Allowing Firms to Choose Between Separate Accounting and Formula Apportionment Taxation¹

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Abstract

This paper analyzes the effect on firm behavior and national tax revenues of allowing multinational firms to choose to be taxed under separate accounting rules or an apportionment formula. Separate accounting always generates more profitable output and conditional labor demand distortions from tax differentials while either method can generate a more profitable income-shifting distortion. Both low-cost and high-cost firms can prefer separate accounting while medium-cost firms prefer formula apportionment. With symmetric countries, a firm's preferred method generates greater tax revenues in the country with the lower tax rate. With asymmetric countries, a firm's preferred method need no longer generate greater tax revenues in the lower-tax-rate country due to the fact that countries and firms now value tax base distortions differently. With this misalignment of preferences, some firms can choose the preferred method of both countries at fixed tax rates.

Keywords: separate accounting, formula apportionment, transfer prices, firm choice

JEL Classifications: H73, H26, K34

¹ I thank Andreas Haufler, Kai Konrad, Wolfgang Schön, Dirk Schindler, participants of the 2013 Skatteforum (Tax Forum) in Norway, the research seminar at the Max Planck Institute for Tax Law and Public Finance in Munich, Germany, and the 8th Norwegian-German Seminar on Public Economics at CESifo, and three anonymous reviewers for all their comments.
1. Introduction.

The OECD and the European Commission (EC) disagree about how countries should tax multinational profits. OECD (2010) favors a traditional separate accounting (SA) approach while EC (2011) favors using formula apportionment (FA). EC (2011) also contains a provision that allows each firm to choose between using a specified apportionment formula or a transfer price method (see page 5 and Chapter III, EC (2011)). In this regard, the EC proposal is similar to current Canadian policy that offers multi-province firms the option of allocating taxable income via an apportionment formula or via transfer prices.\(^2\) The purpose of this paper is to develop a theory to identify what types of firms would prefer each option and to assess the effect of these choices on national tax revenues.

Heterogeneous firms may prefer different options because corporate income tax rates will distort a firm's final good production, conditional factor demand, and income-shifting decisions differently under SA and FA, and these differences can vary with each firm's economic characteristics. Even in relatively simple models, it is possible to generate many different patterns of firm choice. These various patterns arise because the relative magnitudes of the above distortions need not change in a monotonic fashion. Thus, to say anything systematic about the effect of firm choice, one needs to focus not on the final selection patterns of firms but on the economic factors that influence these patterns.

To do this, I analyze a model in which a continuum of heterogeneous multinational firms can sell their final goods in each of two countries. The firms will differ in their economic cost of capital. Countries can differ in terms of their tax rates and their wage rates. Each firm chooses its intermediate good production in country 1 and its final good output and capital and labor demands in each country. Under SA, each firm also chooses a transfer price. With different tax rates, the transfer price for units of the intermediate good shipped to country 2 creates a channel

\(^2\) See Mintz and Smart (2004).
through which a firm can shift taxable income into the country with the lower tax rate, and it can affect production and factor employment margins. FA rules do not use transfer prices but relative country sales and factor employment to determine a multinational's taxable income in each country. Firms can then shift income between the countries through their output and input decisions that are factors in the apportionment formula. Output and input distortions also arise with FA as it makes a firm's marginal tax rate endogenous. Thus, the linkages between marginal tax rates and each firm's output, input, and income-shifting choices are what will be important in determining the more profitable method for each firm.

One implication of the different linkage effects generated by SA and FA can be seen in firms with high capital costs that exclusively or predominately sell their final goods in country 2 (while still producing the intermediate good in country 1). For these firms, SA is more profitable. At a given transfer price, a firm that sells more in country 2 can shift more income into the low-tax country because more country 2 sales requires more units of the intermediate good. With limited or no country 1 final good sales, a firm's ability under FA to change its effective tax rate is attenuated. This translates into limited income-shifting potential under FA. Thus, high capital cost firms operating near their country 1 extensive margin will strictly prefer SA.

For firms with lower capital costs, their choice is determined by how the two methods affect the firms' intensive margins for output, conditional factor demands, and income-shifting. To identify these effects, I exploit the fact that, at equal tax rates, both methods result in identical after-tax firm profit and identical rates at which a change in one country's tax rate affects after-tax firm profit. These baseline facts allow one to focus on how small tax rate and wage rate differences can affect a firm's profits under SA and FA.

Symmetric countries will have identical wage rates. Only their tax rates can differ. With symmetric countries, SA always generates more profitable output and conditional factor demand
distortions because a firm's effective tax rate under FA is an average of the countries' statutory tax rates. This means that an increase in one country's tax rate increases a firm's effective tax rate under FA by a smaller amount than under SA. For a firm to earn greater profit under FA, FA must generate a more profitable income-shifting distortion than SA. However, a firm's ability to shift more income under FA is smallest for firms with very low capital costs. Low capital cost firms will sell more in country 2 and thus can shift more income under SA than firms with higher capital costs. With similar tax rates, the incentives for income-shifting under FA will depend on differences in final good sales in the two countries, which will be smaller for firms with lower capital costs because FA comes closer to being a pure profit tax system for these firms. Thus, a second implication of the difference in linkage effects between SA and FA is that lower cost firms are more likely to prefer SA while firms with intermediate costs are more likely to prefer FA. Combined with the extensive margin results described above, what emerges is a selection pattern that is non-monotonic with respect to a firm's capital costs, even with similar tax rates and symmetric countries.

Wage rate differences alter the linkages between effective tax rates and firm decisions. I will show that a wage rate differential can change the preferences of firms selling significant amounts of their final goods in both countries under FA as wage rates differences can make final good sales in each country more sensitive to tax rate differentials. This increased income-shifting distortion improves firm profitability under FA only if the country with the higher tax rate also has the higher wage rate because both differentials reinforce a firm's incentive to shift income.

Given the non-trivial choice patterns suggested by the above discussion, it is not obvious whether a country with the higher tax rate or the lower tax rate would benefit from firm choice. At equal tax rates, both methods generate the same tax revenues for each country. This means that the main difference in national tax revenues under each method depends on how each
country's tax base responds to increases in the tax and wage rate differentials. With symmetric countries, the tax method that generates larger profit for each intensive margin firm is also the method that generates larger tax revenues for the country with the lower tax rate and smaller tax revenues for the other country. In this case, the high tax rate countries in the EU would be justified in thinking that firm choice would result in lower tax revenues for them. This need no longer be true with asymmetric countries. Now the method preferred by the country with the higher tax rate can be the method some intensive margin firms prefer.

With asymmetric countries, firm choices can be imperfectly aligned with the preferences of the low-tax country. It is also possible, at fixed tax rates, for the choices of some firms to be aligned with each country's preferred method. This alignment can occur if a firm's preferred method increases its taxable income in each country through an efficiency effect. If the distribution of capital costs favors enough of these firms, choice will increase each country's tax revenues.

Firm choice will also have tax competition effects. For symmetric countries, giving firms the ability to choose their tax methods increases the incentive for each country to lower its tax rate to attract a larger tax base and results in lower equilibrium tax rates than would arise under either a system that imposes SA on all firms or a system that imposes FA on all firms.

1.1 Literature Review

The only other paper to study firm choice between SA and FA is Mintz and Smart (2004). Rather than explaining each firm's choice, they take the choices of Canadian firms at the provincial level as given and use the choices to estimate separately the elasticity of taxable income with respect to tax rates for firms who use transfer prices and for firms that use an apportionment formula.

Most often, the literature that compares SA to FA compares the equilibrium allocations that result from a specific formula to those that result from a specific set of transfer price
regulations in representative firm models. Key examples include Nielsen, Raimondos-Møller, and Schjelderup (2003 and 2010), Eichner and Runkel (2008 and 2011), and Runkel and Schjelderup (2011).\(^3\) NRS (2003) shows that a shift from SA to FA can actually exacerbate income-shifting via transfer prices if the firm operates in oligopoly markets, while NRS (2010) shows that tax revenues can either rise or fall from a shift to FA depending on the cost of income-shifting and the magnitude of pure firm profits. ER (2008) provides sufficient conditions for a sales-only formula to increase tax revenues relative to SA. ER (2011) endogenizes interest rates and shows that FA will generate higher tax revenues than SA if the elasticity of substitution between capital and labor is sufficiently large. RS (2011) primarily study the choice of apportionment weights but also show that the optimal apportionment formula can increase tax rates, tax revenues, and national welfare relative to SA.

The main weakness with a representative-firm approach for the purpose of studying firm choice is that it admits no scope for differential firm choice. The only paper of which I am aware that compares SA and FA with heterogeneous multinational firms is Gresik (2010).\(^4\) Although this paper does not allow each firm to choose its method of taxation, it does show that after-tax profit and tax revenue differences between SA and FA vary not only with differences in firm productivity but also with the location of a firm's intermediate good production and its final good sales. Moreover, the profit and tax revenue differences need not be monotonic with respect to firm productivity which suggests the potential for subtle firm selection patterns.

In what follows, section 2 presents the model. Section 3 describes the profit-maximizing

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\(^3\) One exception is Riedel and Runkel (2007) in which a representative multinational must use a specified apportionment formula for income generated within a union of countries and transfer prices for income generated between union and non-union countries.

\(^4\) A number of papers have studied optimal taxation and/or tax competition with heterogeneous firms in recent years. Burbridge, Cuff, and Leach (2006) study profit tax systems with national firms. Becker (2013) and Haufler and Stähler (2013) study tax competition for FDI but do not address the issue of income-shifting. Krautheim and Schmidt-Eisenlohr (2011) study tax competition with income-shifting but do not specifically address the role of SA or FA. Bauer and Langenmayr (2013) formally model SA but not FA.
choices firms make under SA and section 4 presents the same analysis under FA. A firm's optimal choice between SA and FA is studied in section 5. Section 6 describes the effect of firm choice on national tax revenues. I offer concluding remarks in section 7.

2. The model.

In this section, I describe a model in which heterogeneous firms can choose between being taxed under SA or FA and in which each option influences each firm's output, conditional factor demand, and income-shifting decisions. Most of the model's features are standard. There are two countries (1 and 2) and a continuum of firms, each producing a distinct product. Each firm can choose the countries in which it sells its product. To sell its product in a country, a firm must operate a subsidiary in that country, where it produces and sells the product using labor, capital, and a proprietary intermediate good. The intermediate good is produced only in country 1. Without the intermediate good, the subsidiaries are independent firms operating in separate countries for which there is no scope for income-shifting and no reason to apportion income between the two countries. The choice between SA and FA then becomes mute.

One unit of the intermediate good is needed to produce one unit of the final good and requires one unit of capital and one unit of labor. The potential to shift income from one country to another via SA is created by shipping units of the intermediate good from country 1 to country 2. The assumption that every firm produces its intermediate good in country 1 is made to simplify the presentation of the analysis. The main results generalize in a straightforward way if some firms locate their intermediate good production in country 2.

A firm's final good production function in each country is \( \min \{z, F(k^{f}, l^{f})\} \) where \( z \) denotes the quantity of the intermediate good used, \( k^{f} \) denotes the quantity of capital used in country \( j \) for final good production, and \( l^{f} \) denotes the quantity of labor used in country \( j \) for final good production. The sub-production function, \( F \), allows the firm to substitute between capital and labor and exhibits constant returns to scale. The Leontief structure with regard to the
intermediate good simplifies the analysis while the sub-production function maintains a channel through which taxes can distort a firm's conditional factor demands.

