Procurement with Adverse Selection and Dynamic Moral Hazard

Terence R. Johnson*
University of Notre Dame

First draft: 3/2012
This draft: 12/2012

Abstract
Consider a principal seeking to contract with one of many firms to undertake a risky project in an environment that suffers from both adverse selection and dynamic moral hazard. The likelihood of project success at each moment depends on the amount of work already completed, which is observed only by the contracted firm. Firms can *siphon* a portion of the funds intended for the project and use the rest to create an illusion of productivity. Consequently, inefficient firms can bid for contracts simply to siphon, efficient firms who win may shirk, and firms who succeed can siphon funds rather than report success to the principal. I show that under standard assumptions, dynamic contracts can be constructed that mitigate all inefficiency arising from dynamic moral hazard. These contracts can be implemented through a generalization of a procurement auction that features endogenous penalties for failure and time-varying awards for success.

*JEL Classification Codes: D44, C73, D86
Keywords: Auction Theory, Dynamic Mechanism Design, Dynamic Moral Hazard

1 Introduction

In many economic settings, risk evolves over time in ways that dynamically alter the incentives of the participants. Even in relatively mundane procurement settings like highway construction, there are jobs that become unexpectedly complicated, costly, and sometimes end in default. These failures can not only be costly to the principal but also to the firms involved, harming their reputations and financial solvency.

A prominent example of the drawbacks of traditional approaches to procurement is the Big Dig, a massive highway construction project in Boston, Mass. The project was planned to be completed in 1998 at a cost of $6 billion. It was ultimately finished in 2007 at a cost of $14.6 billion, and a number of lawsuits are currently pending against contractors involved in the project. The process by which the project was procured is generally called the *design-bid-build method*, where

---

*I am grateful to Thomas Gresik and participants at the Spring 2012 Midwest Microeconomic Theory Conference for helpful comments. 917 Flanner Hall, University of Notre Dame. tjohns20@nd.edu.*
the principal specifies the design details and issues a request for proposals, firms bid competitively, and a contract is awarded to the lowest responsive bidder. Indeed, this was the method used for the Big Dig: “When the preliminary engineering on the Big Dig started in the 1980s, the Massachusetts Highway Department had strict bidding requirements governed by Massachusetts law,” says Carl Gottschall, project administrator at FHWA’s Massachusetts Division Office [1]. “Design-bid-build was our only option for project delivery.” Gottschall goes on to say, “Building such infrastructure within a dense urban core ... would have made it almost impossible to pin down a price up front.” Pinning down a price upfront, however, is a limitation that contributes to the massive cost overruns and defaults that can accompany risky projects. This paper shows that by incorporating dynamic elements into the economic environment, time-varying contracts can be structured that mitigate misconduct on the side of the contractors and give rise to novel market designs.

Consider an economic environment in which a principal would like to hire one of a number of firms to undertake a project, but the firms’ costs per unit time are privately known only to them. Initially, a “market for contracts” occurs in which the project is allocated to at most one firm, after which the firm dynamically maximizes its payoff from the project. The likelihood of the project’s success is random and commonly known, and depends on the accumulated stock of effort exerted by a winning firm. The principal cannot observe whether the agent is exerting effort or whether the project is complete, but can verify ex post whether success has occurred. Despite having capital for the purposes of collateral, the firms are not sufficiently liquid to fund work on their own, requiring the principal to finance the project. The principal can set a terminal date at which the relationship is terminated, pay the firm a potentially time-varying flow payment, punish the firm for failing to succeed by the terminal date through forfeit of a bond, and pay a time-varying award to the firm if success occurs. The principal’s value for successful completion additionally depends on whether or not the contracted firm receives sufficient funding to covers its cost per unit time or has to operate at a loss, capturing the general problem that firms often underbid to win a project but are then are unable to satisfactorily complete it. If the firm’s marginal cost is fully funded, the principal values a project at \( v \) upon successful completion, but if the firm is underfunded, it is worth less.

Working against the principal, however, is the agent’s ability to siphon funds: Rather than expend effort on work that will lead towards success, the agent can divert funds to private consumption and use the rest to give the illusion of productivity; this is a model of costly state falsification. The
principal’s inability to observe the project’s progress introduces three sources of potential inefficiency: (i) a firm alternates between working and shirking, thereby wasting resources, (ii) a firm succeeds, but then delays revealing this to the principal in order to continue receiving the flow payment, (iii) inefficient firms spuriously bid in hopes of winning and then siphoning for the entire course of the project. For example, a construction firm might succeed at the crux of a large scale project, then delay completion of less demanding tasks over time to stretch out the payments from the principal. Once the firm comes forward to claim success, however, bridge inspectors and civil engineers can judge whether the final result is complete. Or, the firm might work for a time, realize the project is impossible, and siphon the remaining stream of flow payments rather than admit the bad news to the principal and default immediately.

Under the standard assumptions of an increasing virtual cost and a decreasing hazard rate of success, Section 3 shows that an optimal dynamic direct revelation mechanism mitigates all these sources of inefficiency, allowing the principal to implement the same effort level by the firm he would select if he could directly observe the project’s progress. In order to show that the principal’s preferred allocation can be implemented, the behavior of firms who initially lie about their marginal costs must also be characterized: Firms who overstate their true costs exert effort at all dates, but firms who understate their true costs stop work early and siphon the remaining funds. This can be profitable because the firm can improve its expected payoff by understating its costs to get a later terminal date and a higher likelihood of winning, and use the siphoned funds to offset the lower flow payment at all previous dates. Surprisingly, the optimal mechanism bears similarity to a “cost-plus” contract, in which the agent is paid a fee to cover expenses plus a bonus upon success. Such contracts provide the best motivation for firms to keep working late in a contractual agreement by paying the winning firm his stated marginal cost, but pushing all of the informational rents onto the award function, thereby providing positive incentives for effort and immediate revelation of success. To discourage shirking, a bond is selected that punishes the firm for failure, keeping it “on the hook” late in the contractual horizon. It is not enough, however, to set the bond at the maximum amount siphonable from the project, since this ignores incentive constraints across types. One contribution of the paper is to show how the bond plays a non-trivial role not only in preventing spurious bidding, but also in determining incentive compatibility and ensuring that the proper level of effort is exerted.
An indirect implementation is proposed in Section 4 that has a number of interesting features. Firms bid in terms of their unit cost per time, and the lowest bidder who satisfies a reserve price is awarded the project and paid its stated bid for each moment of time over the course of the contract. The bond and award functions are determined endogenously by the winning bid and the next highest bid, exploiting a term that rewards a winning firm for the cost savings it generates relative to the next-highest cost firm eligible to win, as well as the longer time horizon that it brings to the project. This bears similarity to the Vickrey-Clarke-Groves mechanism, but the dynamic procurement auction developed here maximizes the principal’s payoff and is not constructed to internalize each agent’s influence on social welfare. The endogenous determination of the award and bond functions is not only useful in explicating some of the opaque features of the optimal direct revelation mechanism, but necessary, because the bond plays a central role in ensuring incentive compatibility. Firms can optimally deviate by reporting a lower cost than their true one, thereby guaranteeing a longer time horizon before defaulting and a higher likelihood of winning the project. Consequently, it is not enough to set the bond to the maximum amount siphonable across all types, because this fails to deter firms from lying about their type, and then implementing a different strategy than the one that type would have used. While the profit-maximizing outcome could be achieved by posting deterministic bond and award functions as part of the indirect implementation, this is essentially equivalent to the direct revelation mechanism. Furthermore, making the bond endogenous has the added advantage that it reduces the sensitivity of the mechanism to the principal’s beliefs about the distribution of private information. This particular indirect implementation shows how the award equals the cost savings that stems from the winning firm being more efficient than the next-best firm.

