Procurement with Dynamic Uncertainty

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Introduction

How are large or complex projects generally procured?
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- "When the preliminary engineering on the Big Dig started in the 1980s, the Massachusetts Highway Department had strict bidding requirements governed by Massachusetts law," says Carl Gottschall, project administrator at FHWA’s Massachusetts Division Office. "Design-bid-build was our only option for project delivery ...."

This Paper

- Model the project as a risky venture whose likelihood of success varies over time
- Allow the firms to engage in misappropriation of funds and shirking
- Award the project through a competitive bidding system, but tailor the project terms to vary over time to mitigate inefficient behavior
- Shift from thinking about procurement as “cost minimization” to “project management”
Model

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- Let the hazard rate of success be \( h(t) = g(t)/(1 - G(t)) \), where \( t \) is the amount of time actually worked.
If the principal could observe everything or control the project himself, the value of the project would evolve as

\[ J(t) = h(t) \Delta e^{\rho \Delta} v + (1 - h(t) \Delta) e^{-\rho \Delta} J(t + \delta) - c \Delta \]

yielding an objective function

\[ \max_{T(c)} \int_{T(c)}^0 (1 - G(z)) e^{-\rho z} (h(z)v - c) dz \]

with solution \( h(T^*(c)) = c \), as long as \( h(t) \) is decreasing.
Siphoning and moral hazard

Firms can pocket a portion $0 \leq \lambda \leq 1$ of the funds paid from the principal, and spend the $1 - \lambda$ portion in maintaining an illusion of productivity. The principal can only observe whether success has been achieved at the end of the contract, or when a firm comes forward to claim success.
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- A firm bids spuriously, and exerts no effort for the entire contractual horizon.
- A firm works for some amount of time, but upon realizing the project is “doomed”, gives up and starts siphoning funds.
- A firm works, succeeds, and then delays reporting this to the principal until a later, more lucrative date.
Direct Revelation Mechanisms

Let $\hat{C} = (\hat{c}_i, \hat{C}_{-i})$ be any vector of costs that might be realized for the firms. A direct revelation mechanism is a set of functions $(P_i(\hat{C}), T_i(\hat{C}), B_i(\hat{C}), r_i(t, \hat{C}), w_i(t, \hat{C}))$, where

- $P_i(\hat{C})$, the probability that $i$ wins, given announcements $\hat{C}$
- A terminal date, $T_i(\hat{C})$, giving the date at which the winner is declared to be in default
- A (surety) bond, $B_i(\hat{C})$, giving the amount of money forfeited to the principal by a firm who defaults
- A flow payment, $r_i(t, \hat{C})$, giving the payment from the principal to the firm at each date $t$
- An award, $w_i(t, \hat{C})$, giving a payment from the principal to the firm for success at date $t$

By the revelation principle, we can restrict attention to incentive compatible direct revelation mechanisms.
Approach

- Determine the value of winning the project, for any report $c'$ and type $c$, $J_i^*(c', c)$
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- Given the value of winning the project, characterize incentive compatibility in terms of the initial report
- Determine the project terms that induce each type $c$ to report success immediately and work for all dates in $[0, T_i(c)]$, and check that this is incentive compatible for all other types that might deviate and report $c$. 
Profit-maximizing reporting delay

Given that a firm has succeeded at time $t$, it should pick the most profitable date to reveal this information to the principal:

$$\tilde{w}_i(t, c') = \max_{s \geq t} e^{-\rho(s-t)}w_i(s, c') + \lambda \int_t^s e^{-\rho(z-t)}r_i(z, c')dz$$

$$= e^{-\rho(s_i(t, c')-t)}w_i(s_i(t, c'), c') + \lambda \int_t^{s_i(t, c')} e^{-\rho(z-t)}r_i(z, c')dz$$

Delayed award

Misappropriated funds

So $\tilde{w}_{it}(c')$ is the effective award accruing to a firm who reported type $c'$ and succeeded at time $t$. 
Since success is a function of how long the firm has \textit{actually} been working — not the calendar date — let $\epsilon_{it}(c', c)$ be the effort choice of a firm of type $c$ who reported $c'$ at date $t$. Then

$$E_i(t, c', c) = \int_{0}^{t} \epsilon_{iz}(c', c)dz$$

is the \textit{accumulated effort}.
Valuing ongoing contracts

Exerting effort for a moment $\Delta$ at time $t$ for a firm who reported $c'$ with true type $c$ gives an expected payoff

$$J_i(t, E_i(t, c', c), c', c) = e^{-\rho \Delta} \Delta h(E_i(t, c', c)) \tilde{w}_i(t, c')$$

