We do not differentiate between the relative cost of raising debt versus equity, since we have shown in Section 2 that total capital is the constraint that binds.

The optimal conditions for the bank's problem (5) are

\[
\frac{\partial e_i}{\partial r_i} \left[ 1 + \tau - E_i \left\{ \int_{\tilde{r}} m_{t,i} \left( \frac{\partial V}{\partial (q_{s_{t+1}+b_{s_{t+1}}})} \right) dF \left( \theta_{L_{t+1}} \right) \right\} \right] + \lambda_i \theta_2 l_i = 0, \tag{6}
\]

\[
\frac{\partial e_i}{\partial \theta} \left[ 1 + \tau - E_i \left\{ \int_{\tilde{r}} m_{t,i} \left( \frac{\partial V}{\partial (q_{s_{t+1}-b_{s_{t+1}}})} \right) dF \left( \theta_{L_{t+1}} \right) \right\} \right] = 0,
\]

\( \theta_2 L_i \leq q_{t-1} s_i + b_i, \)

and

\[-\tau + E_i \left\{ \int_{\tilde{r}} m_{t,i} \left( \frac{\partial V}{\partial (q_{s_{t+1}+b_{s_{t+1}}})} \right) dF \left( \theta_{L_{t+1}} \right) \right\} = 0. \tag{7}\]

Substituting the optimal decision for bank dividends (7) into the optimal decisions for the loan rate and deposits reveals that this decision can be solved independent of time. Interestingly, the separation of the optimal dividend decision of the bank from the other bank's decisions implies that the behavior of the bank can be generalized to account for dynamic elements in the bank's decision without technical difficulty.\(^{52}\) This separation leads immediately to the following result\(^{53}\)

\[ 51 \text{ It is assumed here that } k_i \text{ is zero for simplicity.} \]


\[ 53 \text{ See the Appendix for proofs.} \]
Proposition 1 The optimal decisions of the bank in each quarter when the total capital constraint is not binding are

\[ r_t^* = \frac{1}{l_t} \left[ l_0 + l_2 M_t + \varepsilon_{L,t} \right] + \frac{1}{2} \left[ c_L + \frac{1}{1-\alpha} \left( r_t^D + c_D \right) \right], \]

\[ L_t = \frac{1}{2} \left[ l_0 + l_2 M_t + \varepsilon_{L,t} \right] - \frac{1}{2} \left[ c_L + \frac{1}{1-\alpha} \left( r_t^D + c_D \right) \right], \]

\[ T_t = \overline{D} + \frac{\overline{D}}{\gamma} \left[ r_t^T - \frac{1}{1-\alpha} \left( r_t^D + c_D \right) \right], \]

and

\[ D_t = \frac{1}{1-\alpha} \left[ \overline{D} + L_t - (q_{t-1}s_t + b_t) \right] + \frac{\overline{D}}{(1-\alpha)\gamma} \left[ r_t^T - \frac{1}{1-\alpha} \left( r_t^D + c_D \right) \right]. \]  

In this case the optimal profit for the bank is

\[ \pi_t = \frac{1}{l_t} \left[ l_0 + l_2 M_t + \varepsilon_{L,t} - \frac{q_{t-1}s_t + b_t}{r_t^T} \right] + \frac{1}{2} \left[ c_L + \frac{1}{1-\alpha} \left( r_t^D + c_D \right) \right] (q_{t-1}s_t + b_t) + \frac{\overline{D}}{\gamma} \left[ r_t^T - \frac{1}{1-\alpha} \left( r_t^D + c_D \right) \right] + \overline{D} \left[ r_t^T - \frac{1}{1-\alpha} \left( r_t^D + c_D \right) \right]. \]  

When the total capital constraint is binding, the bank's optimal decisions in each quarter are

\[ r_t^{*,*} = \frac{1}{l_t} \left[ l_0 + l_2 M_t + \varepsilon_{L,t} - \frac{q_{t-1}s_t + b_t}{\delta_2} \right], \]

\[ L_t = \frac{2q_{t-1}s_t + 2b_t}{\delta_2}, \]

\[ T_t = \overline{D} + \frac{\overline{D}}{\gamma} \left[ r_t^T - \frac{1}{1-\alpha} \left( r_t^D + c_D \right) \right], \]

and

\[ D_t = \frac{1}{1-\alpha} \left[ \overline{D} + L_t - (q_{t-1}s_t + b_t) \right] + \frac{\overline{D}}{(1-\alpha)\gamma} \left[ r_t^T - \frac{1}{1-\alpha} \left( r_t^D + c_D \right) \right]. \]

The profits of the bank, if the total capital constraint is binding, is given by

\[ \pi_t^* = \frac{1}{l_t} \left[ \frac{q_{t-1}s_t + b_t}{\delta_2} \right] \left[ l_0 + l_2 M_t + \varepsilon_{L,t} - \frac{q_{t-1}s_t + b_t}{r_t^T} \right] + \frac{1}{2} \left[ c_L + \frac{1}{1-\alpha} \left( r_t^D + c_D \right) \right] \left( q_{t-1}s_t + b_t \right) + \frac{\overline{D}}{\gamma} \left[ r_t^T - \frac{1}{1-\alpha} \left( r_t^D + c_D \right) \right] + \overline{D} \left[ r_t^T - \frac{1}{1-\alpha} \left( r_t^D + c_D \right) \right]. \]  

This proposition may be illustrated by looking at Figure 2. When the total capital constraint is not binding, the optimal level of loans occurs at point A, where the marginal revenue from loans is equal to the marginal cost of loans, \( c_L + \frac{1}{1-\alpha} \left( r_t^D + c_D \right) \). In this case an increase in the exogenous demand for loans, \( l_0 + l_2 M_t + \varepsilon_{L,t} \), leads to an increase in the quantity of loans and in the loan rate. Moreover, a decrease in the marginal cost of loans causes an increase in the quantity of loans and a decrease in the loan rate. In both cases the profits of the bank increase.
The presence of the capital constraint, however, introduces an asymmetry in the impact of changes in the demand for loans or in the marginal cost of loans, on the price and quantity of bank loans. When the total capital constraint binds, an increase in the demand for loans or a reduction in the marginal cost of loans will not lead to an increase in the supply of loans. At the critical level of loans in (10), any additional demand or decrease in the marginal cost of loans no longer impact the amount of loans. At this level, the loan rate is determined by the point B on the demand for loans curve in Figure 2. As a result, further increases in the demand for loans will increase the loan rate, but not the quantity of loans. This increase in the loan rate does lead to more profits, as can be seen by looking at Figure 2. However, these profits would be higher if the total capital constraint were not binding, since the capital-constrained bank is prevented from increasing loans even though marginal revenue is above the marginal cost of loans.