Denote the marginal economic cost of capital for a firm by $\mu \in [0, \mu]$. I assume firms can have different values of $\mu$ for three reasons. First, although the standard assumption in international taxation papers is that all firms can finance investment at a world interest rate, the corporate finance literature links a firm's economic cost of capital to its CAPM $\beta$ and points out that the cost of capital for an individual project, such as funding a subsidiary, can differ from the firm's market $\beta$ for a number of factors, such as the riskiness of the firm's projects.\(^5\) Second, using the firm's cost of capital as the source of firm heterogeneity simplifies the analysis. Any other source of firm heterogeneity could be used without altering the basic analysis. Third, when $\mu = 0$ both SA and FA are pure profit tax methods at equal tax rates. This provides a convenient baseline for identifying the effects of income-shifting separate from the distortionary effects of the tax rates. Denote the differentiable cumulative distribution function of firm types by $\Phi(\mu)$.

Country $j$'s wage rate is $w_j$. Labor in each country is immobile and each firm takes the wage rate in each country as given. Given the novelty of studying firm choice even in a partial equilibrium setting, I leave the study of general equilibrium effects to future work.

Each firm serves market $j$ with inverse demand function $p_j(q_j)$ where $q_j$ denotes the quantity of the final good produced and sold in country $j$. There exists a quantity $\bar{q}_j$ such that for all $q_j \geq \bar{q}_j$, $p_j(q_j) = 0$. Define a firm's revenue function in each country $j$ by $r_j(q_j)$. $r_j(0) = 0$ and $r'_j(0) = p_j(0) > 0$. Each $r_j(\cdot)$ is strictly concave, which allows for the possibility of firm rents, and each $p_j(0)$ is finite. A firm that sells $q_1$ units of its final good in country 1 and $q_2$ units of its final good in country 2 will require $q_1 + q_2$ units of labor and capital to produce $q_1 + q_2$ units of its intermediate good. Thus, a multinational's pre-tax economic profit in country 1 equals

$$r_1(q_1) - w_1(l_1^f + q_1 + q_2) - \mu(k_1^f + q_1 + q_2)$$

and its pre-tax economic profit in country 2 equals

$$r_2(q_2) - w_2(l_2^f + q_2 + q_1) - \mu(k_2^f + q_2 + q_1)$$

\(^5\) For example, see chapter 10 in Berk and DeMarzo (2011).
$r_2(q_2) - w_2 l_2^f - \mu k_2^f$. Throughout the paper, $r(q) = r_2(q)$ for all $q$, while the wage rates will be equal only for symmetric countries.

2.1 After-tax profit under separate accounting.

With SA, each firm sets a transfer price, $\rho$. It is the unit price the subsidiary in country 2 pays the subsidiary in country 1 for the intermediate good. The countries jointly audit each firm's transfer price and the per-unit audit costs incurred by a multinational are denoted by the strictly convex function $C(\rho - w_i)$ for which $C(\cdot) \geq 0$ and $C(0) = 0$. Positive audit costs can reflect the costs incurred by the firm to defend its transfer price during an audit as well as expected legal penalties. Writing $C(\cdot)$ as a function of $\rho - w_i$ implies an arm's-length price for the intermediate good of $w_i$. If $\rho < w_i$, the multinational is shifting taxable income into country 2 and if $\rho > w_i$, the multinational is shifting taxable income into country 1. An alternative definition of the arm's-length price would be $w_i + \mu$. Because capital costs are generally not 100% tax-deductible, using this alternative definition of an arm's-length price would effectively allow firms to use non-tax-deductible costs to shift some taxable income at no (auditing) cost.

Denote the countries' statutory corporate income tax rates by $t_1$ and $t_2$. For a firm that chooses SA, its global post-tax economic profit equals

$$\pi_{SA}^{\infty}(q_1, q_2, \rho, l_1^f, l_2^f, k_1^f, k_2^f) = (1 - t_1)(r_1(q_1) - w_1(l_1^f + q_1 + q_2) + \rho q_2) + (1 - t_2)(r_2(q_2) - w_2 l_2^f - \rho q_2) - C(\rho - w_i)q_2 - \mu(k_1^f + k_2^f + q_1 + q_2).$$

Note that total audit costs equal $C(\cdot)q_2$. Defining total audit costs in this way emphasizes that the benefits and costs of income-shifting are both proportional to the volume of intermediate good shipments. Also note that most countries do not permit the full tax-deductibility of capital costs.

For simplicity, I assume that capital costs are not tax-deductible.

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*This specification abstracts from actual transfer price regulations under which the auditing costs vary with the tax rates but shares the same qualitative properties. See Gresik and Osmundsen (2008) and Nielson, Raimondos-Møller, and Schjelderup (2010) for models that incorporate this level of detail in their models.*
2.2 After-tax profit under formula apportionment.

With FA, each firm reports its total taxable income to each country. An apportionment formula is then used to divide the total taxable income between the two countries. Rather than focus on just one formula, I consider any formula of the form \( \lambda(q_1, q_2) \), where \( \lambda \) determines the fraction of global taxable income taxed in country 1, \( 0 \leq \lambda(q_1, q_2) \leq 1 \), \( \lambda_1(q_1, q_2) \equiv \partial \lambda / \partial q_1 \geq 0 \) (with equality only if \( \lambda=1 \)), and \( \lambda_2(q_1, q_2) \equiv \partial \lambda / \partial q_2 \leq 0 \) (with equality only if \( \lambda=0 \)). That is, larger sales in country 1 increase the firm’s taxable income in country 1 while larger sales in country 2 decrease the firm’s taxable income in country 1. \( \lambda(q_1, 0) = 1 \) as the firm is not operating in country 2. \( \lambda(0, q_2) \) need not equal zero as the firm is still producing the intermediate good in country 1. Given \( \lambda(q_1, q_2) \), the effective tax rate for a firm will be

\[
T = \lambda(q_1, q_2) t_1 + (1 - \lambda(q_1, q_2)) t_2.
\]

Thus, under FA a firm shifts income between countries by adjusting its output levels in each country in order to alter its effective tax rate.

Two examples of apportionment formulas with this structure are the output formula, 
\( \lambda = q_1 / (q_1 + q_2) \), and the sales-only formula, 
\( \lambda = r_1(q_1) / (r_1(q_1) + r_2(q_2)) \). This specification does not include formulas that depend on inputs such as wages, employment, and/or capital (see also footnote 10).

An apportionment formula will be symmetric if there is a non-negative, non-decreasing function, \( \Lambda(q_i) \) with \( \Lambda(0) = 0 \) so that \( \lambda(q_1, q_2) = \Lambda(q_1) / (\Lambda(q_1) + \Lambda(q_2)) \). For symmetric apportionment formulas, \( \lambda(q_1, q_2) = 1/2 \), \( \lambda_1(q_1, q_2) + \lambda_2(q_1, q_2) = 0 \), \( \lambda_{11}(q_1, q_2) + \lambda_{22}(q_1, q_2) = 0 \). An apportionment formula will be symmetric if it does not bias operations in one country over the other. Examples of symmetric formulas include the output formula and the sales-only formula (when \( r_1(q) = r_2(q) \) for all \( q \)).

Given (2), a firm’s post-tax economic profit is

\[\text{If marginal revenue can be negative, the sales-only formula can violate either } \lambda_1 \geq 0 \text{ or } \lambda_2 \leq 0. \] However, both of these cases can be shown not to be profit-maximizing.
\[
\pi_{SA}^{CA}(q_1, q_2, k^j_1, k^j_2, l^f_1, l^f_2) = (1 - T)(r_1(q_1) + r_2(q_2) - w_1(l^f_1 + q_1 + q_2) - w_2(l^f_2) \\
- \mu(k^j_1 + k^j_2 + q_1 + q_2). \tag{3}
\]

Notice that (2) and (3) imply that the tax base definitions under SA and FA are the same.8 This assumption is consistent with the papers comparing SA to FA discussed in the introduction.

3. Profit-maximization under separate accounting.

Under SA, each multinational chooses its outputs, factor demands, and transfer price to maximize (1) subject to \( q_j \leq F(k^j_1, l^f_j) \) for \( j \in \{1,2\} \), holding tax rates, wages, and \( \mu \) constant. Holding \( q_1, q_2, \text{ and } \rho \) fixed, a firm's conditional factor demands, \( K^S_A(q_j, t_j) \) and \( L^S_A(q_j, t_j) \), are the (after-tax) cost-minimizing values of \( k^j_i \) and \( l^f_i \). These conditional factor demands are defined by the standard equations: \( F(k^j_i, l^f_i) = q_i \) and \( F_k / \mu = F_L / (1 - t_i)w_i \) where the "\( K \)" and "\( L \)" subscripts denote partial derivatives of the sub-production function. \( K^S_A \) and \( L^S_A \) do not depend on \( q_j \) or \( t_j \) and the constant returns to scale assumption means they are linear in \( q_i \).

Given the conditional factor demands, differentiating (1) with respect to \( \rho \) implies that a firm's optimal transfer price satisfies \( t_2 - t_1 = C'(\rho - w_i) \) or \( \rho = w_i + (C')^{-1}(t_2 - t_1) \). As long as \( C(\cdot) \) is sufficiently convex, the firm's optimal transfer price will be well-defined. I assume this to be the case for all firms in the economy.

Substituting the conditional factor demands and the optimal transfer price into (1) implies that a firm solves its profit-maximization problem by choosing \( q_1 \) and \( q_2 \) to maximize

\[
\pi_{SA}(q_1, q_2) = (1 - t_1)(r_1(q_1) - w_1(t^S_A(q_1, t_1) + q_1)) + (1 - t_2)(r_2(q_2) - w_2L^S_A(q_2, t_2) - w_2q_2) \\
+ \Delta(t_1, t_2)q_2 - \mu(K^S_A(q_1, t_1) + K^S_A(q_2, t_2) + q_1 + q_2), \tag{4}
\]

where \( \Delta(t_1, t_2) = (t_2 - t_1)(C')^{-1}(t_2 - t_1) - C((C')^{-1}(t_2 - t_1)) \) is the optimal amount of profit shifted per

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8 Alternative tax base definitions, such as the allowance for capital equity (ACE) and the corporate business income tax (CBIT) are also being debated. ACE provides a deduction for capital expenses while CBIT disallows any deduction for interest payments. See Brekke, Garcia Pires, Schindler, and Schjelderup (2014) for a recent comparison of these methods. Neither method applies to this analysis but could be incorporated into a more general analysis.
unit of country 2 sales. \( \Delta(t_1, t_1) = 0 \) for all \( t_1 \), and for all \( t_2 \neq t_1, \Delta(t_1, t_2) > 0 \) by the strict convexity of \( C(\cdot) \). \( \pi^{SA}(q_1, q_2) \) is strictly concave in \( q_1 \) and \( q_2 \). Denote the profit-maximizing quantities by \( q_1^{SA}(t_1, t_2, w_1, w_2, \mu) \) and \( q_2^{SA}(t_1, t_2, w_1, w_2, \mu) \).

