Problem Set 3

Do problem 4a or 4b or 4c, not all three

(1.) Backwards Induction and Subgame Perfection: Find all subgame-perfect equilibria of the following games. Explicitly write down the behavior strategies that support a subgame-perfect equilibrium (don’t just point out the equilibrium path through the game). Are there any Nash equilibria that aren’t sub-game perfect?

(a.)

```
1
   L  R
 a  2  b  c  d
10,3 2,-1 2,3 4,7
```

(b.)

```
1
   L  R
 a  2  b  c  d
0,10 -3,2 3,1 -6,-2
```

(c.)

```
1
   L  R
 a  2  b  c  d
10,3 -1,3 2,1 5,4
```

(2). Suppose there is a single manufacturer, $M$ and two retailers $R_1$ and $R_2$. First, the manufacturer posts a price $h$ for its good; Second, the two retailers observe $h$ and simultaneously place orders $q_1$ and $q_2$ for the manufacturer’s good (the two retailers participate in a simultaneous Cournot market, where their marginal cost is $h$).

The market price of the good is $p(q_1, q_2) = A - (q_1 + q_2)$. The manufacturer’s total costs of production are $C(q_1 + q_2) = c(q_1 + q_2)$.

(a). Solve for a subgame perfect Nash equilibrium of the game. What are the firms’ profits? What is the market price and market quantity?

(b). Suppose the two retailers merge into a single firm, or horizontally integrate, so that there is a single manufacturer and a single retailer. Is this profitable for the retailers (in the sense that the joint profits are higher after
the merger)?

(c). Suppose the manufacturer decided to *vertically integrate* by buying one of the retailers and refusing to sell its good to the other retailer. Is this profitable for the manufacturer and integrated retailer, in the sense that the vertically integrated monopolist generates more profits than the manufacturer and a single retailer make in the presence of the other firm? (Or, can the manufacturer and retailer make more money by cutting the other retailer “out of the loop”?)

(d). In class, we looked at the case with a single retailer and concluded that welfare for consumers will improve from vertical integration, since price goes down and quantity goes up. Are the same conclusions true in b and c here? When does consumers welfare go up or down? Briefly explain your results.

(By the way, the correct answer for part c can flip when we’re looking at price competition rather than quantity competition, which is another reason to keep track of the difference between strategic complements and strategic substitutes in studying markets).

(3). There is a firm that produces specialized machines, which require a substantial investment in learning the details of the client’s business; think of industrial sewing machines or equipment for physics laboratories. First, the firm invests effort $e$ in producing the machine, at cost $\frac{c}{2}e^2$. The client’s profits are increasing in the effort put into the machine, with $\pi(e) = ae$. Second, the client proposes a trading price $p$. If the firm refuses to sell the machine at the proposed price, the firm can sell the machine on the market for a payoff $be$, $b < a$. If the client has no machine, the client gets a pay-off of $r$, with $r$ small.

(a.) Draw an extensive form representing this game and solve for a subgame perfect Nash equilibrium.

(b.) Solve the social planner’s problem:

$$\max_e (ae - p) + (p - \frac{c}{2}e^2)$$

and thereby find the efficient level of effort. Is it greater or less than the subgame perfect Nash level? Explain your results.

(d.) Does the equilibrium choice of effort, $e^*$, depend on the client’s outside option, $r$? Is this realistic? Briefly explain.

(4a). Imagine an alternating-offer bargain game, but instead of discounting future payoffs by $\delta$ and keeping the size of the pie fixed at 1, assume that in each period the value of the ‘pie’ is reduced by one — In the first period, $v_1 = V$, in the second $v_2 = V - 1$, in the third $v_3 = V - 2$, etc., and there is no discounting.

(a). Find the subgame perfect Nash equilibrium for $V = 3$ and $V = 4$.

(b). Solve for the subgame perfect Nash equilibrium for arbitrary $V$ (you can guess at the solution, then explain why you think it is true).