Given the expected demand for loans and marginal cost of loans, it is possible to solve for a critical shock to the demand for loans, $\varepsilon_{L,t}^*$, that the total capital constraint will just bind, i.e., $L_t = L_t^*$. It turns out that the marginal value of total capital, $\lambda_t = \frac{\varepsilon_t - \varepsilon_{L,t}^*}{\lambda_t^{\kappa_2}} = 2 \frac{\zeta_t - L_t^*}{\lambda_t^{\kappa_2}}$. As a result total capital is only useful to the bank when a shock to loan demand is above this critical value.

The bank views the marginal value of total capital as a call option with strike price, $\varepsilon_{L,t}^*$, as in Figure 3. The marginal payoff, when the option is in the money, is $\frac{1}{\sigma^2}$ for each additional increase in loan demand. Thus, this marginal payoff is smaller when either the degree of monopoly power falls or the total capital ratio increases. In these cases, a marginally higher total capital yields a smaller possible change in the optimal loan rate and profits as a consequence of relaxing the total capital constraint.

In setting the optimal dividends this quarter, the bank realizes that there is a chance that the total capital constraint is binding next quarter. The cost of this constraint binding is $\lambda_{t+1}^*$, which is positive only for $\varepsilon_{t+1} > \varepsilon_{t+1}^*$. The value of this call option is dependent on the degree of uncertainty in loan demand. Let $\sigma$ measure a mean preserving spread of the distribution of the shock to the demand for loans. As the next result shows, $\varepsilon_{t+1}^*$ is positively related to the current quarter's total capital, since more capital makes it less likely the bank would face a limitation on its loans next quarter. This argument leads to the following:

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$^{{54}}$ Following Ljungqvist and Sargent (2000), a mean preserving spread satisfies the conditions $\int_{L}^{\bar{L}} F_0 (\varepsilon_{L,t+1}, \sigma) d\varepsilon_{L,t+1} = 0$ and $\int_{L}^{\bar{L}} F_0 (\varepsilon_{L,t+1}, \sigma) d\varepsilon_{L,t+1} \geq 0$ for $L \leq y \leq \bar{L}$. 

---
Lemma 1 The bank's optimal total capital decision solves the condition

$$\tau = \frac{1}{1 - \alpha} E_t \left[ m_{t,1} \left( r_{t+1}^D + c_D \right) \right] + \frac{1}{\theta_2 l_2} E_t \left\{ \int_{t+1}^{\infty} m_{t,1} \left( \epsilon_{t+1} - \epsilon_{t+1}^* \right) dF(\epsilon_{t+1}, \sigma) \right\},$$

where

$$\epsilon_{t+1}^* = 2 \frac{q_t s_{t+1} + b_{t+1}}{\theta_2} - \left[ l_0 + l_2 M_{t+1} \right] + l_1 \left( c_{t+1} + \frac{1}{1 - \alpha} \left( r_{t+1}^D + c_D \right) \right).$$

In choosing total capital, the bank compares the marginal deadweight cost of total capital with the expected marginal benefit of total capital. The expected marginal benefit of additional total capital has two components. The first component arises because additional equity or bonds reduces the marginal cost of raising deposits in the next quarter. To the extent that the bank has more total capital, the bank does not have to attract as many deposits next quarter. The second marginal benefit of total capital is the expected marginal benefit from the total capital constraint being reduced in the next quarter, \( \lambda_{t+1}^* \).

In making these evaluations, the bank needs to know the stochastic behavior for economic activity and the deposit rate, which are part of the state vector, \( x_t \). Assume that they follow first order autoregressive stochastic processes

$$M_{t+1} = \rho_M M_t + \epsilon_{M,t+1},$$

and

$$r_{t+1}^D = \rho_{r_t} r_t^D + \epsilon_{r_{t+1}},$$

(12)

where \( \epsilon_{M,t+1} \) and \( \epsilon_{r_{t+1}} \) are mean zero stochastic shocks to economic activity and the deposit rate, respectively.

The optimal behavior of the bank's total capital is described by

Proposition 2 If the marginal deadweight cost of total capital is zero, then the bank would raise sufficient capital to completely circumvent the total capital constraint. Otherwise the total capital of the bank satisfies

$$q_t s_{t+1} + b_{t+1} = h \left[ \tau - \frac{1}{1 - \alpha} \left\{ E_t \left[ m_{t,1} \right] (\rho_{s_t} r_{t+1}^D + c_D) + E_t \left[ m_{t,1} \epsilon_{r_{t+1}} \right] \right\}, \right.$$  

$$l_0 + l_2 \rho_M M_t, c_{t+1} = \frac{1}{1 - \alpha} \left( \rho_{s_t} r_{t+1}^D + c_D \right) \sigma, l_1, \theta_2,$$
such that \( h_i < 0, h_1 > 0, h_2 < 0, h_3 > 0, \) and \( h_5 < 0.\) \(^{55}\) In addition, \( h_6 > 0,\) as long as \( L < 2L'.\)

Proposition 2 highlights some interesting characteristics of the option value of the bank's total capital. First, banks would tend to hold more total capital than actually required. Holding a buffer of total capital allows the banks to take advantage of a sudden increase in the desire to issue loans. This is more likely to happen during periods of unanticipated increase in the demand for loans, or when there is a decrease in the marginal cost of loans. This result corresponds with the evidence reported by Jackson et. al. (1999) for 1996. Banks in the G-10 countries had a total capital ratio of 11.2% with a standard deviation of 1.6% (the minimum value was 9.2%, while the maximum value was 13.8%).