This is related to the related literature on firm defaults at auctions, as in Board [3], Calveras et al. [5], Zheng [32], and Waehrer [31]. These papers assume that not all of the information is available at an initial date when contracting takes place, and the resolution of some residual uncertainty makes the initial agreement untenable. The current paper departs from such a framework by imagining that even if the agent worked in good faith, the project might take an arbitrarily long amount of time to finish. In the presence of moral hazard, this means that the principal cannot commit to perpetual funding, or winning firms would have an incentive to siphon forever. The current paper contributes to the literature by providing an environment in which success is fundamentally
uncertain, and future re-contracting is undesirable because it would undermine the principal’s ability to deter spurious bidding and socially shirking. In this sense, defaults are not something that could be avoided through more careful contracting, but necessary evils that result from the informational features of the environment.

The closest literature considers how to design markets for agency contracts, including McAfee and McMillan [19] and [20], and Laffont and Tirole [15]. These papers show how the principal can trade off moral hazard against adverse selection to improve his payoff. In the current paper, however, it is shown that inefficiency arising from moral hazard can be entirely mitigated. This is possible because there is no limit to the amount of collateral or size of the bond that can be demanded of the firm, so that efficient effort provision can be made to maximize the expected payoff of the firm as well as the principal. In the literature that focuses on moral hazard alone, models like Sannikov [28] have a much richer dynamic principal-agent problem, but do not consider the market structure that gives rise to the contractual setting in which the principal and agent bargain. By incorporating the market for contracts, the current paper provides a tractable framework for investigating dynamic principal-agent problems with an adverse selection component.

One of the key features of the model in the current paper is that a winning firm can engage in dynamic costly state falsification, where it sacrifices a portion of the payments received from the principal to given an impression of effort. This captures a dynamic where completely shutting down operations would be too brazen, but the firm can “keep the lights on” at work to hide its shirking and siphoning. This is similar to Lacker and Weingberg [14], Crocker and Morgan [11], and other works that allow an agent to expend effort to keep the principal from learning that they have engaged in inefficient or wasteful behavior. While these papers adopt a static moral hazard framework in which the principal gets a noisy signal that is correlated with the agent’s effort, the signal of effort in the current paper is whether or not the agent has come forward to claim success or defaulted. Another difference is that because the financial penalty of failure can be made sufficiently severe in the current paper that the firm wants to work at all dates, no siphoning actually occurs in the current paper on the equilibrium path.

Within the growing field of dynamic mechanism design, the current paper is most similar to Board [4]. In that paper, the author considers the problem of auctioning an option contract to agents who have a privately known benefit, but a time-varying expected cost. Board shows that
the welfare-maximizing mechanism does not feature payments conditional on exercise, but the revenue-maximizing mechanism does, leading to delays in exercise. By incorporating a dynamic moral hazard problem, so that not only does the agent have an optimal stopping problem once the project is successful, but the agent also invests in the project over time. A characterization of dynamic mechanisms that provide optimal effort provision is the main contribution of this paper. Doepke and Townsend [13] consider a discrete-time framework with dynamic adverse selection and moral hazard, closest in spirit to the optimal taxation or optimal unemployment benefits literatures. They provide a revelation principle in which allocations are implemented by presenting the agents with a “promised utility” that represents the discounted benefit of each actions, and then show how optimal contracts can be computed efficiently. The current paper involves a market for contracts, and involves a single project that either succeeds or fails, rather than an infinitely-lived relationship. However, the firms in the current paper draw only a single type at the beginning of the game, so there is no dynamic adverse selection element, while Doepke and Townsend allow types to evolve according to very general rules. Finally, Pavan et al [?] study a very general dynamic mechanism design framework, exploiting what they call impulse response functions to study how an agent’s current behavior influences the payoffs of future types. The firms’ types in the current paper are static, but the simpler framework allows a study of indirect implementation and the kinds of markets for contracts that exploit competition among the agents to implement the profit-maximizing outcome. In contrast to Board [4], Doepke and Townsend [13], and Pavan et al [?], the current paper exploits a continuous time framework, rather than discrete time. This allows a much more elegant characterization of an agent’s optimal dynamic behavior using the adjoint equation, rather than systems of inequalities. Implementing a particular allocation then hinges on studying the properties of a single function characterized by an ordinary differential equation and an economically meaningful transversality condition. A main contribution of the current paper is also the use of continuous time, rather than discrete time. Most dynamic mechanism design models are formulated in discrete time, such as Pavan et al [?] and Doepke and Townsend [13]. It turns out that by exploiting a continuous time framework, the systems of inequalities that characterize obedience can be characterized by a single function, the adjoint equation. This approach offers another approach to studying dynamic mechanism design by exploiting optimal control, rather than dynamic programming.
A large literature considers procurement auctions, both theoretically and empirically. Dasgupta and Spulber [12] study a procurement model similar to Myerson [25], and show that in the presence of strictly convex costs for the firms there can be dual- and multi-sourcing. The contribution of the current paper is to incorporate a dynamic moral hazard problem, although dual- or multi-sourcing are not studied since there is a single project. Manelli and Vincent [16] show that in a procurement setting where sellers privately know the value of their good to the buyer, making a series of take-it-or-leave-it offers to potential sellers can dominate auctions. This occurs because of the dependence of the buyer’s value for the good on the seller’s private information, which is intrinsically different from a situation in which the sellers all provide the same good but have different, privately known costs of providing it. Since the principal in the current paper receives the value $v$ whenever a fully funded firm completes the project and any winning firm is fully funded, this phenomenon does not appear. In a paper with theoretical and empirical components, Bajari and Lewis [2] look at recent changes in Caltrans’ procurement auctions that incorporate time-varying elements into the contracts, and find evidence that incentivizing firms along a time dimension leads to significant welfare gains. The current paper is a useful step towards thinking about other features of mechanism design that require explicitly incorporating the time horizon into the contractual terms. In addition, the static projects considered in the literature are similar to the dynamic outcome in the current paper when the hazard rate of success is monotone increasing: In this case, all winning firms are contracted to work until they succeed, and the interesting margins on which the optimal mechanism in the current paper differs from previous works vanishes. This nests the regular procurement auction as a special case of the current model in which the longer the firm works, the more likely it is to succeed.