When it is optimal for a firm to sell in both countries, \( q_1^{SA} \) and \( q_2^{SA} \) solve the first-order conditions

\[
(1-t_1)(r_1'(q_1)-w_1(\partial L_1^{SA}(q_1, t_1) / \partial q_1 + 1)) = \mu(\partial K_1^{SA}(q_1, t_1) / \partial q_1 + 1) \quad (5)
\]

and

\[
(1-t_2)(r_2'(q_2)-w_2\partial L_2^{SA}(q_2, t_2) / \partial q_2 + \Delta(t_1, t_2)) = \mu(\partial K_2^{SA}(q_2, t_2) / \partial q_2 + 1). \quad (6)
\]

Eq. (5) equates the firm's after-tax marginal taxable income in country 1 with its marginal economic cost of capital from country 1 sales. Eq. (6) equates the firm's after-tax marginal taxable income in country 2 plus its marginal profits shifted with its marginal cost of capital from country 2 sales. Direct comparative statics calculations imply that

\[
(\partial q_i^{SA} / \partial \mu) = (1 + K_i^{SA} / q_i^{SA}) / ((1-t_i)r_i'') < 0,
\]

which means that firms with higher \( \mu \) sell less in each country. Thus, there exists \( \mu_i^{SA}(t_1, t_2, w_1, w_2) \) such that \( q_i^{SA} = 0 \) for all firms with \( \mu \geq \mu_i^{SA} \). I will refer to a firm with \( \mu = \mu_i^{SA} \) as being on the extensive margin in country \( i \) under SA.

**Lemma 1.** If the countries are symmetric and demand is sufficiently large, then

\[
(\mu_1^{SA} - \mu_2^{SA})(t_1 - t_2) < 0 \text{ for all } t_1 \neq t_2.
\]

The proof of Lemma 1 can be found in the online web supplement. The lemma states that, absent country asymmetries, fewer firms will sell in the high-tax country. If \( t_1 > t_2 \), then any firm with \( \mu < \mu_1^{SA} \) will sell its output in both countries; any firm with \( \mu_1^{SA} < \mu < \mu_2^{SA} \) will sell its output only in country 2; and any firm with \( \mu > \mu_2^{SA} \) will sell in neither country. In the first case, tax rate changes will only have intensive margin effects while in the second case there can also be
extensive margin effects as the set of firms selling in both countries will change. Note that a firm that falls into the second case will still engage in income-shifting via its transfer price because the intermediate good is still produced in country 1.

Evaluating (4) at these optimal quantities yields the indirect profit function, \( \Pi^{S_A}(t_1,t_2,w_1,w_2,\mu) \). By the Envelope Theorem,
\[
\frac{\partial \Pi^{S_A}}{\partial \mu} = -(K_1^{S_A}(q_1^{S_A}) + K_2^{S_A}(q_2^{S_A}) + q_1^{S_A} + q_2^{S_A}) < 0
\]
and by the Theorem of the Maximum, \( \Pi^{S_A} \) is also convex in \( \mu \). Thus, firms with higher values of \( \mu \) earn less after-tax profit and the rate of decline in after-tax profit is falling in \( \mu \).

4. Profit-maximization under formula apportionment.

Under FA each firm chooses its outputs and factor demands to maximize (3) subject to \( q_j \leq F(k_j^F, l_j^F) \) for \( j \in \{1,2\} \), holding tax rates, wages, and \( \mu \) constant. Holding \( q_1 \) and \( q_2 \) fixed, a firm's conditional factor demands, \( K_i^F \) and \( L_i^F \), are solutions to standard after-tax, cost-minimization problems so that \( K_i^F(q_1,q_2) \) and \( L_i^F(q_1,q_2) \) satisfy \( F(k_j^F, l_j^F) = q_j \) and \( F_k / \mu = F_L / (1-T)w_i \). Because both output quantities influence the effective tax rate, the conditional factor demands depend on both quantities.

Substituting these conditional demands into (3) implies that a firm solves its profit-maximization problem by choosing \( q_1 \) and \( q_2 \) to maximize
\[
\pi^F(q_1,q_2) = (1-T)\pi^0(q_1,q_2) - \mu(K_1^F + K_2^F + q_1 + q_2),
\]
where
\[
\pi^0(q_1,q_2) = r_1(q_1) + r_2(q_2) - w_1(l_1^F(q_1,q_2) + q_1 + q_2) - w_2 l_2^F(q_1,q_2)
\]

The convexity of \( \Pi^{S_A} \) is due to similar reasons that indirect profit functions in general are convex in a linear parameter (see Varian (1992), chapter 3). If a firm holds output and input levels constant, its direct profit is linear in \( \mu \). But since a change in \( \mu \) will induce the firm to change its output and input levels to maximize its profit, the new optimized level of profit must exceed its profits under the passive benchmark and implies that \( \Pi^{S_A} \) is convex in \( \mu \).

For apportionment formulas that use input shares or input-cost shares, it need no longer be the case that \( q_j = F(K_j^F, L_j^F) \), as some firms might have an incentive to over-employ capital or labor in order to influence \( T \). Given this added complication, I focus only on output-based formulas in this paper.
denotes a firm’s global taxable income. Unlike with SA, \( \pi^{FA}_{12} \neq 0 \), as now both \( q_1 \) and \( q_2 \) affect \( T \). Moreover, \( \pi^{FA} \) need not be globally concave in \( q_1 \) and \( q_2 \). However, the market bounds, \( \bar{q}_1 \) and \( \bar{q}_2 \), imply that profit-maximizing quantities exist. Denote these quantities by \( q^{FA}_1(t_1,t_2,w_1,w_2,\mu) \) and \( q^{FA}_2(t_1,t_2,w_1,w_2,\mu) \).

When it is optimal for a firm to sell its output in both countries, \( q^{FA}_1 \) and \( q^{FA}_2 \) must satisfy
\[
(1-T)(r^T_i - w_1(\partial L^{FA}_1 / \partial q_i) + 1) - w_2(\partial L^{FA}_2 / \partial q_i) - \mu(\partial K^{FA}_1 / \partial q_i + \partial K^{FA}_2 / \partial q_i) + 1 = \lambda_i(t_1 - t_2)\pi^0 \tag{10}
\]
for \( i = 1,2 \). The left-hand side of (10) is the difference between a multinational's marginal after-tax taxable income and its marginal economic cost of capital. The right-hand side of (10) is the change in the income-shifting benefit a multinational realizes through the effect of \( q_i \) on its effective tax rate. If a firm maximizes its after-tax economic profit by selling \( q_j > 0 \) units in country \( j \) and \( q_k = 0 \) in country \( k \), then (10) must be satisfied with equality for \( i = j \) and with a weak inequality for \( i = k \). Similar to the SA analysis, there exists \( \mu^{FA}_i(t_1,t_2,w_1,w_2) \) such that \( q^{FA}_i = 0 \) for all firms with \( \mu \geq \mu^{FA}_i \). I will refer to a firm with \( \mu = \mu^{FA}_i \) as being on the extensive margin in country \( i \) under FA.

**Lemma 2.** If the countries are symmetric and demand is sufficiently large, then
\[
(\mu^{FA}_1 - \mu^{FA}_2)(t_1 - t_2) < 0 \text{ for all } t_1 \neq t_2.
\]

Lemma 2 is the FA analog to Lemma 1. Its proof is also available in the online web supplement. Under FA, a firm will sell more in the low-tax country than in the high-tax country, and as a result, it will shut down sales first in the high-tax country.

Evaluating (9) at a firm's optimal quantities yields the indirect economic profit function,
\[ \Pi^{FA}(t_1,t_2,w_1,w_2,\mu) \]. By the Envelope Theorem,
\[
\partial \Pi^{FA} / \partial \mu = -(K^{FA}_1 + K^{FA}_2 + q^{FA}_1 + q^{FA}_2) < 0 \tag{11}
\]
By the Theorem of the Maximum, \( \Pi^{FA} \) is convex in \( \mu \). Thus, firms with higher \( \mu \) earn less after-
5. The optimal tax method choice.

The results of sections 3 and 4 raise two challenges for developing a general theory of how a multinational's choice between SA and FA is affected by country-specific and firm-specific economic factors. First, firms operating near an extensive margin can have different preferences between SA and FA than do firms who are responding only to differences in intensive margin incentives. Thus, the analysis in this section will first focus on the preferences of firms that sell primarily in one final good market. Second, the set of firms preferring one method over the other need not be monotonic. To address this challenge without resorting to making strong functional form assumptions, I compare the Taylor series approximations of both indirect profit functions near equal tax rates and symmetric countries.\textsuperscript{11} This approach allows one to identify the dominant marginal differences between the two methods and shows that these differences have a natural economic interpretation.

5.1 Extensive margin effects (Dominant country sales).

SA and FA define two extensive margin types in each country. A firm that operates near an extensive margin will choose to sell almost exclusively in one country. Define 

\[ \overline{\mu}_i = \max \{ \mu_i^{SA}, \mu_i^{FA} \} \quad \text{and} \quad \underline{\mu}_i = \min \{ \mu_i^{SA}, \mu_i^{FA} \}. \]

A firm with \( \overline{\mu}_i \leq \mu < \underline{\mu}_i \) will sell nothing in country \( i \) and a strictly positive amount in country \( j \) regardless of which tax method it chooses. A firm with a value of \( \mu \) just below \( \overline{\mu}_i \) will have sales predominately in country \( j \). In this case \( \overline{\mu}_i \) is the first extensive margin. A sufficient condition for this case to arise with symmetric countries is \( t_i > t_j \). Proposition 1, which applies to symmetric as well as asymmetric countries, describes a firm's preferred tax method for this case.

\textbf{Proposition 1.} Assume that \( \overline{\mu}_i < \underline{\mu}_j \). There exists \( \hat{\mu} < \overline{\mu}_i \) such that a firm with marginal cost of

\textsuperscript{11} Taylor series approximations have also been used in optimal tax studies such as Sørensen (2014) as they identify the primary-order effects of a tax policy.
capital \( \mu \in (\mu, \mu_2) \) strictly prefers SA over FA.

Proposition 1 focuses on the case in which all firms with \( \mu \in [\overline{\mu}_1, \mu_2) \) sell only in country 2, and those with \( \mu \) just below \( \overline{\mu}_1 \) sell predominately in country 2. With \( q_i \) small (or zero), an increase in \( q_2 \) has little effect on the ability of these firms to shift income under FA since the firm's effective tax rate is already close to \( t_2 \). These firms will prefer SA since they can still shift more income for the same transfer price by increasing \( q_2 \).

A definitive analogous result for the case in which a firm sells only or predominately in country 1 does not exist. Now the first extensive margin is \( \overline{\mu}_2 \). Depending on details of a firm's revenue or production functions, a firm operating just below the country 2 extensive margin can prefer either SA or FA. For firms that sell predominately in country 1, their choice will depend on whether a decrease in \( \mu \) below \( \overline{\mu}_2 \) first results in country 2 sales under SA or FA. This ambiguous ranking arises for three reasons that are easiest to see with symmetric countries and \( t_1 < t_2 \).