Second, a bank holds less total capital if the expected net marginal cost of external funds goes up, \( h_1 < 0.\) This result suggests that smaller banks would on average hold less total capital. Fama and French (1996) show that small firms in general incur a higher cost of external funds vis-à-vis large firms so that the small banks would have a higher cost of raising external funds. This result helps to explain the increased concentration in banking, since banks with ready access to the financial markets would have a cost advantage in raising the required total capital.

Third, an increase in the expected demand for loans leads a bank to hold more total capital, \( h_2 > 0.\) In Figure 2 the demand for loans shifts to the right, so that the total capital constraint is more likely to be binding. As a result, the strike price of the call option falls. The bank responds to this possibility by raising its total capital, since the value of the option is higher. This result implies that during an expansion in the economy, the banks hold more total capital.

Fourth, a bank with a higher expected marginal cost of loans holds less total capital, \( h_3 < 0.\) In Figure 2 the marginal cost curve shifts up, such that the total capital constraint is less likely to be binding. As a result, the strike price of the call option increases and the option value of total capital decreases. The bank responds to this situation by holding less total capital. On the other hand, a technological innovation, which reduces the cost of operations, would induce the bank to hold more total capital.

These last two results are key for analyzing the impact of monetary policy under the Basel Accord. When the central bank raises the treasury rate, the deposit rate moves in the same direction.\(^{56}\) This will lead the bank to hold less total capital, so that the loans of the bank are

\(^{55}\) A subscript refers to the partial derivative.

\(^{56}\) For example, the deposit rate would be equal \( r_i^T (1-\alpha) - c_d,\) when the marginal cost of external financing is zero. As a result, a 1% increase in the treasury rate yields a \((1-\alpha)\)% increase in the deposit rate.
more likely to be constrained next quarter. Thus, a tightening of the monetary policy reduces the option value of holding capital and results in a reduction in the bank’s capacity to make loans. This gives rise to a “bank capital accelerator effect” that is distinct from the financial accelerator, which arises from the borrower’s side. In a macromodel, where both effects are present, a tight monetary policy will impact the balance sheets of the borrowers and the lenders, leading to a contraction in loans and to a drop in economic activity. The expected decrease in economic activity will also tend to depress the bank’s total capital this quarter and loans next quarter.

Fifth, higher demand volatility increases the option value of holding capital. Thus, a bank would choose to hold more total capital when there is a mean preserving spread in the distribution of the stochastic shock to loan demand, $h_i > 0$. In this case, the bank realizes that there is a greater chance that the total capital constraint will be binding next period. As a result, the call option of total capital is “in the money.” Thus, by holding additional total capital, the bank is able to maintain the option of increasing lending, in the face of a possible increase in loan demand.

Sixth, a decrease in monopoly power increases the strike price of the call option and decreases the marginal payoff for each possible shock to loan demand. A decrease in monopoly power would be represented by an increase in the elasticity of demand for loans, which occurs when $l_i$ increases. As a result, the value of the call option for total capital decreases. Thus, the bank decides to hold less total capital when there is a decrease in monopoly power.

Finally, an increase in the required total capital ratio, $\theta_{x_i}$, has an ambiguous impact on the amount of total capital the bank holds. Figure 3 demonstrates the two changes in the payoffs of the call option for total capital, which arise due to an increase in the required capital. First, note that an increase in the total capital ratio causes the strike price to fall. As a result, the value of the call option increases. On the other hand, the marginal payoff of the option decreases (note that the slope is $\frac{1}{\theta_{x_i}}$), which leads to a decline in the value of the option. It turns out that the first effect dominates when the unconstrained amount of loans is less than twice the constrained level of loans. As long as the first effect is stronger, the bank will choose to hold more total capital when the required total capital ratio increases.

V. THE BANKING INDUSTRY

We can now discuss the equilibrium in the banking industry. The Bertrand equilibrium would result in the competitive loan rate equal to the marginal cost of loans $c_i + \frac{1}{\lambda_3} \left( r_i^D + c_D \right)$. An alternative trigger strategy equilibrium can be described as follows: each bank charges the monopoly loan rate and issues $\frac{1}{T}$ of the loans, given by (8) or (10), as long as no bank has deviated from this strategy. The individual firm raises total capital $\frac{1}{T} \left[ q_i s_{i+1} + b_{i+1} \right]$, given by
Proposition 2, and earns $\frac{1}{N}$ of the profits, i.e., either (9) or (11).\footnote{The current period profits are given by (11) when the total capital constraint is binding, (9) when it is not.} If a bank follows this strategy, then the value of the bank is

$$\frac{1}{N} v(\theta_{i-1}, s_t + b_i, x_i) = \frac{1}{N} \left\{ \pi_t - \tau_t \left[ q_t s_{t+1} + b_{t+1} \right] \right. + \sum_{t=1}^{\infty} E_t \left[ \int_{t}^{\infty} \pi_{i+1} dF \left( \theta_{i+1, t+1} \right) + \int_{t}^{\infty} \pi_{i+1} dF \left( \theta_{i+1, t+1} \right) \right].$$

(13)

Now an individual firm can renege on the deal by setting the loan rate slightly below the rate charged by every other bank. Interestingly, here, the bank’s benefit from renegeting is restricted by the constraint on loans, $\frac{1}{N} \left[ q_{i-1, t} s_t + b_i \right]$. There is no benefit to undercutting the other banks when the shock to loans is above its critical value, given in Lemma 1, since the bank is not allowed to issue additional loans at the lower rate. The bank would only undercut the other banks when the shock is significantly below this critical value. Even in this case the benefit of undercutting the loan rate is limited by the additional sales being only $\frac{1}{N} \left[ q_{i-1, t} s_t + b_i \right]$ where $L_t$ is given by (8). The total gain from undercutting the loan rate is

$$\pi_{i+1} = \frac{1}{N} \left[ \frac{q_{i-1} s_t + b_i}{\theta} - L_t, \right] \left[ r_{i+1} - \left( c_t + \frac{1}{1-\alpha} (r_{i+1} + c_t) \right) \right],$$

where the loan rate is given by (8). This result is opposite of that reached by Rotemberg and Saloner (1986). In their model, an oligopolistic firm is more willing to undercut the cooperative price when the demand for the product is high, since it is then that the benefit from undercutting the other firms is highest. But, depository institutions face an additional constraint, for under the Basel Accord a bank cannot undercut the cooperative price, because the bank comes up against the total capital constraint. If the bank could secretly raise total capital, then it would be more likely to undercut the loan rate since this would lessen the constraint on total capital. Presumably, the bank would do this in the case of high loan demand as in Rotemberg and Saloner (1986). The problem with this is that secret attempts to raise additional capital could become public through S.E.C. filings in the United States.