Lastly, the current paper assumes that firms can only work sequentially, but a large literature considers optimal contests and tournaments. Particularly, Che and Gale [7] and Moldovanu and Sela [24] consider environments where prizes are offered for success, and the principal must decide on the optimal contest architecture. The results of the current paper would change if firms could work in parallel, since competition could be used to deter shirking behavior. However, in many environments firms cannot feasibly work in parallel; for example, only one construction site typically exists. Similarly, many markets feature the kind of contracting here, where a principal issues a request-for-proposals but does not award the project to multiple firms and force them to compete for a prize.
1.1 Outline

Section 2 provides the details of the model and explains how the revelation principle can be adjusted to use direct revelation mechanisms to maximize the principal’s payoff without loss of generality. Section 3 shows how a firm’s payoff can be computed in this continuous time, dynamic environment, which allows incentive compatibility and individual rationality constraints to be clearly defined. Section 4 then constructs a profit-maximizing mechanism. This process is complicated by the fact that incentive compatibility and individual rationality constraints must be considered not only for honest reports and the subsequent optimal effort provision schedule, but also for out-of-equilibrium deviations. Section 5 then proposes the dynamic procurement auction and shows that it implements the same outcome as the principal-optimal direct revelation mechanism. Section 6 concludes.

2 Model

A principal would like to hire a firm to undertake a risky project, but the market suffers from both adverse selection and dynamic moral hazard. There are \( i = 1, 2, \ldots, n \) firms, who each have a privately known marginal cost per unit time, \( c_i \). Let \( c = (c_1, c_{-i}) \) be the vector of firm marginal costs, with \( c_{-i} = (c_1, c_2, \ldots, c_{i-1}, c_{i+1}, \ldots, c_n) \). Each \( c_i \) is drawn independently from a commonly known probability distribution function \( F(c_i) \) with a strictly positive, continuous density \( f(c_i) \) on \( [\underline{c}, \bar{c}] \), where \( \underline{c} > 0 \) and \( \bar{c} \) is finite. There is an observable fixed cost of attempting the project, \( K \), which is borne by the principal without loss of generality\(^1\). The principal and firms are perfectly patient, maximize their expected utility, and are risk neutral.

Time is the union of an allocation stage, \( \emptyset \), during which the market-for-contracts meets, and a construction stage, \( [0, \infty) \), during which a winning firm solves its project management problem. At each moment during the construction stage, a hired firm either chooses to work and produce a unit of effort \( \epsilon_{it} = 1 \), or shirk and produce no progress, \( \epsilon_{it} = 0 \). The stock of accumulated effort is given by

\[
E_{it} = \int_0^t \epsilon_{iz} dz
\]

For any time-subscripted variable \( x_t \), \( \dot{x}_t \) denotes the partial derivative of \( x_t \) with respect to \( t \), so

\(^1\)See the discussion following Proposition 4.1
that

$$\dot{E}_{it} = \epsilon_{it}$$

The probability that a firm succeeds given accumulated effort $E_{it}$ is $G(E_{it})$, a differentiable distribution function with density $g(E_{it})$. The hazard rate of success is given by

$$h(E_{it}) = \frac{g(E_{it})}{1 - G(E_{it})}$$

The hazard rate is assumed to be decreasing and have strictly positive support on $[0, \infty)$, but the less interesting case in which the hazard rate is strictly increasing and have strictly positive support on $[0, \bar{T}]$, where $\bar{T}$ is finite, can be solved using the same methods, and the differences are mentioned in the text. By relaxing the assumptions on the hazard rate, this stochastic success framework can be made significantly more general than it might appear at first. For example, a project may have a zero probability of success for some time after which the hazard rate increases, and then decreases as the long delay reveals that the project is actually unlikely to succeed at all. Many settings for which a stochastic framework might sound unnatural — building a bridge in fairly straightforward ex ante conditions, for example — are actually very appropriate, given that unexpected delays might reveal that the difficulty of the project was initially underestimated.

Firms own capital which can be used as collateral. For example, a highway construction firm might own valuable machines, buildings, and raw materials which form the basis for a three-way contract between the principal, the firm, and a bank who acts as a surety. However, the firms do not have funding to cover variable costs of the project, requiring financial support from the principal. As compensation for its work, the firm receives a flow payment payment $r_{it}$ for each moment of time in $[0, \infty)$ while success has not been declared, and an award $w_{it}$ if success occurs at time $t$. The principal can choose a terminal date $T_i$ at which time a winning firm is determined to be in default, and must forfeit a bond, $B_i$.

The base value of the project to the principal is $v$, but firms who receive a lower flow payment than their true marginal cost per unit time produce an inferior product of lesser value. A completed project yields $v(r_{it}, c_i)$ to the principal, which equals a constant, $v$, for $r_{it} \geq c_i$ but is less than $v$ and increasing in $r_{it}$ when $r_{it} < c_i$. Thus, underfunding the firm reduces the value of the project to the principal. This allows the model to incorporate on-going or potential future harm to the principal.
as a consequence of firms under-bidding to win the project, and then delivering an inferior product or performance.

The principal cannot observe if the firm is working or not, so the firm can divert funds away from the project by engaging in costly behavior that gives the appearance of productive effort, but does not lead to a higher likelihood of completion. In particular, a firm can *siphon* funds at a rate \(1 \geq \lambda \geq 0\), keeping that fraction of any payments received in cash for itself and setting \(\epsilon_{it} = 0\). From the principal’s perspective, activity appears to be taking place, but this is just to hide the agent’s efforts to extract money from the contract.

This monitoring structure creates three separate challenges. First, firms may attempt to win the contract even though they have no intention of exerting any effort, and are merely planning to siphon all the funds. Second, a firm that has worked and succeeded may then begin siphoning funds, waiting to exercise the option of revealing success at a later, more lucrative date. Finally, late in its contract, a firm might cease exerting effort and begin siphoning funds, since the likelihood of success fails to justify further effort.