Payoff from success

$$+ e^{-\rho \Delta} (1 - \Delta h(E_i(t, c', c))) J_i(t + \Delta, E_i(t, c', c) + \Delta, c', c)$$

Payoff from failure

$$+ \Delta (r_i(t, c') - c)$$

Flow value
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Payoff from failure

$$+ \Delta (r_i(t, c') - c)$$

Flow value

while shirking for a moment $\Delta$ at time $t$ gives an expected payoff

$$J_i(t, E_i(t, c', c), c', c) = e^{-\rho\Delta} J_i(t + \Delta, E_i(t, c', c), c', c)$$

Project value

$$+ \Delta \lambda r_i(t, c')$$

Flow value
Valuing ongoing contracts

By taking the limits as $\Delta \to 0$ and re-arranging, the contracts can be valued recursively as

$$
(1 - G(E_i(t_a, c', c))) e^{-\rho t_a} J_i(t_a, E_i(t_a, c', c), c', c) = \\
\text{Today's value from working on } [t_a, t_b] \\
\int_{t_a}^{t_b} (1 - G(E_i(z, c', c))) e^{-\rho z} \{ h(z) \tilde{w}_i(z, c') + r_i(z, c') - c \} \, dz \\
+ (1 - G(E_i(t_b, c', c))) e^{-\rho t_b} J_i(t_b, E_i(t_b, c', c), c', c) \\
\text{Discounted, expected project value}
$$
Valuing ongoing contracts

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\]

Today's value from working on $[t_a, t_b)$

\[
\int_{t_a}^{t_b} \left(1 - G(E_i(z, c', c)))e^{-\rho z} \{ h(z) \tilde{\omega}_i(z, c') + r_i(z, c') - c \} \right) dz
\]

\[
+ (1 - G(E_i(t_b, c', c)))e^{-\rho t_b} J_i(t_b, E_i(t_b, c', c), c', c)
\]

Discounted, expected project value

and

\[
(1 - G(E_i(t_a, c', c)))J_i(t_a, E_i(t_a, c', c), c', c) =
\]

Today's value from shirking on $[t_a, t_b)$

\[
\int_{t_a}^{t_b} (1 - G(E_i(z, c', c)))e^{-\rho z} \lambda r_i(z)(c')dz
\]

\[
+ (1 - G(E_i(t_b, c', c)))e^{-\rho t_b} J_i(t_b, E_i(t_b, c', c), c', c)
\]
Linear optimal control formulation

By working backwards from the default value,

\[-(1 - G(E_i(T_i(c'), c', c)))e^{-\rho T_i(c')}B_i(c')\]

the firm’s objective is

\[\int_0^{T_i(c')} \epsilon_{it}(c', c) \left(1 - G(E_{it})\right)e^{-\rho t} \left\{ h(E_{it})\tilde{\omega}_{it} + r_{it} - c \right\} y_1(t, c', c, E_{it}) \]
\[+ (1 - \epsilon_{it}(c', c)) \left(1 - G(E_{it})\right)e^{-\rho t} \left\{ \lambda r_{it} \right\} dt y_0(t, c', c, E_{it}) \]
\[-(1 - G(E_{iT_i(c')}))e^{-\rho T_i(c')}B_i(c')\]

subject to \(\epsilon_{it} \in [0, 1], \dot{E}_{it} = \epsilon_{it}, E_{i0} = 0.\)
The Hamiltonian is

\[ H(\epsilon_{it}; t, E_{it}, c', c) = \epsilon_{it} y_1(t, c', c, E_{it}) + (1 - \epsilon_{it}) y_0(t, c', c, E_{it}) + \mu_E(t) \epsilon_{it} \]
The Hamiltonian is

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Then if \( y_1(t, c', c, E_{it}) - y_0(t, c', c, E_{it}) + \mu_E(t) \geq 0 \), the firm exerts effort, but not otherwise.
The Hamiltonian is