In the event that a bank undercutsthe cooperative loan rate the other banks are assumed to follow a punishment strategy. For example the banks could respond by setting the loan rate equal to the Bertrand level indefinitely. In this case the bank would earn zero economic profit. The bank suffers a net penalty of $E_t \left[ m_{t, 1} v^C_{i+1} \right] = \frac{1}{N} E_t \left[ m_{t, 1} v(q_t s_{t+1} + b_{t+1}, x_{i+1}) \right]$. Thus, the
bank undercuts the loan rate only in the case of \( \pi^U_i > \pi_i + E_t[m_{i,t}v^C_{t+1}] \). This discussion leads to the following result:

**Proposition 3** If a bank does not undercut the cooperative loan rate when the quantity of loans is given by \( \frac{1}{4} L'_i \), then a bank will never find it optimal to undercut the cooperative loan rate. In the case where it does, there is a finite neighborhood of this quantity of loans such that the bank reneges on the cooperative solution. In addition, the upper bound on this neighborhood is significantly below, \( L_i = L'_i \).

The above Proposition has a very important implication for collusive behavior among banks, for it implies that the probability of cooperative behavior among the banks is higher under the Basel Accord. The bank would never undercut the loan rate at the constrained level of loans, since the bank cannot expand its loans beyond the regulatory constraint. Thus, the bank does not undercut the loan rate when there is a high demand for loans. In addition, the gain in profits from undercutting is maximized at the quantity of loans, \( \frac{1}{4} L'_i \). If profits from undercutting the loan rate at this shock are not greater than the penalty imposed on the bank, then the bank would always find it optimal to cooperate with the other banks.

**VI. Monetary Policy Under the Basel Accord**

It is now possible to explain how monetary policy impacts the banking industry under the Basel Accord. Suppose the central bank raises the treasury rate. This will decrease the supply of deposits by individuals, such that the deposit rate increases as well. As long as the increase in the treasury rate persists, the deposit rate remains higher. This result is represented by the autoregressive process for the deposit rate in (12). The purpose of this section is to lay out how this change in monetary policy impacts the behavior of banks when they are subject to risk-based capital requirements, à la Basel.

To identify the complete impact of monetary policy on the banks, it is best to first analyze the effect on the choice of optimal total capital, and then to analyze the impact on bank profits, and on the value of the bank. Proposition 2 can be used to highlight how higher deposit rates impact the current total capital as well as all future total capital. Thus, for each quarter \( j = 1, \ldots, \infty \),

\[
\frac{\partial q_{t+1|t} s_{t+1} + b_{t+1}}{\partial r_i^{t+1}} = \left[ -h_t E_{t+1} \left[ m_{i,t} \right] + h_i \right] \frac{1}{1 - \rho_{t,\rho}} \leq 0,
\]

\[\text{58 Note that only the part of profits in (9) or (11) dealing with loans is influenced by the undercutting of the loan rate.}\]

\[\text{59 In the U.S. the Fed would raise the funds rate.}\]
as long as \( \theta_j E_{t+1} \left[ m_{t,j} \right] \leq E_{t+1} \left\{ \int_{\lambda_{t,j}} m_{t,j} dF(e_{t+1}) \right\} \).

The impact of the change in the deposit rate on total capital dies out over time, since the effect of the current deposit rate on future deposit rates declines over time. There are two ways through which the deposit rate can influence the total capital of the bank. First, a higher deposit rate lowers the net cost of total capital, because the bank saves more on the deposits used next quarter. This lower net cost of total capital leads to more total capital. The second effect is to increase the marginal cost of loans next quarter, which decreases the bank’s need for total capital. It is assumed that this latter effect is dominant, so that the total capital of the bank declines this quarter and in future quarters.

Proposition 1 can now be used to see how the bank’s profits change as a result of the contractionary monetary policy. The higher treasury rate impacts both the spread between the treasury and deposit rates and that between the loan and deposit rate. The first effect is dependent on whether or not the bank is a net holder of treasury securities. When the bank holds net positive (negative) balances of treasury securities, the net interest margin between treasury and deposit rates will go up, which increases (decreases) the profits of the bank, as long as there is increasing marginal cost of external financing. In a perfectly competitive market for external finance, this effect would disappear, since the treasury rate would always equal the marginal cost of deposits.

Next, look at the net interest margin between loan and deposit rates, and assume a constant marginal cost of external financing. If the total capital constraint is not binding for each quarter \( j = 1, \cdots, \infty \), then

\[
\frac{\partial \pi_{t+j}}{\partial r^D_t} = \rho^D_t \frac{1}{1-\alpha} \left[ q_{t+j} + s_{t+j} + h_{t+j} - L_{t+j} \right] + \frac{1}{1-\alpha} \left( r^D_t + c_D \right) \frac{\partial q_{t+j} + s_{t+j} + h_{t+j}}{\partial r^D_t} \leq 0,
\]

when \( \theta_j L^* < L \).

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60 In the United States, small commercial banks hold, on net, positive amounts of marketable securities, while the large commercial banks hold, on net, negative amounts. See Koch and MacDonald (2000, pp. 96-97).

61 This effect is emphasized by Kashyap and Stein (1995) and Stein (1998).

62 Note that, as we discussed perviously in Section 2, our results do not depend on the presence of an increasing marginal cost of external funds, as long as banks possess market power.