### 2.1 Direct revelation mechanisms

For the firms to decide whether reporting honestly at the allocation stage is profit-maximizing, they must assess the dynamically evolving value of the project. Since they have private information not only about their initial type, but also about the accumulated level of effort and whether or not the project has succeeded, the incentive constraints are not limited to honesty in initial reporting, but also obedience in the undertaking of the project.

Formally, a *direct mechanism* is a set of functions

\[
m(c) = \{P_i(c), r_{it}(c), w_{it}(c), B_i(c), T_i(c), s_{it}(c), \epsilon_{it}(c)\}_{i, t}
\]

whose domains are the type spaces of the agents, where

1. The *assignment function* \(P_i(c_i, c_{-i})\), giving the probability of assigning the project to firm \(i\)

2. The *flow payment*, \(r_{it}(c_i, c_{-i})\), giving the payment at each date \(t\)

3. The *award*, \(w_{it}(c_i, c_{-i})\), giving the payment to a firm who reports success at time \(t\)
4. The (surety) bond, \( B_i(c_i, c_{-i}) \), specifying a penalty in case of failure.

5. The terminal date, \( T_i(c_i, c_{-i}) \), specifying when flow payments stop and an award is no longer offered; if the contract goes “forever”, set \( T_i(c_i, c_{-i}) = \infty \).

6. The delay function, \( s_{it}(c'_i, c_i, c_{-i}) \), specifying the amount of time a firm should delay after success before reporting it to the principal, given that success occurred at time \( t \), and the agent’s initially reported type is \( c'_i \).

7. The effort function, \( \epsilon_{it}(c'_i, c_i, c_{-i}) \), specifying the effort level the firm for each date \( t \) when the agent’s initial report is \( c'_i \).

A mechanism in which agents have incentives to report truthfully and behave obediently is a direct revelation mechanism.

The functions \( w_{it}(c) \), \( r_{it}(c) \), \( T_i(c) \), and \( B_i(c) \) are all assumed to be piecewise continuously differentiable. This specification of a mechanism is without loss of generality because these functions cover all possible scenarios the principal might witness. In particular, each firm provides at most two pieces of information: The initial report of its type and the date at which it succeeds (and no report at all if it fails). Consequently, the set of functions given above spans all possible decisions and payments contingent on observable events.

In this environment, a mechanism is incentive compatible if (i) each firm finds it optimal to report its marginal cost per unit time honestly at the allocation stage, (ii) the proposed effort functions \( \epsilon_{it}(c'_i, c_i, c_{-i}) \) are optimal for all dates \( t \) for a firm with true type \( c_i \) reported its type as \( c'_i \), and (iii) the proposed delay \( s_{it}(c'_i, c_i, c_{-i}) \) is optimal whenever success occurs at date \( t \) and a firm with true type \( c_i \) reported its type as \( c'_i \). An incentive compatible mechanism is individually rational if a firm that participates honestly at the allocation stage and then adopts the recommended effort and reporting strategies gets at least a payoff of at least zero, its outside option. In short, a mechanism is incentive compatible if it is a Bayesian Nash equilibrium for the firms to be honest about their costs during the allocation stage, and obedient concerning the proposed effort and delay for all type-report pairs and success dates during the construction phase. By an extension of the revelation principle due to Doepke and Townsend [13] that accommodates hidden information and hidden actions, the principal can restrict attention to direct revelation mechanisms that induce honesty at the allocation stage and obedience at the construction phase without loss of generality.
3 Project Value and Incentive Compatibility

A contractual arrangement in many procurement design models is typically static, or involves a small number of periods across time. This section shows how to dynamically value a contract where the likelihood of success changes across the project horizon, thereby characterizing the agent’s payoffs in the construction phase. A small amount of discounting of the form $e^{-\rho t}$ is assumed, and the payoffs are then derived as the time preference parameter $\rho$ goes to zero.$^2$

After winning the project, a firm’s behavior is characterized by two functions: The effort that the firm exerts at each date, $\epsilon_{it}(c'_i, c_i, c_{-i})$, and the delay between success and reporting this to the principal, $s_{it}(c'_i, c_i, c_{-i})$, where $c'_i$ is the agent’s report, $c_i$ is the agent’s true type, and $c_{-i}$ are the reports of the other firms. Since the winning firm receives a flow of payments over time, it may be to his advantage to wait to exercise the option of revealing success to the principal. Define the optimal delay given an initial report type $c'_i$ and success at time $t$, $s_{it}(c'_i, c_i, c_{-i})$, as the solution to

$$\tilde{w}_{it}(c'_i, c_{-i}) = \max_s e^{-\rho(s-t)}w_{ts}(c'_i, c_{-i}) + \lambda \int_t^\infty e^{-\rho(z-t)}r_{iz}(c'_i, c_{-i})dz$$

Let $\tilde{w}_{it}(c'_i, c_{-i})$ be the effective award. Note that the true $c_i$ appears nowhere in the maximization problem, so the optimal delay depends only on the firm’s reported type and the terms of the contract, not on the firm’s true costs. A sufficient condition for the solution to be well-defined is that the functions $r_{it}(c'_i)$ and $w_{it}(c'_i, c_{-i})$ be continuous and the terminal dates $T_i(c)$ be bounded for all reports $c$. Since the objective function is continuous in the delay, by the theorem of the maximum the value function is continuous, so the effective award $\tilde{w}_{it}(c'_i, c_{-i})$ is a continuous function.

Given the effective award, the value of a given effort plan can be characterized. The expected value to the agent of exerting effort for a moment of time $\Delta$ is given by

$$J_i(t, E_{it}(c'_i, c), c'_i, c) = h(E_{it}(c'_i, c))\Delta e^{-\rho\Delta} \tilde{w}_{it}(c'_i, c_{-i}) + \Delta (r_{it}(c'_i, c_{-i}) - c_i) + (1 - h(E_{it}(c'_i, c))\Delta) e^{-\rho\Delta} J((t + \Delta, E_{it}(c'_i, c) + \Delta, c'_i, c)$$

The first term is the current probability of success times the effective award, the second term is the net flow profits, and the third term is the current probability of failure times the discounted value

$^2$ All results in the paper can be understood as holding in a model with discounting for $\rho$ sufficiently close to zero. Allowing arbitrary discounting complicates the analysis since sufficiently impatient firms ($\rho \to \infty$) will always shirk.
of reaching time $t + \Delta$ with a stock of accumulated effort of $E_{it}(c_i', c) + \Delta$. This expresses the value of exerting effort at time $t$ as a discounted expected value. Similarly, the payoff to exerting no effort is

$$J_i(t, E_{it}(c_i', c), c_i', c) = e^{-\rho \Delta} J_i(t + \Delta, E_{it}(c_i', c), c_i', c) + \Delta \lambda r_{it}(c_i', c_{-i})$$

so that the firm siphons the current flow payment at rate $\lambda$ and enters the next moment $t + \Delta$ without accumulating any additional effort.