(4b). As a regulator, you might be worried about *predatory investment*: When an incumbent firm over-invests to keep entrants out of the market. But what does predatory investment even mean?
In our model, we have an incumbent firm $i$ and an entrant $e$. First, the incumbent chooses either to invest in cost-reducing research that reduces its marginal costs to zero at a cost of 5, or no investment in cost-reducing research, in which case its marginal cost is 2. Second, the entrant observes the investment choice and decides whether to pay a fixed cost $1/2$ to start its business and enter the market with a marginal cost of 2, or not. Third, if entry occurs, they become Cournot duopolists. If entry does not occur, the entrant gets a payoff of zero, and the incumbent becomes a monopolist.

The market price is $p(q_e, q_i) = 5 - q_e - q_i$. The total costs for the incumbent if it invested is 5; the total costs for the incumbent if it didn’t invest are $2q_i$. The total costs of production for the entrant if entry occurs are $2q_e + 1/2$.

(a). Solve for the firms’ equilibrium choice of quantity in the Cournot Duopoly, given that the entrant chose to enter (note that this depends the investment level of the incumbent). Solve for the incumbent’s monopoly strategy, given that the entrant chose not to enter (note that this depends the investment level as well). Compute the profits of the firms in each scenario.

(b). Solve for a subgame perfect equilibrium of the entry game, using the payoffs you computed in part (a). What happens in equilibrium?

(c). Consider the following definition:

Investment is predatory if (i) the incumbent chooses a level of investment in the presence of the potential entrant that results in the entrant staying out of the market and (ii) in the absence of the potential entrant, the incumbent would have chosen a lower level of investment.

Does predatory investment occur in our model? Is this a good definition of predatory investment? Identify one practical or theoretical problem with using it, and briefly explain either how your problem can be solved or why it is impossible to fix the definition. Should predatory investment be illegal? Briefly explain.

(d). Briefly explain how investment in this model can behave as a commitment device for the incumbent to be more aggressive in dealing with any potential entrants.

(4c). We mentioned that if the buyer privately knew his value of the good, the seller would have a more difficult time capturing all the value as happens in the dictator game. This question explores this idea.

There is a single buyer and a single seller. The buyer’s value is distributed uniformly between zero and 1, so $Pr[v_b < x] = F(x) = x$ and $Pr[v_b > x] = 1 - F(x) = 1 - x$. The buyer knows her value, but the seller does not. The seller has zero value for the good. The buyer’s utility is $u_b = \delta^k(v - t)$ if they trade at price $t$ at date $k$, and zero otherwise. The seller’s utility is $u_s = \delta^k t$ if they trade at price $t$ at date $k$, and zero otherwise. There are potentially two trading dates: $k = 0, 1$.

(a). If the seller gets to make a single offer to the buyer — announces a $t$ which the buyer can accept or reject — what should it be? What is the probability that trade occurs?
(b). If the seller gets to make two offers — $t_0$ and $t_1$ — and both players discount the second trade by $\delta$ as described above, what should the offers be? Think of it like this: There is a range of values $[\hat{v}, 1]$ for which the buyer accepts right away, and a range $[\bar{v}, \hat{v}]$ where the buyer accepts in the second period, and a range $[0, \bar{v}]$ where the buyer rejects both offers (why?). So, there is an agent $\hat{v}$ for whom $\hat{v} - t_0 = \delta(\hat{v} - t_1)$, and an agent $\bar{v}$ for whom $\delta(\bar{v} - t_1) = 0$. Solve backwards.

(c). How do your equilibrium $t_0$ and $t_1$ depend on $\delta$? As $\delta \to 1$, what happens to $t_0$ and $t_1$?

(d). Does the seller make more profits from making two offers as $\delta \to 1$, or a single offer? Coase conjectured that as the time between offers went to zero, a monopolist would be unable to maintain high prices, and would end up with low profits. What do you observe here?