63 In the initial quarter, the deposit rate does not affect total capital, since it is determined in the previous quarter.
There are three ways that the deposit rate affects the bank’s profits. The first effect is that the bank earns direct interest savings as a result of having total capital rather than deposits for the funding of assets. The second effect comes from the reduction in the net interest margin on loans, which reduces the profits of the bank each quarter. The overall effect is a reduction in the profits of the bank (since the total capital ratio $\theta_2 = 8\%$). The final consequence is the reduction in profits coming from the marginal reduction in total capital that occurs, because the bank finds it optimal to hold less total capital. Thus, when deposit rates rise, the bank’s profits decrease, if the total capital constraint is not binding. On the other hand, if the total capital constraint is binding, then

\[
\frac{\partial \pi^*_{t+j}}{\partial r^D_t} = P_1 + P_2 \leq 0,
\]

where

\[
P_1 = -\rho^D_t \frac{1-\theta_2}{\theta_2(1-\alpha)} \left[ q_{t+j-1}s_{t+j} + b_{t+j} \right] + \frac{1}{1-\alpha} \left( r^D_{t+j} + c_D \right) \frac{\partial q_{t+j-1}s_{t+j} + b_{t+j}}{\partial r^D_t},
\]

and

\[
P_2 = \frac{1}{\theta_2} \frac{\partial q_{t+j-1}s_{t+j} + b_{t+j}}{\partial r^D_t} \left( r^D_{t+j} - \left[ c_L + \frac{1}{1-\alpha} \left( r^D_{t+j} + c_D \right) \right] - \frac{q_{t+j-1}s_{t+j} + b_{t+j}}{l_i \theta_2} \right).
\]

This effect is negative as long as the term inside the curly brackets in $P_2$ is positive. $P_1$ contains the three effects discussed in the unconstrained case. $P_2$ contains two parts, which can be seen by referring to Figure 2. The first part is represented by the spread between the loan rate and the marginal cost of loans. When the deposit rate increases, the bank holds less total capital, leading to a loss of profits measured by the distance from A to B times the decrease in total capital. The second part arises from the movement along the demand for loan curve. Starting from point B, the loans decrease from the lower total capital, which contributes to higher profits from the higher loan rate. The total effect of a change in the deposit rate is negative as long as the second part is small relative to the first part. Thus, the profits of the bank decrease under contractionary monetary policy when the banks are subject to the Basel Accord.

Finally, these results can be used to see how the value of the bank changes as a result of an increase in the treasury rate.

\[
\frac{\partial V}{\partial r^D_t} = \frac{\partial \pi^*}{\partial r^D_t} - \tau \frac{\partial q_{t+j}s_{t+j} + b_{t+j}}{\partial r^D_t} + \sum_\tau E_\tau \left[ m_{\tau} \left( \int_{\tau}^{\infty} \frac{\partial \pi_{t+j}}{\partial r^D_t} dF(e_{L,\tau}) + \int_{e_{L,\tau}}^{\infty} \frac{\partial \pi^*_{t+j}}{\partial r^D_t} dF(e_{L,\tau}) \right) \right].
\]
All the effects are negative except for the savings from the lower deadweight cost of total capital, since the bank economizes on the holding of total capital. More generally, higher treasury rates reduce the net interest margin on loans both now and in future quarters. As a result, current and future profits of the bank would be lower. Thus, under the Basel capital requirements, a contractionary monetary policy will reduce the market value of a bank’s stock.

Using similar arguments, it is possible to find the impact of changes in economic activity, uncertainty in the demand for loans, and the deadweight cost of total capital. These are summarized in the following result.

**Proposition 4** If the marginal impact on the deadweight cost of additional total capital is relatively small, then under the Basel Accord, the total capital, profits and market value of a bank will increase in response to 1) an increase in economic activity, 2) a mean preserving spread of the distribution of loan demand or 3) a decline in the deadweight cost of total capital.

The first result in Proposition 4 implies that total capital, profits, and the market value of a bank are all procyclical. During periods of economic expansion the bank faces a stronger demand for loans and raises total capital so that it can meet this extra demand. In addition, the bank is able to increase the net interest margin on loans as well as the amount of loans because of the stronger demand for loans. Thus, the “bank capital financial accelerator” implies that the banking industry tends to reinforce the expansion in the economy. Monetary policy in this case may mitigate this effect by raising the treasury rate during these expansionary periods in the economy so as to limit the increase in the quantity of loans.

The effect of a mean preserving spread of the demand for loans initially is surprising. More uncertainty in the demand for loans leads to higher profits and a higher stock market value for the bank. Again, the reason is that total capital, in a sense, represents an option to the bank. The bank decides to hold more total capital because it sees a greater chance that the total capital constraint would be binding. In response to this possibility the bank holds more total capital. Yet, more total capital increases the bank’s future profit and current stock market value, since it lessens the chance that the total capital constraint will be binding.

**VII. Conclusion**

With the recent trend toward replacing reserve requirements with capital requirements, the question arises as to what, if any, are the implications for monetary policy. This paper shows that monetary policy, in the presence of risk-based capital requirements, a la Basel, will impact the capacity of banks to supply loans. The monetary effect works through the “bank balance-sheet channel” by impacting the bank’s capital, its profitability and the value of its stock. Thus, capital requirements present a significant impact on the banking industry and on the economy as a whole.
We show that holding capital endows the bank with a call option whose value is affected not only by the monetary policy, but also by the level of economic activity, the structure of the banking industry, technological shocks to banking services, and by changes in the level of regulatory capital. Thus, we use a contingent-claim contract approach to highlight the impact of the aforementioned factors on the supply and pricing of loans, on the bank’s profitability, and on the value of its capital. Capital requirements are shown to introduce asymmetries into the effects of these variables on the banking industry. Basel capital requirements also have an interesting and important impact on the banking industry. In economies where there is bank concentration and market power is present, capital requirements are shown to maintain, and perhaps enhance, collusive behavior among the banks. Capital requirements, when present, reduce the expected profits to cheating banks, and as a result, reduce the incentive for individual banks to renge on cooperative agreements.