By standard limiting arguments (see the proof of Proposition 3.1), these can be converted into differential equations that describe the evolution of the value of the project:

$$\frac{dJ_i(t, E_{it}(c_i', c), c_i', c)}{dt} = (\rho + h(E_{it}(c_i', c)))J(t, E_{it}(c_i', c), c_i', c) - (h(E_{it}(c_i', c))\tilde{w}_{it}(c_i', c_{-i}) + r_{it}(c_i', c_{-i}) - c_i)$$

and

$$\frac{dJ_i(t, E_{it}(c_i', c), c_i', c)}{dt} = \rho J(t, E_{it}(c_i', c), c_i', c) - \lambda r_{it}(c_i', c_{-i})$$

In the differential equation corresponding to exerting effort, the change in the current value of the project is equal to the discounted expected value, less a term that reflects the lost opportunity for success at time $t$. In the differential equation corresponding to shirking, the firm has not accumulated any additional effort, so the “opportunity to succeed” at $E_{it}(c_i', c)$ is still available, but some time has been lost and some value siphoned out of the contract. This provides another interpretation for the decreasing hazard rate framework: There are a limited number of ways to succeed, with some more likely than others. If the firm approaches the project by trying the most likely approaches first, it exhausts opportunities upon failure, and the likelihood of ultimate success declines.

Since the differential equations characterize the evolution of the value of the contract, a value function for any candidate policy $\epsilon_{it}(c_i', c)$ can be constructed by working backwards from the terminal value, $-(1 - G(E_{it}(c_i', c_{-i}))(c_i', c))B_i(c_i', c_{-i})$. 

13
Proposition 3.1 Any winning firm solves the linear optimal control problem

\[
\max_{\epsilon_{it}(c', c)} J_t(c'_i, c, \epsilon_{it}(c'_i, c)) = \\
\max_{\epsilon_{it}(c'_i, c)} \int_0^{T_i(c'_i, c_{-i})} \epsilon_{it}(c'_i, c) \left\{ g(E_{it}(c'_i, c))w_{it}(c'_i, c_{-i}) + (1 - G(E_{it}(c'_i, c))(r_{it}(c'_i, c_{-i}) - c_i) \right\} \\
\quad + (1 - \epsilon_{it}(c'_i, c)) \left\{ \lambda r_{it}(c'_i, c_{-i}) \right\} dt - (1 - G(E_{iT}(c'_i, c_{-i}))(c'_i, c_{-i}))B_i(c'_i, c_{-i}) \\
\quad \text{subject to } \epsilon_{it}(c'_i, c) \in \{0, 1\} \text{ and } \dot{E}_{it}(c'_i, c) = \epsilon_{it}(c'_i, c). \text{ The functions} \\
y_{it}^1(c'_i, c, E_{it}(c'_i, c)) = g(E_{it}(c'_i, c))w_{it}(c'_i, c) + (1 - G(E_{it}(c'_i, c))(r_{it}(c'_i, c_{-i}) - c_i) \\
\quad \text{and} \\
y_{it}^0(c'_i, c, E_{it}(c'_i, c)) = \lambda r_{it}(c'_i, c_{-i}) \\
give the current flow value of exerting effort and siphoning, respectively. The payoff-maximizing effort plan \(\epsilon_{it}(c'_i, c)\) necessarily satisfies \\
\[\epsilon_{it}(c'_i, c) = \begin{cases} 
1 & \text{if } y_{it}^1(c'_i, c, E_{it}(c'_i, c)) - y_{it}^0(c'_i, c, E_{it}(c'_i, c)) + \mu_{it}(c'_i, c) \geq 0 \\
0 & \text{otherwise} 
\end{cases} \]

\text{where} \\
\mu_{it}(c'_i, c) = \epsilon_{it}(c'_i, c) \left\{ -g'(E_{it}(c'_i, c))w_{it}(c'_i, c_{-i}) \\
\quad + g(E_{it})(r_{it}(c'_i, c_{-i}) - c_i) \right\} + (1 - \epsilon_{it}(c'_i, c))g(E_{it}(c'_i, c))\lambda r_{it}(c'_i, c_{-i}) \\
\text{with transversality condition } \mu_{iT_i(c'_i, c_{-i})}(c'_i, c) = g(E_{iT_i(c'_i, c_{-i})})B_i(c'_i, c_{-i}). \\

This converts the problem of choosing the effort policy that maximizes the discounted expected value of the contract into a linear optimal control program, where \(E_{it}(c'_i, c)\) is the state variable, and \(\epsilon_{it}(c'_i, c) \in \{0, 1\}\) is the control, which switches back and forth between exerting effort and siphoning. The co-state variable \(\mu_{it}(c'_i, c)\) denotes the discounted value to the winning firm of increasing the stock of accumulated effort at date \(t\), given its report \(c'_i\) and the true types \(c\). The
transversality condition reflects the fact that at the terminal date, the marginal benefit of a higher stock of accumulated effort is the expected gain of avoiding default. The key condition is that the firm exerts effort \( \epsilon_{it}(c'_i, c) = 1 \) only if

\[
y^1_{it}(c'_i, c, E_{it}(c'_i, c)) - y^0_{it}(c'_i, c, E_{it}(c'_i, c)) + \mu_{it}(c'_i, c) \geq 0
\]

This characterization of effort provision is the main advantage of using a continuous time framework rather than discrete time. Incentive compatibility can now be characterized by focusing on the properties of a single function \( \mu_{it}(c'_i, c) \) rather than a larger number of truth-telling and obedience inequalities.