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- Then if \( y_1(t, c', c, E_{it}) - y_0(t, c', c, E_{it}) + \mu_E(t) \geq 0 \), the firm exerts effort, but not otherwise.
- From Pontryagin’s necessary conditions, the value of effort at time \( t \) given accumulated effort profile \( E_{it} \) is

\[
\mu_E(t) = \int_t^{T_i(c')} -\epsilon_{it} \frac{\partial y_1}{\partial E_{it}} - (1 - \epsilon_{iz}) \frac{\partial y_0}{\partial E_{iz}} \, dz + g(E_{iT_i(c')} (c', c)) e^{-\rho T_i(c')} B_i(c')
\]

“Salvage value” transversality condition.
The Value Function

Lemma

Let $J_i^*(c', c)$ be the optimized value of the value function. Then $J_i^*(c', c)$ is upper semi-continuous in the report $c'$ and the true type $c$. Where differentiable,

$$
\frac{\partial J_i^*(c', c)}{\partial c} = \int_0^{T_i(c')} \epsilon_{it}(c', c) \frac{\partial y_1(t, E_{it}(c', c), c', c)}{\partial c} dt
$$

$$
= - \int_0^{T_i(c')} \epsilon_{it}(c', c)(1 - G(E_{it}(c', c))) e^{-\rho t} dt
$$

Assume the value function is almost everywhere differentiable.
Expected Utility

Then a firm’s expected payoff from participating in the mechanism is

\[ U_i(c', c) = E_{C_{-i}} \left[ P_i(c', C_{-i}) J^*_i(c', c) \right] \]

A mechanism is individually rational if

\[ U_i(c, c) \geq 0 \]

and incentive compatible if, for all \( c' \neq c \),

\[ U_i(c, c) \geq U_i(c', c) \]
Incentive Compatibility

Let $c_i^*$ be the worst-off $i$-type firm that participates in the mechanism. Then a mechanism is incentive compatible iff the envelope theorem

$$U_i(c_i) = \int_{c_i}^{c_i^*} \mathbb{E} \left[ P_i(x, C_{-i}) \int_0^{T_i(x)} \epsilon_{iz}(x, x) \frac{\partial y_1(z, E_i(z, x, x), x, x)}{\partial c} \partial y \right] dz \] \) dx

and $\mathbb{E}_{C_{-i}} [P_i(c', C_{-i}) J_i^*(c', c)]$ is supermodular in $(c', c)$. 
Implementing immediate reporting

Consider the maximization problem

\[
\tilde{w}_i(t, c') = \max_{0 \leq s \leq T_i(c')} e^{-\rho(s-t)} w_i(s, c') + \lambda \int_t^s e^{-\rho(z-t)} r_i(z, c') dz
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\]

A necessary condition at any optimal \( s_{it}^*(c') < T_i(c') \) is

\[
e^{-\rho(s_{it}^*(c')-t)} \left( -\rho w_i(s_{it}^*(c'), c') + \frac{\partial w_i(s_{it}^*(c'), c')}{\partial t} + \lambda r_i(s_{it}^*(c'), c') \right) \, dz = 0
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\]

Solving the differential equation in parentheses yields

\[
w_i(t, c') = e^{\rho t} K_i(c') - \int_0^t e^{-\rho(z-t)} \lambda r_i(z, c') dz
\]

Substituting this into the objective yields a function independent of \(s\)
Minimizing siphoning incentives

Since the principal is constrained to offer \( r_i(t, c') \geq c' \), setting 
\( r_i(t, c') = c' \) minimizes siphoning incentives. Then the award function becomes

\[
\psi_i(t, c') = e^{\rho t} K_i(c') - \int_0^t e^{-\rho(z-t)} \lambda c' \, dz
\]
Pinning down $K_i(c')$ in terms of $B_i(c')$

Equating the direct and indirect utility functions gives an expression of $K_i(c')$ in terms of $B_i(c')$:

$$p_i(c') \int_0^{T_i(c')} e^{-\rho z} g(z) \left\{ e^{\rho z} K_i(c') - \lambda c \int_0^z e^{-\rho(x-z)} dx \right\} dz$$

$$- (1 - G(T_i(c'))) e^{-\rho T_i(c')} B_i(c')$$

$$= \int_{c'}^{c_i^*} p_i(x) \int_0^{T_i(x)} e^{-\rho z} (1 - G(z)) dz dx$$

$$U_i(c')$$
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$$= \int_{c'}^{c_i^*} p_i(x) \int_0^{T_i(x)} e^{-\rho z} (1 - G(z)) dz dx$$

from which it follows that

$$K_i(c') =$$

$$\frac{U_i(c')}{p_i(c')} + e^{-\rho T_i(c')} (1 - G(T_i(c'))) B_i(c') + \lambda c' \int_0^{T_i(c')} g(z) \int_0^z e^{-\rho x} dx dz$$