First, firms that sell only in country 1 (\( \mu \geq \overline{\mu}_2 \)) are indifferent between SA and FA because their entire operations are in country 1. Second, with \( q_2 \) close to zero, the effective tax rate on country 2 operations is \( t_2 \) under SA and close to \( t_1 \) under FA. Thus, after-tax marginal profit is larger under FA than under SA. Third, SA provides more favorable income shifting opportunities. Under SA, an increase in \( q_2 \) shifts income into the lower-tax country, country 1, at a rate of \( \Delta \) while under FA, an increase in \( q_2 \) shifts income into the higher-tax country, country 2. Both of these income shifting effects favor SA. If the higher marginal after-tax operating profits from country 2 under FA dominate the higher marginal income shifting profits under SA, then a firm with \( \mu \) just below \( \overline{\mu}_2 \) will prefer FA and \( \mu_2^{SA} < \mu_2^{FA} \). If the marginal income shifting profits dominate, then a firm with \( \mu \) just below \( \overline{\mu}_2 \) will prefer SA and \( \mu_2^{FA} < \mu_2^{SA} \). Examples exist in which either ranking arises.
With these results established for firms that operate near its first extensive margin, I now consider the selection patterns driven by firms operating inside (below) their first extensive margin.

5.2 Intensive margin effects.

For a firm that sells in both countries, its indirect profits under SA and FA are identical when the countries have equal tax rates, regardless of any differences in wage rates. Thus, some difference in tax rates is needed to generate a strict preference for one method over the other. One can exploit this equivalence at equal tax rates to identify the economic factors that will determine a firm's choice when the countries' tax and wage rates are similar, by calculating the Taylor series approximations for $\Pi^{SA}(t_1, t_2, w_1, w_2, \mu)$ and $\Pi^{FA}(t_1, t_2, w_1, w_2, \mu)$ near equal tax rates and equal wage rates. The role of large tax rate and wage rate differences will be discussed in section 7.

At equal tax rates, the Envelope Theorem implies that a firm's marginal indirect profit under each method, $\partial\Pi^{SA}(t_1, t_2, w_1, w_2, \mu) / \partial t_2$ and $\partial\Pi^{FA}(t_1, t_2, w_1, w_2, \mu) / \partial t_2$, equals the negative of its taxable income or tax base in country 2. With equal wage rates and a symmetric apportionment formula these marginal indirect profits are equal, that is,

$$\frac{\partial\Pi^{SA}(t_1, t_2, w_1, w_2, \mu)}{\partial t_2} = \frac{\partial\Pi^{FA}(t_1, t_2, w_1, w_2, \mu)}{\partial t_2}.$$ 

Therefore, a firm's choice between SA and FA when the countries' tax rates and wage rates are similar will be determined by how a change in $t_2$ or $w_2$ affects the firm's taxable income in country 2.

The first-order difference in taxable income with respect to $t_2$ is $\partial^2 (\Pi^{SA} - \Pi^{FA}) / \partial t_2^2$ and the first-order difference in taxable income due to a change in $w_2$ is $\partial^2 (\Pi^{SA} - \Pi^{FA}) / \partial w_2 \partial t_2$. Combining the marginal effects of tax rate and wage rate differences, relative to $t_1$ and $w_1$ implies that

$$\Pi^{SA} - \Pi^{FA} \approx \frac{(t_2 - t_1)^2}{2} \cdot \frac{\partial^2 (\Pi^{SA} - \Pi^{FA})}{\partial t_2^2} + (t_2 - t_1)(w_2 - w_1) \cdot \frac{\partial^2 (\Pi^{SA} - \Pi^{FA})}{\partial w_2 \partial t_2}, \quad (12)$$

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where all the derivatives are evaluated at equal tax rates and equal wage rates.\textsuperscript{12}

Figures 1 and 2 illustrate the error in this approximation when \( F(k_i^f, l_i^f) = \min \{k_i^f, l_i^f \} \), \( w_1 = w_2 = 1 \), \( r_i(q) = r_2(q) = (20 - q)q \), \( C(\rho - w_i) = (\rho - w_i)^2 \), and \( t_1 = .3 \). In Figure 1, \( t_2 = .25 \), and in Figure 2, \( t_2 = .35 \). The solid curve in each figure illustrates the actual value of \( \Pi^{SA} - \Pi^{FA} \) as a function of \( \mu \) while the dashed curve illustrates the approximated value from (12). The approximation is very accurate for low-\( \mu \) firms. The accuracy decreases as the tax rate differential increases for high-\( \mu \) firms due to the extensive margin effects. Figure 1 suggests that when \( t_1 > t_2 \), if SA is preferred by a firm with \( \mu = 0 \), then both the highest and lowest \( \mu \) firms will prefer SA and that the intensive margin approximation accurately explains the preferences of low-\( \mu \) firms while the extensive margin results of Proposition 1 accurately explains the preferences of high-\( \mu \) firms. In Figure 2 \( (t_1 < t_2) \), the intensive margin approximation alone explains the preferences of both low and high-\( \mu \) firms. When one also considers wage rate differences, the approximation errors from (12) are larger for low-\( \mu \) firms but the qualitative properties of the actual and approximated difference in indirect firm profit remain very similar.

Since the main benefit of using (12) is to identify the economic factors that influence each firm's choice of SA over FA, I now turn to Proposition 2 which restates (12) in a way that helps identify the key economic trade-offs associated with a firm's choice between SA and FA.

\textsuperscript{12} The error in this approximation is bounded by \( (t_2 - t_1)^3 \) times a constant that depends on the third derivatives of \( \Pi^{SA} - \Pi^{FA} \).
Proposition 2. Assume that at \( t_2 = t_1 \) and \( w_2 = w_1 \), a firm with marginal cost of capital \( \mu \) produces strictly positive output in both countries under SA and FA. For \( t_2 \) sufficiently close to \( t_1 \) and \( w_2 \) sufficiently close to \( w_1 \), country and formula symmetry imply that

\[
\Pi^{SA}(t_1, t_2, \cdot) - \Pi^{FA}(t_1, t_2, \cdot) \approx \frac{(t_2 - t_1)^2}{2} \cdot \frac{q_2^{SA}}{C''(0)} - \frac{-2\lambda_2^2 \pi^0}{(1-t_1)r_1''} + \frac{-\mu^2}{2(1-t_1)^3 r_1''} \cdot (K_2^{St} / q_2^{St} + 1)^2 + \frac{w_1}{2} \frac{\partial L_2^{SA}}{\partial t_2} \\
+ (t_2 - t_1)(w_2 - w_1) \cdot \left[ \frac{\lambda_2 \cdot \pi^0}{r_2''} \cdot \frac{L_2^{SA}}{q_2^{SA}} - \frac{\mu(K_2^{St} / q_2^{St} + 1)L_2^{SA} / q_2^{SA}}{2(1-t_1)r_2''} + \frac{1}{2} \left( L_2^{St} + w_1 \frac{\partial L_2^{SA}}{\partial w_2} \right) \right]
\]

(13)

where all output and input quantities and derivatives are calculated at \( t_2 = t_1 \) and \( w_2 = w_1 \).

First, to focus on the effect of tax differences assume that \( w_2 = w_1 \). The tax-only effect corresponds to the first line of (13) as the bracketed term equals

\[
\tau(\mu; t_1, w_1) \equiv \partial^2 \Pi^{SA}(t_1, t_1, w_1, w_1, \mu) / \partial t_2^2 - \partial^2 \Pi^{FA}(t_1, t_1, w_1, w_1, \mu) / \partial t_2^2.
\]

Notice that a firm's preferred tax method depends only on the sign of \( \tau \). \( \tau \) identifies three ways in which \( t_2 \) distorts country 2's tax base: through an income-shifting distortion (from the direct effect of a change in \( t_2 \) holding the output quantities and the conditional factor demands fixed), an output distortion (due to the indirect effect of a change of \( t_2 \) on \( q_1 \) and \( q_2 \) holding the conditional factor demands fixed), and a conditional factor demand distortion (due to the indirect effect of a change of \( t_2 \) on the conditional factor demands holding \( q_1 \) and \( q_2 \) fixed).

The first two terms of \( \tau \) equal the difference in the marginal income-shifting effects under SA and FA. Under SA, the income-shifting effect is positive as a larger difference between \( t_2 \) and \( t_1 \) creates a larger incentive for a firm to use its transfer price to shift income into the country with the lower tax rate. Recall from (4) that \( \Delta \) is the after-tax profit earned from income-shifting per unit of country 2 output. At equal tax rates, \( \partial^2 \Delta / \partial t_2^2 = 1 / C''(0) \). As a result, the optimal amount of income shifted per unit of country 2 output is increasing at an increasing rate as \( |t_2 - t_1| \) increases. The income-shifting effect under FA is also positive as the income-shifting
opportunities under FA increase with $|t_2 - t_1|$. However, a firm's ability to shift income under FA depends on $\lambda_1$ (or $\lambda_2$ since $\lambda_1 = -\lambda_2$ with formula symmetry), the rate at which country 1 output changes the fraction of total income taxed in country 1. Less convex transfer price auditing costs make SA more effective at shifting income while a more responsive formula (larger $\lambda_1$) makes FA more effective.

The third term in $\tau$ is positive (favoring SA) and represents the difference in country 2 output distortions between SA and FA. For both methods, a higher tax rate in country 2 lowers country 2 output which in turn increases marginal profit. However, the output distortion under SA is twice as large as the output distortion under FA because, with country and formula symmetry, an increase in $t_2$ increases the firm's effective tax rate, $T$, by a factor of $1/2$. Notice that this term is zero when $\mu = 0$ as both methods become pure profit taxes when $t_2 = t_1$.

The fourth term in $\tau$ is positive (favoring SA) and represents the difference in the conditional labor demand distortions under SA and FA. This term arises because an increase in $t_2$ gives firms the incentive to substitute tax-deductible labor for non-tax-deductible capital. The larger conditional labor demand distortion under SA arises for two reasons. First, with constant returns to scale in production, equal tax and wage rates imply that a change in $t_2$ has the same effect on conditional labor demands in country 2 under both methods, that is, $\partial L_2^{SA} / \partial t_2 = \partial L_2^{FA} / \partial t_2 \partial L_2^{FA} / \partial t_2$. Second, as with the third term, the effect of a change in $t_2$ on the effective tax rate under FA is only $1/2$ the change under SA. This term also equals zero when $\mu = 0$.

Since the output and conditional labor demand distortions created by a change in $t_2$ always favor SA, the only way for a change in $t_2$ to generate higher profit under FA is if the apportionment formula generates a larger income-shifting effect.

**Corollary 1.** Assume country symmetry and formula symmetry. For a firm with $\mu$ sufficiently
close to zero and \( t_2 \) sufficiently close to \( t_1 \), decreases in \( C''(0) \) and/or \( \lambda_1(q_1^{FA}, q_1^{FA}) \) strengthen the income-shifting distortion under SA.

Corollary 1 focuses on firms with low \( \mu \) because these are the firms for whom the relative income-shifting distortions will be the dominant effect in determining a firm's choice. It emphasizes that changes in auditing costs that decrease \( C''(0) \) encourage more income-shifting under SA. For example, if auditing costs were equal to \( \alpha C \) for \( \alpha > 0 \), smaller values of \( \alpha \) would imply a stronger income-shifting distortion under SA. It also emphasizes that the magnitude of the income-shifting distortion under FA is proportional to \( \lambda_1 \). With symmetric countries and equal tax rates, \( \lambda_1 = (1/4q_2) \cdot (r_2'/p_2) < 1/4q_2 \) under a sales-only formula while \( \lambda_1 = 1/4q_2 \) under an output formula.\(^{13} \) Thus, (13) implies that firms with a low \( \mu \) have a stronger incentive to prefer SA over a sales-only formula than over an output formula because the proportion of income taxed in country 1 is less elastic under the sales-only formula.