Define \( \epsilon_i(c'_i, c) = \{\epsilon_{it}(c'_i, c)\}_{t \in [0, T_i(c'_i, c-i))] \) as the optimal plan given the initial report \( c'_i \) and true types \( c \). Let \( J_i(c'_i, c) = J_i(c'_i, c, \epsilon_i(c'_i, c)) \) denote the optimized value of the project for a firm of type \( c_i \) reporting \( c'_i \). A firm’s payoff from submitting report \( c'_i \) with true type \( c_i \) is given by the direct utility function

\[
U_i(c'_i, c_i) = E(c_{-i}) \left[ P_i(c'_i, c_{-i}) J_i(c'_i, c) \right].
\]

A mechanism is incentive compatible for type \( c_i \) if, for all \( c'_i \neq c_i \),

\[
U_i(c_i, c_i) \geq U_i(c'_i, c_i),
\]

and an incentive compatible mechanism is individually rational for type \( c_i \) if

\[
U_i(c_i, c_i) \geq 0.
\]

The indirect utility function is given by

\[
U_i(c_i) = \max_{c'_i} E(c_{-i}) \left[ P_i(c'_i, c_{-i}) J_i(c'_i, c) \right].
\]

Since the optimized value of the project is determined through \( s_{it}(c'_i, c_{-i}) \) and \( \epsilon_{it}(c'_i, c) \), it is of central interest whether various derivatives of \( J_i(c'_i, c, \epsilon_i(c'_i, c)) \) with respect to \( c_i \) and \( c'_i \) can be computed, and what consequences this has for incentive compatibility of a mechanism.
Proposition 3.2 Where differentiable, the value function $J_i(c'_i, c)$ satisfies

$$\frac{\partial J_i(c'_i, c_i, c_{-i})}{\partial c_i} = \frac{\partial J_i(y, y, c_{-i})}{\partial c}$$

At a point of non-differentiability, the above expression is well-defined as a sub-gradient in $c_i$. $J_i(c'_i, c_i, c_{-i})$ is upper semi-continuous in $c'_i$.

Let $c^*_i$ be the worst-off type that participates in the mechanism. Then a mechanism is incentive compatible iff for all types $c_i$,

$$U_i(c_i) = u^*_i + \int_{c_i}^{c^*_i} \mathbb{E}_{c_{-i}} \left[ P_i(y, c_{-i}) \frac{\partial J_i(y, y, c_{-i})}{\partial c} \right] dy, \quad (2)$$

for all types $c_i$ and deviations $c'_i$,

$$\int_{c'_i}^{c_i} \mathbb{E}_{c_{-i}} \left[ P_i(y, c_{-i}) \frac{\partial J_i(y, y, c_{-i})}{\partial c} \right] - \int_{c'_i}^{c_i} \mathbb{E}_{c_{-i}} \left[ P_i(c'_i, c_{-i}) \frac{\partial J_i(c'_i, y, c_{-i})}{\partial c_i} \right] dy \leq 0, \quad (3)$$

and the worst-off type for firm $i$, $c^*_i$ receives a payoff $u^*_i \geq 0$.

This proposition characterizes incentive compatibility, and how the the value of the project varies in each firm’s cost and report. In particular, since $J_i(c'_i, c_i, c_{-i})$ is upper semi-continuous in $c'_i$, an optimal report will always exist for the firms. While differentiability of $J_i(c'_i, c_i, c_{-i})$ in $c_i$ cannot be guaranteed, the optimal mechanism is sufficiently well-behaved under our assumptions to guarantee that the envelope payoff representation in (2) holds, and methods like those of Carbajal and Ely [6] are not necessary. It turns out that while there is a kink at the true type in the value function associated with the optimal direct revelation mechanism, this doesn’t affect calculations of the payoff to deviating, so the above characterizations of $J_i(c'_i, c_i, c_{-i})$ and $U_i(c'_i, c_i)$ are sufficient for what follows.

4 Optimal Contracting

By the revelation principle, the principal can focus on direct revelation mechanisms that induce honesty at the allocation stage for all types $c_i$ and obedience at the construction phase for all
(c′ i, c i, c_{−i}). Consequently, the principal’s full problem can be stated as

$$\max_{m(c)} \mathbb{E}_{c} \left[ \sum_{i} P_{i}(c) \left\{ \int_{t=0}^{T_{i}(c)} (1 - \epsilon_{it}(c_{i}, c))(1 - G(E_{it}(c_{i}, c)))(-r_{it}(c)) \\
+ \epsilon_{it}(c_{i}, c)(1 - G(E_{it}(c_{i}, c)) \left( h(E_{it}(c_{i}, c))(v(r_{it}(c), c_{i}) - w_{it}(c)(c) - \int_{0}^{s_{it}(c)} r_{iz}(c)dz - r_{it}(c)) \right) dt \\
+ (1 - G(E_{iT_{i}(c), c}))B_{i}(c) - K \right\} \right]$$

subject to the envelope payoff representation (2), monotonicity condition (3), and for all (c′ i, c),

$$\dot{E}_{it}(c_{i}', c) = \epsilon_{it}(c_{i}', c)$$ (4)

$$\mu_{it}(c_{i}', c) = \epsilon_{it}(c_{i}', c) \left\{ -g'(E_{it}(c_{i}', c))\dot{w}_{it}(c_{i}', c_{−i}) + g(E_{it})(r_{it}(c_{i}', c_{−i}) - c_{i}) \right\}$$

$$+ (1 - \epsilon_{it}(c_{i}', c))g(E_{it}(c_{i}', c))\lambda r_{it}(c', c_{−i})$$ (5)

$$\epsilon_{it}(c_{i}', c) = \begin{cases} 
1, & y_{it}^{1}(c_{i}', c, E_{it}(c_{i}', c)) - g_{it}^{1}(c_{i}', c, E_{it}(c_{i}', c)) + \mu_{it}(c_{i}', c) \geq 0 \\
0, & \text{otherwise} \end{cases}$$ (6)

$$s_{it}(c_{i}', c_{−i}) \in \text{Argmax} \ w_{it}(c_{i}', c) + \lambda \int_{0}^{s} r_{iz}(c_{i}', c)dz$$ (7)

where (4) is the law of motion for the state variable $E_{it}(c_{i}', c)$, (5) is the law of motion for the co-state variable $\mu_{it}(c_{i}', c)$, (6) is the obedience constraint on the optimal effort plan, and (7) is the obedience constraint associated with the success revelation strategy.