Market power in the banking industry has significant implications for the transmission of monetary policy. Thus, monetary policy impacts the value of holding capital through its effect on the bank’s net interest margin. A reduction in the net interest margin, say, due to a tightening of monetary policy, will reduce the bank’s profitability and the value of its capital. As a result, a bank is less likely to hold capital, which, in turn, will constrain the supply of loans in the future. Thus, we identify a “bank capital financial accelerator” which is distinct from the demand-driven financial accelerator. The latter arises due to the impact of monetary policy on the balance sheet and creditworthiness of borrowers. Interestingly, the presence of asymmetries in the impact of the monetary policy and the other factors, mentioned earlier, imply procyclical impact on the banking industry and on the economy. How the two financial accelerator effects interact, and their implications for the economy, as a whole, remains a topic for future research.
Figure 1. The Capital Constraints under Basel.
Figure 2. The Optimal Loan Rate Decision

\[ L = l_0 - l_1 r^* + l_2 M + \varepsilon_L^* \]

\[ MC_L = c_L + \frac{r^0 + c_D}{1 - \alpha} \]
Figure 3. The Option Value of Bank Capital

\[ \text{Slope} = \frac{1}{l_i \theta_2} \]

\[ \xi_{i,t+1} \rightarrow \xi_{i,t+1} + 2L^* \]
Appendix I

Proof that only the total capital constraint is binding

Suppose (1) is binding, so that (2) becomes

\[ [\theta_{2} - \theta_{1}] L_t \leq b_t. \]

Now, apply (1) to replace loans with equity

\[ \frac{[\theta_{2} - \theta_{1}]}{\theta_t} q_{t-1} s_t \leq b_t. \]

It follows that this equation is inconsistent with the Tier 2 constraint when \( \theta_{2} = 2\theta_{1} \).

Proof of Proposition 1

If (7) is substituted into (6), then the optimal conditions of the bank is reduced to

\[ L_t - l_t \left[ r^c_t - r^\tau_t - c_L - \frac{r^p_t}{D} (\bar{D} - T_t) - \lambda_t \theta_{2} \right] = 0, \]

\[ r^\tau_t (1 - \alpha) - r^b_t - c_D + (1 - \alpha) \frac{r^p_t}{D} (\bar{D} - T_t) = 0, \]

and

\[ \theta_{2} L_t \leq q_{t-1} s_t + b_t. \]

To solve for (8), set the Lagrange multiplier, \( \lambda_t \), equal to zero. Substitute the second Euler condition into the first condition and solve for the loan and loan rate given the demand for loans in (5). Next, solve the second condition for treasury securities. To solve for deposits, substitute these two results into the balance sheet constraint in (5). The expression for profits (9) is found by substituting the solution (8) into the expression for profits in (5) and combining common terms. To solve for (10) assume the total capital constraint is binding and solve for loans. The loan rate is found by setting this supply for loans equal to the demand for loans. The treasury securities are the same whether or not the total capital constraint is binding. The deposits are found by taking the solution for loans and treasury securities and substituting them into the balance sheet constraint. The expression for profits (11) is found by substituting the solution under the total capital constraint (10) into the expression for profits in (5).
Proof of Lemma 1

To find the critical shock, assume that the constraint is binding so that \( L_t^* = \frac{q_{t,t-1} + b_t}{\theta_2} \). This expression can be substituted into the optimal loan decision in (8), which can be solved for the critical

\[
\epsilon^*_{t,t} = 2 \frac{q_{t,t-1} s_t + b_t}{\theta_2} - \left[ I_0 + l_2 M_t \right] + l_1 \left[ c_L + \frac{1}{1 - \alpha} (r_t^D + c_D) \right].
\]

If the optimal decisions (10) are substituted into the Euler condition for the loan rate, it is possible to solve for the marginal value of total capital.

\[
\lambda_t^* = \frac{\epsilon^*_{L,t} - \epsilon^*_{L,t}}{l_t \theta_2} \text{ for } \epsilon^*_{L,t} \leq \epsilon^*_{L,t} \leq \overline{L}.
\]

Applying the envelope condition to the value function, (5), yields

\[
\frac{\partial V}{\partial (q_{t,t-1} s_t + b_t)} = \lambda_t + \frac{1}{1 - \alpha} \left( r_t^D + c_D \right) = \begin{cases} \frac{1}{1 - \alpha} \left( r_t^D + c_D \right) & \text{for } L \leq \epsilon^*_{L,t} \leq \epsilon^*_{L,t}^* \text{.} \\ \frac{1}{1 - \alpha} \left( r_t^D + c_D \right) + \frac{\epsilon^*_{L,t} - \epsilon^*_{L,t}}{l_t \theta_2} & \text{for } \epsilon^*_{L,t} \leq \epsilon^*_{L,t} \leq \overline{L}. \end{cases}
\]

Thus, the optimal condition for total capital (7) becomes

\[
\tau - \frac{1}{1 - \alpha} E_t \left[ m_{t,t} \left( r_t^D + c_D \right) \right] - \frac{1}{\theta_2 l_1} E_t \left[ \int^{t-1}_{t} m_{t,s} \left( \epsilon^*_{L,t+1} - \epsilon^*_{L,t} \right) dF \left( \epsilon^*_{L,t+1} \right) \right] = 0. \tag{14}
\]

Proof of Proposition 2

It is possible to show that there is a unique solution to the total capital held by the bank.\(^{64}\)

The total capital of the bank influences the critical shock to the demand for loans, \( \epsilon^*_{L,t-1} \).

From the proof of Lemma 1, the critical shock updated one quarter is

\[
\epsilon^*_{t,t+1} = 2 \frac{q_{t,t+1} s_{t+1} + b_{t+1}}{\theta_2} - \left[ I_0 + l_{2,1} M_{t+1} \right] + l_1 \left[ c_L + \frac{1}{1 - \alpha} (r_{t+1}^D + c_D) \right].
\]

\(^{64}\) The composition of the total capital is not known since there is no difference in the relative price of each capital within the model.
This critical shock is a function of the future level of economic activity and the deposit rate. Given the stochastic processes, (12), for these variables, the critical shock is

\[
\varepsilon_{e,t+1}^* = \frac{q_{f,t+1}^* + b_{f,t+1}^*}{\theta_2} - \left[l_0 + l_2 \rho_{M_f} M_{f,t} \right] + l_1 \left[ c_{f,t} + \frac{1}{1 - \alpha} \left( \rho_{e,f} \sigma_{e,f,t+1} + c_D \right) \right]
\]

\[+ \frac{l_1}{1 - \alpha} \varepsilon_{e,f,t+1}^* - l_2 \varepsilon_{M,e,t+1}^*.
\]