Among low- \( \mu \) firms, a preference for SA will also depend on the curvature of the apportionment formula. For example, with quadratic revenue functions and the perfect complements production function used to generate Figures 1 and 2,

\[
\frac{\partial \tau}{\partial \mu} = -\frac{1}{2(1-t_1)} + \frac{2\lambda_1(\pi')^2\lambda_{q_1}}{(1-t_1)^2}
\]

at \( \mu = 0 \). Thus, for \( \mu \) close to zero, the strongest preference among low- \( \mu \) firms for SA will occur at \( \mu = 0 \) if \( \lambda(\cdot,\cdot) \) is strictly concave in \( q_1 \) because \( \tau \) will be locally decreasing. Both the sales-only and the output formulas are concave in \( q_1 \).

Now consider how a small difference in wage rates modifies a firm's preference in the presence of a small difference in tax rates. Denote the bracketed term in line 2 of (13) by

\(^{13} \) One could also write the expressions for \( \lambda_1 \) in terms of \( q_1 \) as all sales quantities are equal with country symmetry and equal tax rates.
\( \omega(\mu; t, w) \). The sign of the wage effect depends on the sign of \( \omega \) and the signs of the tax and wage rate differentials because any wage effects require different tax rates. The first term in \( \omega \) is the income-shifting distortion under FA due to a difference in wage rates. This term is negative (favoring FA) as there is no income-shifting distortion under SA, because a change in \( w_2 \) has no effect on a firm's arm's-length price, \( w_1 \). The second term of \( \omega \) is the net output distortion. It is always positive (favoring SA). An increase in \( w_2 \) induces a firm to substitute away from sales in country 2 to mitigate the effect of a higher tax rate in country 2. This substitution effect is stronger under SA because FA reduces the impact of a higher tax by a factor of 1/2, as discussed above. The third term of \( \omega \) is the net distortion from a firm's conditional cost of labor in country 2. It is negative (favoring FA) only if the conditional demand for labor in country 2 is elastic, as an increase in \( w_2 \) induces a lower total conditional wage bill in country 2 and increases taxable income in country 2. With higher taxable income, the marginal effect of \( t_2 \) on firm profit is exacerbated but weaker under FA. Notice that for the lowest- \( \mu \) firms, the output distortion will be close to zero while the income-shifting distortion and the conditional country 2 labor cost distortion will both be negative if the conditional demand for labor by the firm in country 2 is elastic. This discussion then implies Corollary 2.

**Corollary 2.** Assume country and formula symmetry. Suppose that at equal tax rates and wages, a firm's conditional demand for labor in country 2 is elastic. If the higher-tax country also has the higher wage, then the wage difference will increase the incentive for firms with a sufficiently low cost of capital to choose FA.

For low- \( \mu \) firms, the effect of a wage differential on their choice is unambiguous with elastic conditional demand for labor in country 2 – the set of firms favoring FA will increase. However, if the conditional demand for labor in country 2 is inelastic, the wage effect will
generate opposing income-shifting and conditional demand distortions. For example, if
\[ F = \min \{k_i^f, l_i^f\} \] (as in the above examples), the conditional demand for labor in country 2 will be inelastic and the conditional demand distortion will dominate the income-shifting distortion for low-\( \mu \) firms.

6. Tax revenues.

I now turn to the issue of how a firm's optimal choice affects tax revenues in each country, for fixed tax rates, and tax competition incentives. In order to use earlier results without introducing additional notation, I will focus on country 2 tax revenues.

The tax revenue collected by country 2 from a firm that elects SA equals
\[
TR_2^{SA}(t_2, t_2) = t_2[r_2(q_2^{SA}) - w_2L_2^{SA}(q_2^{SA}, t_2) - w_1q_2^{SA} - \partial \Delta / \partial t_2 \cdot q_2^{SA}]
\] (15)
while the tax revenue collected by country 2 from a firm that elects FA equals
\[
TR_2^{FA}(t_1, t_2) = t_2(1 - \lambda(q_1^{FA}, q_2^{FA}))\pi^0(q_1^{FA}, q_2^{FA}).
\] (16)
For each method, country 2's tax revenues equal \( t_2 \) times its tax base, which by the Envelope Theorem is equal to \( \partial II^{SA} / \partial t_2 \) or \( \partial II^{FA} / \partial t_2 \). The formulas for country 1's tax revenues are analogous.

This connection between a country's tax revenues and a firm's profit is what allows us to determine the effect of firm choice on each country's tax revenues. Proposition 3 below is thus the analog to Proposition 2 for tax revenues. At equal tax rates, both methods generate the same tax revenue for each country. This means each country's preference between SA and FA in the presence of small tax rate and wage differences can be approximated by the first-order effects of tax rate and wage differentials on each country's tax base. To determine if a firm's preferred method is aligned with the preferred method of the low-tax or the high-tax country, define
\[
\Delta II \equiv (t_2 - t_1)^2 \tau(\mu; t_1, w_i) / 2 + (t_2 - t_1)(w_2 - w_1)\omega(\mu; t_1, w_i).
\] Recall from (13) that a firm prefers SA when \( \Delta II \) is positive and FA when it is negative.
Proposition 3. Assume that at \( t_2 = t_1 \) and \( w_2 = w_1 \), a firm with marginal cost of capital \( \mu \) produces strictly positive output in both countries. Then formula symmetry implies that

\[
TR_1^{St}(t_1, t_2, w_1, w_2, \mu) - TR_1^{FA}(t_1, t_2, w_1, w_2, \mu) \approx \frac{t_1}{t_2 - t_1} \left( \Delta II + \frac{(t_2 - t_1)^2}{2} \tau(\mu; t_1, w_1) \right) 
\]

and

\[
TR_2^{St}(t_1, t_2, w_1, w_2, \mu) - TR_2^{FA}(t_1, t_2, w_1, w_2, \mu) \approx \frac{-t_1}{t_2 - t_1} \left( \Delta II + \frac{(t_2 - t_1)^2}{2} \tau(\mu; t_1, w_1) \right). 
\]

With symmetric countries, so that \( w_2 = w_1 \), \( \Delta II \) and \( \tau \) will have the same sign,

\[
\frac{\partial TR_2^{St}}{\partial t_2} - \frac{\partial TR_2^{FA}}{\partial t_2} \bigg|_{t_2=t_1} \approx -t_1 \tau(\mu; t_1, w_1), 
\]

and (18) simplifies down to

\[
TR_2^{St}(t_1, t_2) - TR_2^{FA}(t_1, t_2) \approx (t_2 - t_1) t_1 \tau(\mu; t_1, w_1). 
\]

According to (20), country 2's preference between SA and FA is the same as each firm's preference on the intensive margin if, and only if, it has the lower tax rate. Since (17) simplifies down to \( TR_1^{St}(t_1, t_2) - TR_1^{FA}(t_1, t_2) \approx (t_2 - t_1) t_1 \tau(\mu; t_1, w_1) \), the same relationship exists for country 1.

Thus, in the absence of country or formula asymmetries, we have the following proposition.

Proposition 4. With country and formula symmetry, each firm on the intensive margin will prefer the same method as the (tax-revenue-based) preference of the low-tax country and the opposite method of the high-tax country. If firm choice increases the tax revenues one country collects from an intensive margin firm, it decreases the tax revenues collected by the other country.

Eq. (19) also has implications for the tax competition incentives under firm choice with country and formula symmetry. To calculate symmetric equilibrium tax rates, we first partition...
firms into three sets according to \( \mu \). For \( \mu \in \Omega^{SA}(t_1, t_2) \), a firm has positive sales in some country and weakly prefers SA to FA. For \( \mu \in \Omega^{FA}(t_1, t_2) \), a firm strictly prefers FA to SA. Any remaining firms sell in neither country (and are indifferent between SA and FA). Then country 2's total tax revenues under firm choice are defined as

\[
TR_2 = \int_{\mu \in \Omega^{SA}} TR_{2,SA} d\Phi(\mu) + \int_{\mu \in \Omega^{FA}} TR_{2,FA} d\Phi(\mu).
\]  

(21)

Because \( \Pi^{SA} = \Pi^{FA} \) at \( t_1 = t_2 \) implies that \( TR^{SA}_2 = TR^{FA}_2 \), Leibniz's Rule implies that

\[
\frac{\partial TR_2(t_1, t_2)}{\partial t_2} = \int_{\mu \in \Omega^{SA} \cup \Omega^{FA}} \frac{\partial TR_{2,SA}}{\partial t_2} d\Phi(\mu) + \int_{\mu \in \Omega^{FA}} \frac{\partial TR_{2,FA}}{\partial t_2} d\Phi(\mu) 
= \int_{\mu \in \Omega^{SA}} \frac{\partial TR_{2,SA}}{\partial t_2} d\Phi(\mu) - \int_{\mu \in \Omega^{SA}} t_1 \tau d\Phi(\mu) \leq \int_{\mu \in \Omega^{FA} \cup \Omega^{FA}} \frac{\partial TR_{2,FA}}{\partial t_2} d\Phi(\mu)
\]

(22)

where the second equality comes from (19) and the inequality is due to the fact that \( \tau > 0 \) for firms that prefer SA. In an equilibrium in which all firms must use FA, the integral on the right side of the inequality will be zero. If at the symmetric equilibrium tax rate under FA-only some firms strictly prefer SA, the inequality in (22) will be strict and the symmetric equilibrium tax with choice must be smaller. By the same reasoning, \( \partial TR_2 / \partial t_2 \leq \int \partial TR^{SA}_2 / \partial t_2 \cdot d\Phi(\mu) \) and \( \partial TR_1 / \partial t_1 \leq \int \partial TR^{m}_1 / \partial t_1 \cdot d\Phi(\mu) \) for \( m \in \{SA, FA\} \), which gives us Proposition 5.

**Proposition 5.** With country and formula symmetry, equilibrium tax rates under firm choice are strictly lower than the equilibrium tax rates under SA and under FA as long as at the SA-only and the FA-only equilibrium tax rates some firms prefer SA and some prefer FA.

If all firms prefer the mandated tax method under an SA-only or FA-only regime, the equilibrium tax rate with firm choice will not change. Because \( \partial TR^{SA}_2 / \partial t_2 > 0 \) and \( \partial TR^{FA}_2 / \partial t_1 > 0 \) at equal tax rates, the equilibrium tax rate in an SA-only regime must also be
smaller than the rate that would maximize joint tax revenues. Hence, by Proposition 5, firm choice would result in even lower equilibrium tax revenues. As in Nielsen, Raimondos-Møller, and Schjelderup (2010), who study a homogeneous firm model, the signs of $\partial TR^E_1 / \partial t_2$ and $\partial TR^E_2 / \partial t_1$ are ambiguous because a change in a country's tax rate will change both the global tax base as well as the fraction taxed by the country. This means the equilibrium tax rate in an FA-only regime could be either smaller or larger than the joint tax revenue maximizing rate. In the latter case, firm choice could result in higher joint tax revenues than under an FA-only system. Table 1 illustrates some of these relationships. It compares equilibrium tax rates and country tax revenues for the same wage rates and production, revenue, and auditing cost functions used to generate Figures 1 and 2. Identical patterns arise for examples with both smaller and larger demand. Decreasing auditing costs makes taxable income under SA more elastic and can flip the tax rate and tax revenue ranking between SA and FA but will not change the bottom tax-rate ranking of choice.