$$\frac{G(T_i(c'))}{G(T_i(c'))}$$
The minimal bond

Returning to the necessary condition to exert effort,

\[ y_1(t, E_{it}) - y_0(t, E_{it}) + \mu_E(t) \geq 0 \]
The minimal bond

Returning to the necessary condition to exert effort,

\[ y_1(t, E_{it}) - y_0(t, E_{it}) + \mu_E(t) \geq 0 \]

with \( E_i(t, c, c) = t \) and \( \epsilon_{it}(c, c) = 1 \), the inequality is equivalent to

\[
g(T_i(c)) \left( e^{-\rho T_i(c)} w_{iT_i(c)}(c) + e^{-\rho T_i(c)} B_i^{min}(c) \right) \geq (1 - G(t)) e^{-\rho t} \lambda c - \lambda c \int_t^{T_i(c)} e^{-\rho z} g(z) dz
\]

This pins down \( B_i^{min}(c) \).
A “no arbitrage” condition

- If exclusion of some types is desirable ex ante, or if high-cost firms can make profits by pretending to be low-cost firms, there will be deviations.
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To stop these kinds of deviations, consider the **no arbitrage bond**

\[
B^\text{NA}_i(c) = e^{\rho T_i(c)} \int_0^{T_i(c)} e^{-\rho x} \lambda cdx
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\]

Then to ensure that firms all work the intended horizon and no arbitrage is possible, set the bond to

\[
B_i(c) = \max \left\{ b_i^{NA}(c), b_i^{min}(c) \right\}
\]

which pins down the arbitrary constant \( K_i(c) \).
The optimal project horizon

The principal’s objective is

$$
\mathbb{E} \left[ \sum_i P_i(C) \left\{ \int_0^{T_i(c_i)} e^{-\rho t} (1 - G(t)) (h(t)(v - w_{it}(c_i)) - c_i) \, dt \right. \\
+ (1 - G(T_i(c_i)) e^{-\rho T_i(c_i)} B_i(c_i)) \right\} \right]
$$
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\left. + (1 - G(T_i(c_i)))e^{-\rho T_i(c_i)} B_i(c_i) \right\} \right]
\]

Substituting in previous work and identifying like-terms between the principal’s and firms’ payoffs yields

\[
\mathbb{E} \left[ \sum_i P_i(c) \int_0^{T_i(c_i)} e^{-\rho t} \left\{ h(t)\nu - \left( c_i + \frac{f_i(c_i)}{F_i(c_i)} \right) \psi_i(c_i) \right\} (1 - G(z)) \, dz \right]
\]

which has a solution \( h(T_i(c_i)) = \frac{\psi_i(c_i)}{\nu} \) if \( h'(t) < 0 \)
“Filling in” the rest of the $\epsilon_{it}(c', c)$

- By construction, the $(c, c)$ types work for the entire horizon and report success immediately.
- For $c' > c$, the condition $y_1 - y_0 + \mu_E(t) \geq 0$ will still be satisfied for all dates with continuous effort, so firms that overstate their marginal cost will work the entire horizon.
- For $c' < c$, firms siphon for the entire contract since they cannot afford to work at any date\(^1\).

So with the mechanism so characterized, it can be verified that $U_i(c', c) = p_i(c')J^*_i(c', c)$ is supermodular.

\(^1\)In a model with secret saving, the condition $y_1 - y_0 + \mu_E(t) \geq 0$ is violated for early dates, so firms procrastinate before working.
Main Results

- The monotonicity constraint fails to bind at the proposed mechanism \((U_i(c', c))\) is supermodular since \(T_i(c')\) and \(P_i(c', C_{-i})\) are decreasing and for \(c' > c\), \(E_{it} = t\), so the solution is unconstrained optimal.

- An indirect implementation exists resembling a second-price auction, where the firms are compensated for (i) the value of their private information and (ii) the extra efficiency that they bring to the project, compared to the next-best firm. So rents arise for two reasons.

- If the hazard rate \(h(t)\) is increasing, a simple first- or second-price procurement auction with a bond is optimal (The principal’s payoff becomes a convex function in \(T_i(c_i)\)).

- If firms are constrained in how large a bond they can post, the optimal mechanism will involve “contingency contracting”, where alternates are queued up in case the first firms fail.