By identifying like-terms between the principal’s objective and the agent’s direct utility function, the envelope payoff representation can be used to simplify the problem. We further simplify by dropping not only the monotonicity condition — which is standard — but all obedience constraints that correspond to dishonest reports and those that correspond to the delay strategies $s_{it}(c_{i}', c_{−i})$, yielding a relaxed program

$$\max_{m(c)} \mathbb{E}_{c} \left[ \sum_{i} P_{i}(c) \left\{ \int_{t=0}^{T_{i}(c)} (1 - \epsilon_{it}(c_{i}, c))(1 - G(E_{it}(c_{i}, c)))(-r_{it}(c)) \\
+ \epsilon_{it}(c_{i}, c)(1 - G(E_{it}(c_{i}, c)) \left( h(E_{it}(c_{i}, c))(v(r_{it}(c), c_{i}) - \psi(c_{i}) - (1 - \lambda) \int_{0}^{s_{it}(c)} r_{iz}(c)dz \right) dt - K \right\} \right]$$
subject to

\[
\begin{align*}
\dot{E}_{it}(c_i, c) &= \epsilon_{it}(c_i, c) \\
\dot{\mu}_{it}(c_i, c) &= \epsilon_{it}(c_i, c) \left\{ -g'(E_{it}(c_i, c))\bar{w}_{it}(c_i, c_{-i}) + g(E_{it})(r_{it}(c_i, c_{-i}) - c_i) \right\} \\
&\quad+ (1 - \epsilon_{it}(c_i, c))g(E_{it}(c_i, c))\lambda r_{it}(c, c_{-i}) \\
\epsilon_{it}(c_i, c) &= \begin{cases} 
1, & y_{it}^1(c_i, c, E_{it}(c_i, c)) - y_{it}^0(c_i, c, E_{it}(c_i, c)) + \mu_{it}(c_i, c) \geq 0 \\
0, & \text{otherwise}
\end{cases}
\end{align*}
\]

where \(\psi_i(c_i) = c_i + F(c_i)/f(c_i)\) is the firm’s virtual marginal cost.

While it is standard procedure to drop a number of constraints in mechanism design problems at this stage of the analysis, ignoring the omitted obedience constraints threatens to render any solution to the relaxed problem useless. However, this approach turns out to be successful under the standard conditions of regular virtual marginal cost and a decreasing hazard rate: As the paper shows, the degrees of freedom in contracting afforded by the award and bond functions afford enough slack in the mechanism design problem to ensure the dropped constraints can be satisfied.

**Proposition 4.1** Suppose the virtual marginal cost is increasing for all \(c_i\) and the hazard rate is decreasing for all \(t\). In the solution to the relaxed program, none of \(T^*_i(c_i)\), \(r^*_i(c_i)\), \(e^*_i(c_i)\), \(s^*_i(c_i)\), or \(c^*_i\) depend on \(c_{-i}\). Let

\[
\phi_i(c_i) = \int_0^{T^*_i(c_i)} (1 - G(t)) (h(t)v(c_i, c_i) - \psi_i(c_i)) dt - K
\]

The solution to the relaxed program is \(r^*_i(c_i) = c_i\), \(s^*_i(c_i) = 0\),

\[
P^*_i(c_i, c_{-i}) = \begin{cases} 
1, & \text{if } c_i = \min_k c_k \text{ and } \phi_i(c_i) \geq K \\
0, & \text{otherwise}
\end{cases}
\]

\[
T^*_i(c_i) = \begin{cases} 
(h)^{-1} \left( \frac{\psi(c_i)}{v(c_i, c_i)} \right), & \text{if } \phi(c_i) \geq K \\
0, & \text{otherwise}
\end{cases}
\]
\[ e_{it}^*(c_i) = \begin{cases} 1 & \text{if } 0 \leq t \leq T_i(c_i) \\ 0 & \text{otherwise} \end{cases}, \]

and the worst-off type \( c_i^* \) is defined by \( \phi_i(c_i^*) = K \).

Since the principal is the residual claimant on social surplus, he suffers any inefficiencies arising from breaks in effort provision, reporting delays, or inefficient funding are then borne directly by the principal. Consequently, in the solution to the relaxed problem, the project is awarded to the firm with the lowest marginal cost that justifies incurring the fixed cost \( K \), the winning firm works continuously and reporting success immediately, and a winning firm’s flow payment is equal to the its true marginal cost.

In the case of a strictly increasing hazard rate with support \([0, \bar{T}]\), the solution is slightly different: Any firm contracted to work is asked to work until \( \bar{T} \). This is because an increasing hazard rate implies that the principal’s relaxed problem in \( T_i(c_i, c_{-i}) \) is convex, and the solution is at a corner of the interval \([0, \bar{T}]\). Since this case removes one of the margins of the contract, the analysis largely simplifies to a regular procurement model, nesting a standard private-values procurement problem in this framework. This also illustrates that for a non-monotone hazard rate, those firms whose optimal terminal date is on an interval where the hazard rate is decreasing will receive different contracts, while those whose optimal terminal date is on an interval where the hazard rate is increasing will receive the same deadline but not necessarily the same compensation. These features illustrate how markets might arise where, for instance, relatively inefficient firms all receive the same fixed contract, but elite firms receive differentiated contracts that give them more time to succeed.

To proceed, assume that the remaining functions of the contract, \( w_{it}(c_i', c_{-i}) \) and \( B_i(c_i', c_{-i}) \), do not depend on \( c_{-i} \). This is without loss of generality since the solution to the relaxed problem can be implemented under this assumption. To ensure that the unconstrained solution can be implemented, it must be shown that continuous effort, immediate revelation of success, and honest reporting at the allocation stage are mutually achievable. Section 4.1 shows what restrictions must be placed on the award and bond functions so that continuous effort and immediate revelation are achieved for honest reports, Section 4.2 then characterizes behavior for dishonest reports \((c_i', c_i, c_{-i})\) with \( c_i' \neq c_i \), and Section 4.3 uses these characterizations to complete the construction of an incentive
compatible mechanism.

4.1 Strategic delay

Once a firm is awarded the project, the game moves to a new phase in which the firm’s costs are known, but there is incomplete information about whether or not success has occurred and how much effort has been accumulated. Since the firm cannot fool the principal by reporting success early, it can only delay revelation until the most lucrative date. This is equivalent to a secondary mechanism design problem, in which the success date \( t \) is privately known by the firm, and the principal would like to select the delay schedule \( s_{it}(c_i) \) that maximizes his payoff:

\[
\max_{s_{it}(c_i)} \int_0^{T_i(c_i)} (1 - G(t)) \left\{ h(t) \left( v(c_i, c_i) - w_{is_{it}(c_i)} \right) - s_{it}(c_i)c_i - c_i \right\} dt + (1 - G(T_i(c_i)))B_i(c_i)
\]

subject to incentive compatibility for every \( t \),

\[
s_{it}(c_i) \in \text{Argmax}_{t \leq s \leq T_i(c_i)} w_{is}(c_i) + \lambda(s - t)c_i
\]

and ex ante individual rationality for each \( c_i \) in \([c; c^*_i]\),

\[
p_i(c_i) \left\{ \int_0^{T_i(c_i)} g(t) \left( w_{is}(c_i) + \lambda c_i s_{it}(c_i) \right) - (1 - G(T_i(c_i)))B_i(c_i) \right\}
\]

\[
= \int_{c_i}^{c^*_i} p_i(x) \int_0^{T_i(x)} (1 - G(z))dzdx
\]

where \( \Lambda_i(c_i) \) is the agent’s informational rent and \( p_i(c_i) = \mathbb{E}_{c_{-i}} [P_i(c_i, c_{-i})] \).