<table>
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<th>Tax System</th>
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<td>Joint Tax Revenue Maximization</td>
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<td>13.5</td>
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<tr>
<td>SA – only</td>
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<td>13.4985</td>
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<tr>
<td>FA – only</td>
<td>.491206</td>
<td>13.4977</td>
</tr>
<tr>
<td>Choice</td>
<td>.490744</td>
<td>13.4972</td>
</tr>
</tbody>
</table>

Table 1. Symmetric equilibrium tax rates and tax revenues

When $w_2 \neq w_1$, $\Delta IT$ and $\tau$ can have different signs. This means that the firm and each country value changes in taxable income differently. Thus, for some firms, their preferred method can differ from the preferred method of the low-tax country and be the same as the preferred method of the high-tax country. Despite this non-uniform alignment/misalignment of a country's preference with the preferences of all firms operating on the intensive margin, Proposition 3 implies that if one country's tax revenues increase with firm choice, the other country's tax revenues must decrease.
This strictly competitive implication of firm choice can break down when one (i) takes account of both intensive and extensive margin effects or (ii) introduces more dimensions of country asymmetries. To see the impact of (ii), assume that \( r_i(q_i) = (a_i - q_i)q_i \), so that countries can differ in terms of market size as well as wages. Propositions 2 and 3 generalize in natural ways. Figure 3 plots the actual (and not the approximated) SA-FA differences between indirect firm profit, country 1 tax revenues, and country 2 tax revenues when \( w_1 = 1, w_2 = .9, a_1 = 22, a_2 = 20, t_1 = .30, \) and \( t_2 = .50 \). All three curves reflect a preference for SA when \( 1 < \mu < 1.4 \) and all three curves reflect a preference for FA when \( 4 < \mu < 4.5 \). In this example, introducing wage and market size asymmetries increases the size of these regions in which the preferences of some firms are aligned with the preferences of both countries.

![Figure 3](image)

**Figure 3:** The SA-FA difference of indirect firm profit (thin line), country 1 tax revenues (thick line), and country 2 tax revenues (dashed line) as a function of \( \mu \) when \( w_1 = 1, w_2 = .9, a_1 = 20, a_2 = 18, t_1 = .30, \) and \( t_2 = .50 \). Tax revenue differences are scaled by .2 for visualization purposes.

If the countries initially require SA and the distribution of capital costs is concentrated about \((4,4.5)\), switching to firm choice would result in most firms choosing FA and higher tax revenues in both countries. The choices of firms with \( \mu \in (4,4.5) \) are aligned with each country's preferred method because the extensive margin under SA is smaller than under FA. For firms close to the SA extensive margin, a shift to FA will result in increased country 2 sales and increased tax revenues in each country – in country 1 from increased intermediate good

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14 Details are available from the author on request.
shipments and in country 2 from increased revenues. If the countries instead switch to an FA system without choice and if the distribution of capital costs is concentrated about (1,1.4), permitting firm choice would result in most firms choosing SA. Relative to the initial SA-only system, the countries would experience a smaller decrease in tax revenues with choice than without choice. In both scenarios, choice makes all firms weakly better off while a preponderance of firms whose optimal choice is aligned with each country's choice implies higher aggregate tax revenues for each country, at the given tax rates.

7. Summary and conclusions.

SA and FA represent two distinct methods for allocating a multinational's global taxable income to calculate national tax liabilities. In this paper, I have focused on how these two methods generate different income-shifting, output, and conditional labor demand distortions and how these distortions influence a firm's preference between these two methods if the apportionment formula depends solely on final good quantities. Proposition 2 reveals that, when symmetric countries have similar tax rates, the output and conditional labor demand distortions on the intensive margin always favor SA while the (intensive margin) income-shifting distortions can favor either method. At the same time, tax rate differences create extensive margin effects in which firms operating near the extensive margins strictly prefer SA to FA when the downstream country has the lower tax rate. In addition, Proposition 3 shows that the intensive margin incentives align firm preferences with the method that raises more tax revenues in the low-tax country and fewer tax revenues in the high-tax country. Extensive margin incentives, caused by large tax rate or wage rate differences, can introduce some misalignment of preferences between the firm and the low-tax country because firm choice affects the tax revenues through both tax-revenue-shifting behavior and tax-base efficiency effects. In some examples, this partial alignment can result in choice increasing the tax revenues in each country. Thus, Proposition 3 tells us that similar countries will not both favor a policy of choice.
In future research, it will be important to determine whether analogous results emerge for apportionment formulas that use factor shares as well as output shares. As noted above, introducing factor shares in apportionment formulas can create an incentive for firms to employ more capital and/or labor than they would need to meet output goals. It is also the case that permitting firm choice can affect where firms locate their intermediate good production. I leave these topics for future research.

References
Haufler, A. and F. Stähler, 2013, Tax Competition in a Simple Model with Heterogeneous Firms:


Appendix

Proof of Proposition 1. If one can show that SA is strictly preferred to FA on $[\bar{\mu}_1, \underline{\mu}_2)$, then the preference order can be extended to $\mu$ slightly less than $\bar{\mu}_1$ by continuity of the indirect profit function under SA.

Since $q_i^{SA} = q_i^{FA} = 0$ for all $\mu \in [\bar{\mu}_1, \underline{\mu}_2)$, we need to compare $\pi^{SA}(0, q_2^{SA})$ with $\pi^{FA}(0, q_2^{FA})$. Note that $\pi^{SA}(0, q_2^{SA}) - \pi^{FA}(0, q_2^{FA}) = \pi^{SA}(0, q_2^{SA}) - \pi^{SA}(0, q_2^{FA}) + \pi^{SA}(0, q_2^{FA}) - \pi^{FA}(0, q_2^{FA})$. Then note that $\pi^{SA}(0, q_2^{SA}) \geq \pi^{SA}(0, q_2^{FA})$ because $q_2^{SA}$ is profit-maximizing under SA while $\pi^{SA}(0, q_2^{SA}) - \pi^{FA}(0, q_2^{FA}) = \Delta(t_1, t_2)q_2^{FA} > 0$ because for all $q_2$, $L_2^{SA}(q_2) = L_2^{FA}(0, q_2)$, $K_2^{SA}(q_2) = K_2^{FA}(0, q_2)$, and $C(\cdot)$ is strictly convex. Thus, any firm with $\mu \in [\bar{\mu}_1, \underline{\mu}_2)$ will strictly prefer SA over FA. 

Q.E.D.

Proof of Proposition 2.

Recall that $\Pi^{SA}(t_1, t_2, w_1, w_2, \mu) = \Pi^{FA}(t_1, t_2, w_1, w_2, \mu)$ for all $t_1$. Consider values of $t_1$ and $w_2 = w_1$ for which a firm with capital cost $\mu$ produces strictly positive quantities in each country when $t_2 = t_1$. Note also that as in the proof of Lemma 2, that $\partial(\mu K_i^m + w_i(1 - t_i)L_i^m) / \partial x = 0$ for $m \in \{SA, FA\}$ and $x \in \{t_1, t_2, w_1, w_2, q_j\}$.

Then given (4),

$$\partial \Pi^{SA} / \partial t_2 = -(r_i + r_j - w_i(L_i^{FA} + q_i^{FA} + q_j^{FA}) - w_j q_j^{SA}) + \partial \Delta / \partial t_2 \cdot q_2^{SA}$$

(A.1)

where $\partial \Delta / \partial t_2 = (C')^{-1}(t_2 - t_1) = 0$ at $t_2 = t_1$, and given (9)

$$\partial \Pi^{FA} / \partial t_2 = (\lambda - 1)(r_1 + r_2 - w_1(L_1^{FA} + q_1^{FA} + q_2^{FA}) - w_2 L_2^{FA}) = (\lambda - 1)\pi^0.$$  

(A.2)

With $t_2 = t_1$, $q_i^{SA} = q_i^{FA}$, $L_i^{SA}(q, t, w) = L_i^{FA}(q, q_j, t, t_j, w)$, and $K_i^{SA}(q, t, w) = K_i^{FA}(q, q_j, t, t_j, w)$. Furthermore, country and formula symmetry implies that at $t_2 = t_1$, $\lambda = 1/2$ and

$$\partial \Pi^{SA}(t_1, t_1) / \partial t_2 = \partial \Pi^{FA}(t_1, t_1) / \partial t_2.$$

Next, for $t_2 = t_1$, $\partial \Pi^{SA} / \partial w_2 = -(1 - t_2)L_2^{SA}$ and $\partial \Pi^{FA} / \partial w_2 = -(1 - T)L_2^{FA}$ so for all $w_1, w_2$, and $\lambda$, $\partial \Pi^{SA} / \partial w_2 = \partial \Pi^{FA} / \partial w_2$ and $\partial^2 \Pi^{SA} / \partial w_2^2 = \partial^2 \Pi^{FA} / \partial w_2^2$. Thus, the second-order Taylor series expansion of $\Pi^{SA}(t_1, t_2, w_1, w_2, \mu) - \Pi^{FA}(t_1, t_2, w_1, w_2, \mu)$ about $t_2 = t_1$ and $w_2 = w_1$ implies
\[ \Pi^{SA} - \Pi^{FA} \approx \frac{(t_2 - t_1)^2}{2} \cdot \frac{\partial^2 (\Pi^{SA} - \Pi^{FA})}{\partial t_2^2} + (t_2 - t_1) (w_2 - w_1) \cdot \frac{\partial^2 (\Pi^{SA} - \Pi^{FA})}{\partial w_2 \partial t_2} \]  
(A.3)

where all the derivatives on the right side of (A.3) are evaluated at equal tax rates and wage rates.

As in the text, \( \tau(\mu; t_1, w_1) = \frac{\partial^2 (\Pi^{SA} - \Pi^{FA})}{\partial t_2^2} \bigg|_{t_1, t_2 = w_2 = w_1} \) and
\[ \omega(\mu; t_1, w_1) = \frac{\partial^2 (\Pi^{SA} - \Pi^{FA})}{\partial w_2 \partial t_2} \bigg|_{t_1, t_2 = w_2 = w_1}. \]

\( \tau(\mu; t_1, w_1) \) derivation: Differentiating (A.1) implies
\[ \frac{\partial^2 \Pi^{SA}}{\partial t_2^2} = \left[ -(r'_2 - w_2 \cdot \partial L^{SA}_2 / \partial q_2) - w_1 \right] \cdot \frac{\partial q^{SA}_2}{\partial t_2} + \frac{\partial^2 \Delta}{\partial t_2^2} \cdot q^{SA}_2 + w_2 \frac{\partial L^{SA}_2}{\partial t_2} \]  
(A.4)

where \( \frac{\partial^2 \Delta(t_1, t_2)}{\partial t_2^2} = 1/C''(0) \) and the first-order condition with respect to \( q_2 \) implies
\[ (1 - t_2) (r'_2 - w_2 \cdot \partial L^{SA}_2 / \partial q_2) = \mu(\partial K^{SA}_2 / \partial q_2 + 1) \]  
at \( t_2 = t_1 \).