Notice that we do not know the expected value of the critical shock since we have not found the fixed point of (14) yet. This critical value is like the reservation value in search theory, except it is subject to the random disturbances to the deposit rate and economic activity. To prove the existence of a unique total capital, use the optimal total capital decision to rewrite (14) as

\[
\tau - \frac{1}{1 - \alpha} \left[ m_{t+1} \left( r_{e,t+1}^D + c_D \right) \right] = \frac{1}{\theta_2} E_t \left\{ 1 \int_{t+1}^{T} m_{t+1} \left( \varepsilon_{e,t+1}^* - \varepsilon_{e,t+1}^* \right) dF(\varepsilon_{e,t+1}) \right\}
\]

(16)

The left hand side is the net marginal cost of total capital which consists of the marginal deadweight cost of total capital minus the expected marginal cost of deposits in the next quarter. This net marginal cost is given to the bank, while the right-hand side starts at a positive amount when \( \varepsilon_{e,t+1}^* = L \). If the net marginal cost of raising total capital is high enough relative to the marginal cost of deposits, then the critical value is at a corner solution. In this case, the bank does not expect to have enough total capital to meet the demand for loans. The more likely scenario is an interior solution for total capital. In this case the RHS of (16) is greater than the net marginal cost of raising total capital at \( \varepsilon_{e,t+1}^* = L \). It follows from Leibniz’s rule that

\[
\frac{\partial \text{RHS}}{\partial \left( q_{f,t+1}^* + b_{f,t+1}^* \right)} = -\frac{2}{\theta_2^2} E_t \left\{ 1 \int_{t+1}^{T} m_{t+1} dF(\varepsilon_{e,t+1}) \right\} \leq 0.
\]

In addition, the RHS of (16) is zero at the other extreme \( \varepsilon_{e,t+1}^* = \bar{L} \). Thus, by the Mean Value Theorem there exist an optimal level of total capital which is unique.

Given the existence of an optimal level of total capital, the response of total capital can be calculated in the neighborhood of the optimal level. Use (14) and (15) to define the function

\[
H(q_{f,t+1}^* + b_{f,t+1}^*; \tau - \frac{1}{\alpha} \left[ E_t \left[ m_{t+1} \left( \rho_{e,f} r_{e,t+1}^D + c_D \right) \right] + E_t \left[ m_{t+1} \varepsilon_{e,f,t+1}^* \right] \right]),
\]

\[l_0 + l_2 \rho_{M_f} M_{f,t}, c_{f,t} + \frac{1}{1 - \alpha} \left( \rho_{e,f} \sigma_{e,f,t+1} + c_D \right) \sigma_{e,f,t}, \theta_2 \right) = 0.
\]

(17)

Here \( \sigma \) is a parameter which represents a mean preserving spread of the distribution of \( F(\varepsilon_{e,t+1}, \sigma) \). It follows that
\[ H_t = \frac{\partial H}{\partial (q_s + b_{t+1})} = \frac{2}{(\theta_2)^2} E_t \left\{ \int_{L_{t+1}} m_{t,i} dF(\varepsilon_{L_{t+1}}) \right\} > 0. \]

As a result, the Implicit Function Theorem implies the existence of an implicit function

\[ q_s + b_{t+1} = h(\tau - \frac{1}{1-\alpha} [E_t \left\{ m_{t,i} \left( \frac{r_{t+1}}{L_{t+1}} \right) + c_o \right\} + E_t \left\{ m_{t,i} e_{t+1} \right\}]), \]

\[ l_o + l_2 p_M c_L + \frac{1}{1-\alpha} \left( \frac{r_{t+1}}{L_{t+1}} + c_o \right) \sigma, l_1, \theta_2, \]

such that

\[ h_1 = -\frac{2}{(\theta_2)^2} E_t \left\{ \int_{L_{t+1}} m_{t,i} dF(\varepsilon_{L_{t+1}}) \right\} < 0, \quad h_2 = \frac{\theta_2}{2} > 0, \quad h_3 = -\frac{l_2 \theta_2}{2} < 0; \]

\[ h_5 = -\frac{2}{(\theta_2)^2} E_t \left\{ \int_{L_{t+1}} m_{t,i} \left[ \frac{e_{t+1}}{L_{t+1}} - \frac{e^*_t}{L_{t+1}} \right] + \left[ c_L + \frac{1}{1-\alpha} \left( r_{t+1}^* + c_o \right) \right] \right\} dF(\varepsilon_{L_{t+1}}) < 0, \]

and

\[ h_6 = \frac{E_t \left\{ \int_{L_{t+1}} m_{t,i} \left( L_{t+1} - 2L_{t+1}^* \right) dF(\varepsilon_{L_{t+1}}) \right\}}{E_t \left\{ \int_{L_{t+1}} m_{t,i} dF(\varepsilon_{L_{t+1}}) \right\}} > 0 \]

for \( L_{t+1} < 2L_{t+1}^* \).

It is possible to analyze the impact of a mean preserving spread when the shock to the demand for loans is independent of the stochastic discount factor of the investor. Again, think of this as unsystematic changes in demand, since the bank would tend to hedge the systematic risk. In this case the stochastic discount factor can be moved outside the integral so that

\[ \tau - \frac{1}{1-\alpha} E_t [m_{t,i} \left( r_{t+1}^* + c_o \right)] - \frac{1}{\theta_2} E_t \left\{ m_{t,i} \int_{L_{t+1}} \left( e_{t+1} - e^*_{t+1} \right) dF(\varepsilon_{L_{t+1}}) \right\} = 0. \]

Next integration by parts can be used to find

\[ \int_{L_{t+1}} \left( e_{t+1} - e^*_{t+1} \right) dF(\varepsilon_{L_{t+1}}, \sigma) = \bar{L} - e^*_{t+1} - \int_{L_{t+1}} F(\varepsilon_{L_{t+1}}, \sigma) d\varepsilon_{L_{t+1}}. \]
Therefore,

$$\frac{\partial}{\partial \sigma} \int_{\mathcal{L}_{t+1}} (e_{L,t+1} - e_{L,t+1}^*) dF(e_{L,t+1}, \sigma) = -\int_{\mathcal{L}_{t+1}} F_0(e_{L,t+1}, \sigma) de_{L,t+1} \geq 0.$$  

The last inequality uses the conditions for a mean preserving spread of the distribution of $e_{L,t+1}$.\(^{65}\) Thus, by Leibniz's rule

$$H_5 = \frac{1}{\theta_2 l_1} E \left\{ m_{L,t+1} \int_{\mathcal{L}_{t+1}} F_0(e_{L,t+1}, \sigma) de_{L,t+1} \right\} \leq 0,$$

so that $h_4 = \frac{-H_5}{H_1} \geq 0$.

