8. In all 4 graphs, the black budget line is the initial budget line and the initial utility-maximizing bundle is bundle A. The blue budget line reflects an increase in the price of food and the red budget line reflects an increase in the price of food and the compensating increase in income needed to offset the drop in real income caused by the higher price of food. The utility-maximizing bundle given the compensating budget line is bundle B. The quantities $F_A$ and $F_B$ denote the utility-maximizing amounts of food at these two bundles. Finally, bundle C represents the utility-maximizing bundle given the uncompensated blue budget line.

a. If food is a normal good, the income effect (from B to C) must result in less food at C than at B.

b. If the income elasticity of demand for food is zero, the income effect must result in the same amount of food at C and at B.
c. If food is an inferior good but not a Giffen good, the income effect must result in more food at C than at B but less food than at A.

d. If food is a Giffen good, then the income effect must result in more food than in bundle A.
10. a. We need to solve the marginal value equation and the budget line equation simultaneously for $x$ and $y$.

Marginal Value equation: $2xy/8 = x^2/2$ implies $y = 2x$

Budget Line equation: $8x + 2y = 240$

Substituting the marginal value equation into the budget line equation implies $8x + 4x = 240$ or $x^* = 20$ and $y^* = 40$. Utility at this bundle equals 16,000.

b. We need to find the value of $P_x$ for which 3 conditions are satisfied:

(1) $2xy/P_x = x^2/8$ (Marginal Value)

(2) $P_x x + 8y = 240$ (Budget Line)

(3) $x^2 y = 16000$ (16000 unit indifference curve equation)

(1) and (2) imply $x^* = 160/P_x$ and $y^* = 10$. Plugging these values into (3) yields $(160/P_x)^2 = 16000$ or $P_x = 4$.

15. a. The consumer’s optimal bundle must satisfy the budget equation

$$P_x x + P_y y = I \quad (1)$$

and the Marginal Value equation

$$\frac{y}{P_y} = \frac{x + 10}{P_x} \quad (2)$$

Solving (1) and (2) simultaneously for $x$ and $y$ yields

$$x = I/(2P_y) - 5 \quad (3)$$

and

$$y = (P_x/P_y)(I/(2P_y)+5) \quad (4)$$

Eq. (3) is the demand schedule for good $x$.

b. Given (3), $I = 100$ implies that the demand for good $x$ is described by the equation
\[ x = 50/P_x - 5. \]

Thus, the quantity demanded of \( x \) will be greater than 0 only if \( P_x < 10 \).

c and d. Note that in the figure below, at bundle A the MRS\(_{xy} \) (=1/2) is less than the relative price of \( x \) (=1) suggesting that the consumer should buy more of good \( y \) and less of good \( x \). Because bundle A consists of no units of good \( x \), further substitution is not possible.
This demand curve does not depend on the value of $P_y$.

19 a. Budget Line equation: $4x + y = 120$
Marginal Value equation: $\frac{y}{4} = x$ or $y = 4x$.
Solved simultaneously, these two equations imply $x^* = 15$ and $y^* = 60$. Total utility equals 900.

b. When $P_x$ falls to 3, the budget line equation becomes $3x + y = 120$ and $y = 3x$. Now the optimal bundle is $x^{**} = 20$ and $y^{**} = 60$. To find the income and substitution effects we need to find the bundle for which $y = 3x$ and utility equals 900, the utility of $(x^*, y^*)$ from (a).

$xy = 900$ and $y = 3x$ implies $3x^2 = 900$ or $x^{***} = 300^{1/2} = 17.32$ and $y^{***} = 2700^{1/2} = 51.96$.

Thus, the substitution effect from the decrease in the price of pizza increases pizza consumption by 17.32 - 15 = 2.32 units and decreases consumption of other goods by 60 - 51.96 = 8.04 units. The income effect from the decrease in the price of pizza increases pizza consumption by another 20 - 17.32 = 2.68 units and increases the consumption of other goods by 60 - 51.96 = 8.04 units.

c. The compensating variation of this price change is the amount of money Lou would be willing to pay to be able to buy pizzas at $3/unit. This is the difference between his current income of $120 and the cost of buying 17.32 units of pizza and 51.96 units of other goods or $(3)(17.32) + 51.96 = 103.92$. Thus, the compensating variation equals $16.08$. Note
that the compensating variation is greater than the cost savings implied by the answer to (a) of $15. Make sure you can explain why.

d. The equivalent variation for this problem is the amount of extra income we could give Lou in lieu of reducing the price of pizza that would make him as well off as at \((x^{**}, y^{**})\). The utility of \((x^{**}, y^{**})\) is 1200. We need to find the bundle for which our original marginal value equation is satisfied \((y = 4x)\) and \(xy = 1200\) or \(x' = 300^{1/2} = 17.32\) and \(y' = 4800^{1/2} = 69.28\). \((x', y')\) costs \(4)(17.32) + 69.28 = \$138.56\). Thus, the equivalent variation equals $18.56. Note that the equivalent variation is less than the extra cost of purchasing \((x^{**}, y^{**})\) when the price of pizza is $4. Make sure you can explain why.

14.a and b. The graph illustrates the situation in which Dave consumes the same amount of coffee and more doughnuts when the price of coffee falls assuming that he has quasi-linear preferences. Since quasi-linear preferences have the property that the slope of the indifference curves stays the same as you move in a vertical direction, it cannot be possible for bundle B to be a point of tangency on the new budget line (BL2) given that A is a point of tangency on the original budget line (BL1).

22. a. Jim’s optimal bundle must satisfy the marginal value equation

\[
\frac{y}{P} = \frac{x}{1}
\]

and the budget line equation

\[Px + y = 100.\]

Solving these equations simultaneously shows that Jim’s optimal bundle is

\[x = 50/P\]

and

\[y = 50.\]
Using the same procedures, Donna’s optimal bundle is \( x = 100/P \) and \( y = 50 \).

b.

![Diagram of Donna’s and Jim’s demand schedules]

When Jim and Donna are the only customers, the market demand function is
\[ x_T = 150/P. \]

When there is a third customer with the same preferences as Donna, the market demand function is
\[ x_T = 250/P. \]

24. Let’s start by solving for their individual demand curves (i.e. solve for \( Q \) in terms of \( P \)):

\[
Q_b = \frac{10 - P}{4} \quad \text{Bart}
\]

\[
Q_H = \frac{25 - P}{2} \quad \text{Homer}
\]

By calculating the choke-off prices, we find that when \( P \) is greater than 10, Bart’s quantity demanded is 0; and when \( P \) is greater than 25, Homer’s quantity demanded is 0. Therefore,

\[
Q_b + Q_H = \begin{cases} 
\frac{10 - P}{4} + \frac{25 - P}{2} & \text{if } P \leq 10 \\
\frac{25 - P}{2} & \text{if } 10 < P \leq 25 \\
0 & \text{if } P > 25.
\end{cases}
\]