Under constant returns to scale, comparative statics calculations imply that
\[ \partial K^{SA}_2(q_2; t_1, t_2) / \partial q_2 = K^{SA}_2 / q^{SA}_2, \]  
(A.6)
\[ \partial L^{SA}_2 / \partial t_2 = \frac{-w_2 F^2_k (K^{SA}_2)^2}{F_{LL} q^{SA}_2 (\mu K^{SA}_2 + (1 - t_1) w_2 L^{SA}_2)}, \]  
(A.7)

and
\[ \partial q^{SA}_2(t_1, t_2) / \partial t_2 = \mu(\partial K^{SA}_2 / \partial q_2 + 1) / ((1 - t_2)^2 r''_2). \]  
(A.8)

Therefore,
\[ \frac{\partial^2 \Pi^{SA}}{\partial t_2^2} \bigg|_{t_1, t_2 = w_2 = w_1} = -\frac{\mu^2}{(1 - t_1)^3 r''_2} \left( \frac{K^{SA}_2}{q^{SA}_2} + 1 \right)^2 + \frac{w_1 \partial L^{SA}_2}{\partial t_2} + \frac{q^{SA}_2}{C''(0)}. \]  
(A.9)

Differentiating (A.2) implies
\[ \frac{\partial^2 \Pi^{FA}}{\partial t_2^2} = (\lambda - 1)(r'_2 - w_1 (\partial L^{FA}_2 / \partial q_1 + 1) - w_2 \partial L^{FA}_2 / \partial q_2) \cdot \partial q^{FA}_2 / \partial t_2 + \\
(\lambda - 1)(r'_2 - w_1 (\partial L^{FA}_2 / \partial q_2 + 1) - w_2 \partial L^{FA}_2 / \partial q_2) \cdot \partial q^{FA}_2 / \partial t_2 + \\
(\lambda_1 \cdot \partial q^{FA}_1 / \partial t_2 + \lambda_2 \cdot \partial q^{FA}_2 / \partial t_2) \pi^0 + \\
(1 - \lambda)[w_1 \partial L^{FA}_1 / \partial t_2 + w_2 \partial L^{FA}_2 / \partial t_2]. \]  
(A.10)

where the first-order conditions with respect to \( q_1 \) and \( q_2 \) are
(1 - T)(r' - w_i(\partial L_i^{FA} / \partial q_i + 1) - w_2 \partial L_2^{FA} / \partial q_i) + \lambda_i(t - t_i)\pi^0 - \mu(\partial K_i^{FA} / \partial q_i + \partial K_2^{FA} / \partial q_i + 1) = 0 \tag{A.11}

and

(1 - T)(r' - w_i(\partial L_i^{FA} / \partial q_i + 1) - w_2 \partial L_2^{FA} / \partial q_i) + \lambda_2(t - t_i)\pi^0 - \mu(\partial K_i^{FA} / \partial q_2 + \partial K_2^{FA} / \partial q_2 + 1) = 0. \tag{A.12}

Under constant returns to scale, comparative statics calculations imply that at \( t_2 = t_1 \)

\[ \partial K_i^{FA} / \partial q_i = K_i^{FA} / q_i, \] \tag{A.13}

\[ \partial L_i^{FA} / \partial t_2 = -(1 - \lambda)w_iF_i^K(\partial K_i^{FA})^2 \] \[ F_i^{q_i} (\mu K_i^{FA} + (1 - t_i)w_i L_i^{FA}), \] \tag{A.14}

\[ \partial K_i^{FA} / \partial q_j = 0, \] \tag{A.15}

and

\[ \partial q_i^{FA} / \partial t_2 = \frac{(1 - \lambda)\mu \left( \frac{K_i^{FA}}{q_i^{FA}} + 1 \right) - \lambda_i \pi^0}{(1 - t_i)r_i''}. \] \tag{A.16}

Note: In general, the numerator for \( \partial q_i^{FA} / \partial t_2 \) also includes the terms

\( (1 - t_i)w_i \cdot \partial^2 L_j^{FA} / \partial t_2 \partial q_i + \mu \partial^2 K_j^{FA} / \partial t_2 \partial q_i \) for both \( j=i \) and \( j \neq i \). When \( j \neq i \), the expression equals zero at \( t_2 = t_1 \). When \( j=i \), the expression equals zero at \( t_2 = t_1 \) under constant returns to scale.

Therefore, given country and formula symmetry, substituting (A.11) – (A.16) into (A.10) implies that

\[ \frac{\partial^3 \Pi^{FA}(t_i, t_i, w_i, w_1, \mu)}{\partial t_1^3} = \frac{-\mu^2}{2(1 - t_i)^3 r_i''} \left( \frac{K_i^{FA}}{q_i^{FA}} + 1 \right)^3 + \frac{1}{2} \left[ w_i \partial L_i^{FA} / \partial t_2 + w_i \partial L_2^{FA} / \partial t_2 \right] - \frac{(\lambda_i^2 + \lambda_2^2)\pi^2}{(1 - t_i)r_i''}. \] \tag{A.17}

Using (A.7) and (A.14), the symmetry assumptions also imply that

\[ \partial L_2^{SA} / \partial t_2 = \partial L_1^{SA} / \partial t_2 + \partial L_2^{FA} / \partial t_2. \] \tag{A.18a}

Thus, (A.9) and (A.17) implies that

\[ \tau(\mu; t_i, w_i) = \frac{-2\lambda_i^2 \pi^2}{C''(0)} + \frac{-\mu^2}{2(1 - t_i)^3 r_i''} \cdot (K_2^{SA} / q_2^{SA} + 1)^2 + \frac{w_i \partial L_2^{SA} / \partial t_2}{2}. \] \tag{A.18b}

\( \omega(\mu; t_i, w_i) \) derivation: Differentiating (A.1) with respect to \( w_2 \), applying (A.5), and evaluating at equal tax rates and wage rates yields
\[
\frac{\partial^2 I^{SA}}{\partial w_2 \partial t_2} = L^{SA}_2 + w_2 \frac{\partial L^{SA}_2}{\partial w_2} - \frac{\mu}{(1-t_2)} \left( \frac{\partial K^{SA}_2}{\partial q_2} + 1 \right) \frac{\partial q^{SA}_2}{\partial w_2}.
\]

Similarly, differentiating (A.2) with respect to \( w_2 \), applying (A.11) and (A.12), and evaluating at equal tax rates and wage rates yields

\[
\frac{\partial^2 I^{FA}}{\partial w_2 \partial t_2} = (1-\lambda) \left( L^{FA}_2 + w_2 \frac{\partial L^{FA}_2}{\partial w_2} \right) + \left( \lambda_1 \frac{\partial q^{FA}_1}{\partial w_2} + \lambda_2 \frac{\partial q^{FA}_2}{\partial w_2} \right) \pi^0
\]

\[
+ (\lambda - 1) \frac{\mu}{(1-t_2)} \left( \frac{\partial K^{FA}_1}{\partial q_1} + 1 \right) \frac{\partial q^{FA}_1}{\partial w_2} + (\lambda - 1) \frac{\mu}{(1-t_2)} \left( \frac{\partial K^{FA}_2}{\partial q_2} + 1 \right) \frac{\partial q^{FA}_2}{\partial w_2}
\]

since \( \partial q^{FA}_i / \partial q_j = \partial K^{FA}_i / \partial q_j = 0 \) for \( j \neq i \) at equal tax rates and wage rates.

Next, totally differentiating (A.5) with respect to \( w_2 \) shows that

\[
\frac{\partial q^{SA}_2}{\partial w_2} = \left( L^{SA}_2 / q^{SA}_2 \right) / r_2^{''}
\]

while totally differentiating (A.11) and (A.12) with respect to \( w_2 \) shows that \( \frac{\partial q^{FA}_1}{\partial w_2} = 0 \) at \( t_2 = t_1 \) while \( \frac{\partial q^{FA}_2}{\partial w_2} = \left( L^{FA}_2 / q^{FA}_2 \right) / r_2^{''} \), also at \( t_2 = t_1 \). Furthermore at \( t_2 = t_1 \), \( \frac{\partial L^{SA}_2}{\partial w_2} = \partial L^{FA}_2 / \partial w_2 \). Finally, using the fact that for symmetric formulas, \( \lambda_1 = -\lambda_2 \), and that at equal tax rates and wage rates, \( q^{SA}_2 = q^{FA}_2 \) yields

\[
\omega(\mu; t_1, w_1) = \frac{\lambda_1 \pi^0}{r_2^{''}} \cdot \frac{L^{SA}_1}{q^{SA}_2} + \frac{1}{2} \left( L^{SA}_2 + w_1 \frac{\partial q^{SA}_2}{\partial w_2} \right) \frac{L^{SA}_2}{q^{SA}_2} - \frac{\mu}{2(1-t_1) r_2^{''}} \left( \frac{K^{FA}_2}{q^{FA}_2} + 1 \right) \frac{L^{SA}_2}{q^{SA}_2}.
\]

Substituting (A.18) and (A.21) into (12) yields (13).  \( Q.E.D. \)

**Proof of Proposition 3.**

Total revenue paid to country 1 from a firm operating under SA equals

\[
TR^{SA}_1 = t_1 \left[ t_1 - w_1 (L^{SA}_1 + q^{SA}_1) + \partial \Delta / \partial t_2 \cdot q^{SA}_2 \right]
\]

and total revenue paid to country 1 from a firm operating under SA equals

\[
TR^{FA}_1 = t_1 \lambda \pi^0 = t_1 \lambda \left[ r_1 + r_2 - w_1 (q^{FA}_1 + q^{FA}_2) - w_1 L^{FA}_1 - w_2 L^{FA}_2 \right].
\]

Eqs. (15) and (16) define the analogous country 2 functions. This proof will derive and compare the first-order Taylor series expansions of (15), (16), (A.22) and (A.23).

At \( t_2 = t_1 \), country and formula symmetry imply \( \lambda = 1/2 \), so

\[
TR^{SA}_1 (t_1, t_1, w_1, w_1, \mu) = TR^{FA}_1 (t_1, t_1, w_1, w_1, \mu) \quad \text{and} \quad TR^{SA}_2 (t_1, t_1, w_1, w_1, \mu) = TR^{FA}_2 (t_1, t_1, w_1, w_1, \mu).
\]

Beginning with country 2, the Envelope Theorem implies that \( \partial IT^{FA} / \partial t_2 = -(1-\lambda)\pi^0 \) so
\[ TR_{2}^{FA} = -t_{2} \cdot \partial II^{FA} / \partial t_{2} \cdot \partial t_{2}. \] Analogously, \( \partial II^{SA} / \partial t_{2} = -(r_{2} - w_{2}L_{2}^{SA} - w_{1}q_{2} - \partial \Delta / \partial t_{2} \cdot q_{2}) \) so 

\[ TR_{2}^{SA} = -t_{2} \cdot \partial II^{SA} / \partial t_{2}. \] Therefore, the first-order Taylor Series approximation of \( TR_{2}^{SA} - TR_{2}^{FA} \) about \( t_{2} = t_{1} \) is 

\[
TR_{2}^{SA}(t_{1}, t_{2}, w_{1}, w_{2}, \mu) - TR_{2}^{FA}(t_{1}, t_{2}, w_{1}, w_{2}, \mu) \approx \frac{t_{2}(t_{2} - t_{1})}{t_{2}} \left( \frac{\partial^{2} II^{FA}}{\partial t_{2}^{2}} - \frac{\partial^{2} II^{SA}}{\partial t_{2}^{2}} \right) \bigg|_{t_{2} = t_{1}, w_{2} = w_{1}} + t_{1}(w_{2} - w_{1}) \left( \frac{\partial^{2} II^{FA}}{\partial w_{2} \partial t_{2}} - \frac{\partial^{2} II^{SA}}{\partial w_{2} \partial t_{2}} \right) \bigg|_{t_{2} = t_{1}, w_{2} = w_{1}} 
\]

(A.24)

which is (18).