**Proof of Proposition 3**

The bank undercut the loan rate only in the case of

$$\pi_{i}^{**} - \pi_i > \mathbb{E} \left[ m_{i,t} v_{i,t+1}^{L_i} \right]$$  

(18)

The right hand side of (18) is independent of the current shock to loan demand. Conversely, the left hand side of (18) is dependent on the loans.

$$\pi_{i}^{**} - \pi_i = \left[ q_{i,t} S_i + b_i \right] I_{i} - \frac{L_i^2}{l_i}.$$

This gain in profit is negative at $L_i = L_{i}^*$ and zero at $L_i = 0$. This gain in profit from undercutting the loan rate is a quadratic function in loans; its maximum may be found by taking the derivative with respect to loans. The derivative of the left hand side of (18) with respect to loans is

$$\frac{L_i^*}{2} - 2L_i.$$

Thus, the left hand side of (18) is maximized at

\(^{65}\) See footnote 57.
\[ L_i = \frac{1}{4} L_i^* . \]

The left hand side of (18) at this level of loans is

\[ \pi_i^L - \pi_i = \frac{1}{8 i_1} (L_i^*)^3 . \]

If the left hand side of (18) is less than the right hand side at this level of loans, then the bank will not find it profitable to renege on the loan rate under any circumstances. On the other hand, if the left hand side is above the right hand side of (18) at this level of loans, then there exists an interval around this level of loans during which the bank finds it optimal to renege on the loan rate. In addition, this interval is significantly below the constrained level of loans.

**Proof of Proposition 4**

First, look at the bank profits in response to an increase in economic activity. When the loan constraint binds in any period

\[ \frac{\partial \pi_{i+1}}{\partial M_i} = \rho_M \frac{L_{i+1}^*}{2 L_i} \left[ L_{i+1}^* + L_{i+1} \left[ \pi_i^L - \left[ \alpha r + \frac{1}{1-\alpha} (r_i + c_D) \right] \right] + \frac{1}{1-\alpha} (r_i + c_D) \right]. \]

In addition, when the loan constraint does not bind

\[ \frac{\partial \pi_{i+1}}{\partial M_i} = \rho_M \frac{L_{i+1}}{2 M_i} \left[ L_{i+1} \left[ \pi_i^L - \left[ \alpha r + \frac{1}{1-\alpha} (r_i + c_D) \right] \right] + \frac{1}{1-\alpha} (r_i + c_D) \right]. \]

\[ j = 0, \cdots, \infty. \] As a result, the change in the value of the bank when the economic activity increases is

\[ \frac{\partial V}{\partial M_i} = \frac{\partial \pi_i}{\partial M_i} - \frac{\partial q_i s_{i+1} + b_{i+1}}{\partial M_i} + \sum_{j=i}^{\infty} E_i \left[ m_{i+1} \left( \int_{r_i + c_D}^{\pi_i^L} \frac{\partial \pi_i}{\partial M_i} dF(e_{L,i+1}) + \int_{r_i + c_D}^{\pi_i^L} \frac{\partial \pi_i}{\partial M_i} dF(e_{L,i+1}) \right) \right] \geq 0. \]

The next issue is what happens if there is a mean preserving spread of the distribution.

\[ \frac{\partial \pi_i^L}{\partial \sigma} = \frac{h_i}{L_i} \left[ e_{L,i+1} - e_{L,i+1}^* \right] + \frac{1}{1-\alpha} (r_i + c_D) h_i \geq 0. \]

\[ j = 1, \cdots, \infty. \]
\[ \frac{\partial V}{\partial \theta} = -\tau \frac{\partial q_s \theta + b_{s+1}}{\partial \sigma} + \sum_{i=1}^{n} E_i \left[ m_{i,i} \left( \int_{l_{i,i}}^{l_{i,i+1}} \frac{\partial \pi^{*,i}}{\partial \sigma} dF(e_{l_{i,i}+1}) \right) \right] + \text{xxx}. \]

The first term represents the deadweight cost of additional total capital because of the higher perceived risk. The second term represents the higher future profits from the bank holding more total capital as a result of the higher risk in the demand for loans. There is a third effect which represents the increased uncertainty in future profit. This third effect, \( \text{xxx} \), is positive since both constrained and unconstrained profits are convex in the shock to the demand for loans. As long as the cost of raising total capital is not too large, the value of the bank goes up and the stock price of the bank goes up.

The final impact deals with what happens if there is an increase in the cost of raising external funds. First, in each period the profits change by

\[ \frac{\partial \pi^{*,i}}{\partial \tau} = \frac{h_i}{l_i \theta_2} \left[ e_{L,i+1} - e_{L,i+1} \right] + \frac{1}{1-\alpha} \left( r_{i+1} + c_D \right) h_i \leq 0, \]

\[ j = 1, \ldots, \infty. \] As a result the total impact on the value of the bank is

\[ \frac{\partial V}{\partial \tau} = -\left[ q_s \theta + b_{s+1} \right] - \tau h_i + \sum_{i=1}^{n} E_i \left[ m_{i,i} \left( \int_{l_{i,i}}^{l_{i,i+1}} \frac{\partial \pi^{*,i}}{\partial \tau} dF(e_{l_{i,i}+1}) \right) \right] \leq 0. \]

If the direct impact of an increase in the cost of raising total capital in the current period dominates, \( -\left[ q_s \theta + b_{s+1} \right] < \tau h_i \), then the value of the bank's stock goes down when there is an increase in the cost of raising external funds.
REFERENCES