For country 1, an analogous derivation applies with regard to changes in \( t_{1} \) and not \( t_{2} \). Thus,

\[
TR_{1}^{SA}(t_{1}, t_{2}, w_{1}, w_{2}, \mu) - TR_{1}^{FA}(t_{1}, t_{2}, w_{1}, w_{2}, \mu) \approx t_{2}(t_{2} - t_{1}) \left( \frac{\partial^{2} II^{SA}}{\partial t_{1}^{2}} - \frac{\partial^{2} II^{FA}}{\partial t_{1}^{2}} \right) \bigg|_{t_{2} = t_{1}, w_{2} = w_{1}} + t_{1}(w_{2} - w_{1}) \left( \frac{\partial^{2} II^{FA}}{\partial w_{2} \partial t_{1}} - \frac{\partial^{2} II^{SA}}{\partial w_{2} \partial t_{1}} \right) \bigg|_{t_{2} = t_{1}, w_{2} = w_{1}} 
\]

(A.25)

Under formula symmetry, the right-hand side of (A.25) is equivalent to (17). \( Q.E.D. \)
Proof of Lemma 1. As preliminary information, recall that an interior solution to a firm’s profit-maximization problem under SA requires that \( q_i^{SA} \) solve (5) and \( q_2^{SA} \) solve (6). Thus, \( \mu_i^{SA} \) is defined by

\[
(1-t_i)(r_i'(0) - w \partial L_1^{SA}(0,t_i) / \partial q_i - w_i) = \mu_i^{SA} (\partial K_1^{SA}(0,t_i) / \partial q_i + 1)
\]  

(W.1)

and \( \mu_2^{SA} \) is defined by

\[
(1-t_2)(r_2'(0) - w_2 \partial L_2^{SA}(0,t_2) / \partial q_2 - w_i) + \Delta(t_1,t_2) = \mu_2^{SA} (\partial K_2^{SA}(0,t_2) / \partial q_2 + 1),
\]  

(W.2)

where the constant returns to scale assumption on final good production implies that

\[
\partial L_i^{SA} / \partial q_i = L_i^{SA} / q_i^{SA} \quad \text{and} \quad \partial K_i^{SA} / \partial q_i = K_i^{SA} / q_i^{SA}.
\]

Both derivatives are constants with respect to \( q_i^{SA} \) and thus are well-defined at \( q_i^{SA} = 0 \). As long as demand is sufficiently large, so that the firm will sell its output in both countries for some tax rates when \( \mu = 0 \), \( \mu_i^{SA} \) and \( \mu_2^{SA} \) are well-defined by (W.1) and (W.2).

Notice that \( \mu_2^{SA} \) depends on \( t_2 \) and \( t_1 \) while \( \mu_i^{SA} \) only depends on \( t_1 \). Thus, we can sign \( \mu_i^{SA} - \mu_2^{SA} \) by calculating \( \partial \mu_2^{SA} / \partial t_2 \) and using the fact that \( \mu_1^{SA} = \mu_2^{SA} \) when \( t_2 = t_1 \).

Differentiating (W.2) with respect to \( t_2 \) yields

\[
-(r_2'(0) - w_2 L_2^{SA} / q_2^{SA} - w_i) - w_2 (1-t_2) \partial (L_2^{SA} / q_2^{SA}) / \partial t_2 + \Delta / \partial t_2 = \mu_2^{SA} \partial (K_2^{SA} / q_2^{SA}) / \partial t_2 + (K_2^{SA} / q_2^{SA} + 1) \partial \mu_2^{SA} / \partial t_2.
\]  

(W.3)

Applying the Envelope Theorem to the firm’s cost-minimization problem implies that

\[
(\partial / \partial t_2)(w_2(1-t_2)L_2^{SA} / q_2^{SA} + \mu_2^{SA} K_2^{SA} / q_2^{SA}) = -w_2 L_2^{SA} / q_2^{SA},
\]

which is equivalent to

\[
w_2(1-t_2) \partial (L_2^{SA} / q_2^{SA}) / \partial t_2 + \mu_2^{SA} \partial (K_2^{SA} / q_2^{SA}) / \partial t_2 = 0.
\]  

(W.4)

Substituting (W.4) into (W.3) and imposing country symmetry then implies that

\[
-(r'(0) - w L_2^{SA} / q_2^{SA} - w) + \partial \Delta / \partial t_2 = (K_2^{SA} / q_2^{SA} + 1) \cdot \partial \mu_2^{SA} / \partial t_2,
\]  

(W.5)

where \( (t_2 - t_1) \partial \Delta / \partial t_2 > 0 \). Since \( \partial \Delta / \partial t_2 \), \( L_2^{SA} / q_2^{SA} \), and \( K_2^{SA} / q_2^{SA} \) are bounded and independent
of demand, for sufficiently large demand $\partial \mu_1^{SA}/\partial t_2$ will be strictly negative for all $t_1$ and $t_2$.

Because $\mu_1^{SA} = \mu_2^{SA}$ when $t_2 = t_1$, $\partial \mu_2^{SA}/\partial t_2 < 0$ implies that $(\mu_1^{SA} - \mu_2^{SA})(t_1 - t_2) < 0$ for all $t_2 \neq t_1$.

**Q.E.D.**

**Proof of Lemma 2.** The assumption of sufficiently large demand is needed to guarantee that for some tax rates, there exists a $\mu \geq 0$ for which the firm is willing to produce in at least one country.

Step 1. Factor demand comparative statics. $(L_i^{FA}, K_i^{FA})$ solves (i) $F(k_i^f, l_i^f) = q_i$ and (ii) $F_L(k_i^f, l_i^f)/(w_i(1 - T)) = F_K(k_i^f, l_i^f)/\mu$.

First, differentiating (i) and (ii) with respect to $q_i$ yields $F_K \cdot \partial K_i^{FA}/\partial q_i + F_L \cdot \partial L_i^{FA}/\partial q_i = 1$ and $(\mu F_{LL} - w_i(1 - T)F_{KL}) \cdot \partial L_i^{FA}/\partial q_i + (\mu F_{LK} - w_i(1 - T)F_{KK}) \cdot \partial K_i^{FA}/\partial q_i = -w_iT_iF_K$. Since $F(K, L)$ exhibits constant returns to scale, for all $K$ and $L$, $F_{KL} = -(K/L)F_{KK}$ and $F_{LL} = (K^2/L^2)F_{KK}$. Solving this system of equations for $\partial K_i^{FA}/\partial q_i$ and $\partial L_i^{FA}/\partial q_i$ implies that $\partial K_i^{FA}/\partial q_i = (-w_iT_iF_KF_L - \mu F_{LL} + w_i(1 - T)F_{KL})/X$ and $\partial L_i^{FA}/\partial q_i = (\mu F_{KL} - w_i(1 - T)F_{KK} + w_iT_iF_K^2)/X$ where $X > 0$ from the second-order conditions associated with each firm's cost-minimization problem. Second, differentiating (i) and (ii) with respect to $q_j$ yields $F_K \cdot \partial K_i^{FA}/\partial q_j + F_L \cdot \partial L_i^{FA}/\partial q_j = 0$ and $(\mu F_{LL} - w_i(1 - T)F_{KL}) \cdot \partial L_i^{FA}/\partial q_j + (\mu F_{LK} - w_i(1 - T)F_{KK}) \cdot \partial K_i^{FA}/\partial q_j = -w_iT_iF_K$. This system of equations implies that $\partial K_i^{FA}/\partial q_j = -w_iT_iF_KF_L/X$ and $\partial L_i^{FA}/\partial q_2 = w_iT_iF_K^2/X$.

These comparative statics imply (a) $w_i(1 - T) \cdot \partial l_i^{FA}/\partial q_i + \mu \cdot \partial K_i^{FA}/\partial q_i = \mu / F_K(K_i^{FA}, l_i^{FA})$, (b) $w_i(1 - T) \cdot \partial l_i^{FA}/\partial q_j + \mu \cdot \partial K_i^{FA}/\partial q_j = 0$, (c) $\partial F_K(K_i^{FA}, l_i^{FA})/\partial q_i = w_iT_iF_K(F_KF_{KL} - F_LF_{KK})/X$, and (d) $\partial F_K(K_i^{FA}, l_i^{FA})/\partial q_j = w_iT_iF_K(F_KF_{KL} - F_LF_{KK})/X$.

Step 2. Without loss of generality, assume $t_1 < t_2$. This assumption implies that $T_1 < 0$, $T_2 > 0$, $\partial F_K(K_i^{FA}, l_i^{FA})/\partial q_1 < 0$, and $\partial F_K(K_i^{FA}, l_i^{FA})/\partial q_1 > 0$. If $q_i^{FA} > 0$, then the results (a) and (b) from Step 1 allow one to write (10) as
\[(1 - T)(r'_i(q^{FA}_i) - w_i) - \mu = \mu / F_K(K^{FA}_i, L^{FA}_i) + T_i\pi_0.\] 

(W.6)

Thus, \(1 - T)(r'_i(q^{FA}_i) - w_i) - \mu > 0\). By country symmetry, \(r'_2(q^{FA}_2) = r'_i(q^{FA}_2)\) so the concavity of \(r(\cdot)\) then implies that \(1 - T)(r'_i(q^{FA}_i) - w_i) - \mu > 0\).

Contrary to the statement of the lemma, now assume that \(q^{FA}_1 \leq q^{FA}_2\). By country symmetry and the concavity of the \(r(\cdot)\), it must then be that

\[(1 - T)(r'_i(q^{FA}_i) - w_i) - \mu \geq (1 - T)(r'_i(q^{FA}_i) - w_i) - \mu.\]

With \(T_1 < 0\) and \(T_2 > 0\), this can only happen for strictly positive \(q^{FA}_i\) if \(F_K(K^{FA}_1, L^{FA}_i) < F_K(K^{FA}_2, L^{FA}_2)\). By country symmetry, \(q^{FA}_1 = q^{FA}_2\) implies that \(F_K(K^{FA}_1, L^{FA}_i) = F_K(K^{FA}_2, L^{FA}_2)\). If \(q^{FA}_1 < q^{FA}_2\), then \(\partial F_K(K^{FA}_1, L^{FA}_i) / \partial q_i < 0\) and

\(\partial F_K(K^{FA}_2, L^{FA}_2) / \partial q_i > 0\) imply \(F_K(K^{FA}_1, L^{FA}_i) > F_K(K^{FA}_2, L^{FA}_2)\). Thus, the assumption that \(q^{FA}_1 \leq q^{FA}_2\) when \(t_1 < t_2\) is contradicted. An analogous argument applies when \(t_1 > t_2\).

Q.E.D.
Highlights

- Multinationals can choose between separate accounting (SA) or formula apportionment (FA).
- Firms producing an intermediate good predominately for export prefer SA.
- A firm's preference for SA over FA can be non-monotonic in its cost of capital.
- Choice lowers symmetric equilibrium tax rates.
- Choice can increase tax revenues with asymmetric countries and fixed tax rates.