This problem is equivalent to asking the firm to reveal the date at which success occurs, \( t \), and then suggesting a time to delay before reporting this to the principal. The obedience constraints require that the suggestion is actually optimal, so that a winning firm behaves as intended. The ex ante individual rationality constraint appears because at the allocation stage, the agent must receive a payoff of \( \Lambda_i(c_i) \), before finding out whether he will succeed or not. The success time \( t \) can then be treated as a piece of private information that is distributed \( g(t) \), turning this into an essentially static mechanism design problem subject to an ex ante participation constraint that must match up with the expected payoffs at the end of the allocation stage, before \( t \) is revealed to the firm.
Note that if the type \( t \) firm delays until \( s_{it}(c_i) \), then all types who succeed between \( t \) and \( s_{it}(c_i) \) must also delay until \( s_{it}(c_i) \), since the payoff function \( w_{is}(c_i) + \lambda(s-t)c_i \) is linear in the type, and the cross-partial is zero. Consequently, delay of revelation for any given type \( t \) implies pooling. These atoms gives rise to complications when considering an optimal control approach to the principal’s problem rather than the pointwise maximization approach pursued here, since any non-zero delay for one type implies a delay for a measure of types.

**Proposition 4.2** Let

\[
w_{it}(c_i) = w_{i0}(c_i) - \lambda c_i t
\]

where the base award is given by

\[
w_{i0}(c_i) = \frac{\Lambda_i(c_i)/p_i(c_i) + (1 - G(T_i(c_i))B_i(c_i) + \lambda c_i \int_0^{T_i(c_i)} g(z)dz}{G(T_i(c_i))},
\]

1. Suppose the award function \( w_{it}(c_i) \) is positive at all dates. If the bond satisfies

\[
B_i(c_i) \geq G(T_i(c_i)) \left( \lambda c_i T_i(c_i) + \frac{\lambda c_i}{h(T_i(c_i))} \right) - \Lambda_i(c_i) \frac{1}{p_i(c_i)} - \lambda c_i \int_0^{T_i(c_i)} z g(z)dz,
\]

then continuous effort and immediate revelation are incentive compatible in the secondary problem.

2. Suppose the award function \( w_{it}(c_i) \) is negative for some dates near \( T_i(c_i) \). If the bond satisfies

\[
B_i(c_i) \left( 1 - \frac{1 - G(T_i(c_i))}{G(T_i(c_i))} \right) \geq \lambda c_i \frac{1}{h(T_i(c_i))} + \frac{\Lambda_i(c_i)}{p_i(c_i)} + \lambda c_i \int_0^{T_i(c_i)} g(z)dz
\]

then continuous effort and immediate revelation are incentive compatible in the secondary problem.

This proposition characterizes the award and bond functions that achieve immediate revelation in the secondary problem and well as continuous effort. As time goes on, the award function decrements the prize for completion, and this erosion is what drives firms to report success immediately. To discourage shirking, the bond is chosen in tandem to keep the firm on the hook late in the contractual horizon. This develops a careful carrot-and-stick balance between the reward and the
punishment. The key is to find the date at which the firm is most likely to give up, and ensure that the award and bond functions satisfy (1). Doing this provides two additional constraints on the bond functions that must be satisfied to ensure the unconstrained solution is a constrained optimum.

This proposition shows that “late penalties” can be an endogenous phenomenon that arises when the award function becomes negative. This shows how the contracts used by Caltrans in Bajari and Lewis [2] could be close to optimal, and why many other markets employ late fees or other punishments when firms fail to reach deadlines. Despite receiving a negative award for success, firms continue to work in fear of the prospect of an even more significant financial loss from the bond. This can lead to violations of the incentive constraints, however, since if

\[ 1 - \frac{1 - G(T_i(c_i))}{G(T_i(c_i))} < 0 \]

then \( G(T_i(c_i)) < 1/2 \), and the left-hand side of (11) is negative while the right-hand side is positive, leading to a contradiction.

**Corollary 4.3** If the award function becomes negative and a firm is more likely to fail than succeed at time zero, it is impossible to simultaneously implement continuous effort and immediate reporting.

This implies that late penalties should be used with caution, since they are guaranteed to lead to inefficiency with high-cost firms. Moreover, the unconstrained solution will no longer be optimal if the award function becomes negative, and the primary problem must be solved using optimal control methods in the primary problem, rather than pointwise maximization.

### 4.2 Optimal plans after dishonest reports

To ensure incentive compatibility at the allocation stage, the optimal effort strategies for dishonest reports must be characterized as well. In particular, there are two qualitatively different types of dishonest reports: Those in which the firm has no intention of exerting any effort at any date, and those in which the deviating firm intends to exert some effort at some date.

To deter spurious bidding in which firms intend never to exert any effort, it must be unprofitable to bid, win, and siphon for all dates. A mechanism *admits arbitrage* if there exists a type \( c'_i \) such
that \( E_{c_{-i}}[P^*_i(c'_i, c_{-i})] > 0 \) and

\[
T^*_i(c'_i) \lambda c'_i - B_i(c'_i) > 0
\]

so that any economic agent could conceivably bid and win, siphon funds for the entire project horizon and then profitably default. If there are some inefficient types who are excluded ex ante this is certainly a concern, but from a broader perspective, arbitrage invites socially inefficient entry by insincere agents. To eliminate these sources of inefficiency, the bond must satisfy the additional restriction

\[
B_i(c_i) \geq \lambda c_i T^*_i(c_i)
\] (12)

This deters the participation of any firms who plan to siphon for the entire horizon, leaving only the behavior of those firms who over-state or under-state their costs, but then exert effort during some period of the contract.

**Proposition 4.4** Suppose the award function satisfies (8), (9) and (10), and the bond function satisfies (12). Then at the relaxed solution,

1. A firm reporting \( c'_i > c_i \) exerts effort for all dates \( t \in [0, T^*_i(c'_i)] \)

2. A firm reporting \( c'_i < c_i \) exerts effort for all dates \( 0 < t \leq \sigma^*_i(c'_i, c_i) \leq T^*_i(c'_i) \) and exerts no effort between \( \sigma^*_i(c'_i, c_i) < t \leq T^*_i(c'_i) \), where the stopping time \( \sigma^*_i(c'_i, c_i) \) is the unique solution to the equation

\[
(1 - G(\sigma^*(c'_i, c_i))(c'_i - c_i - \lambda c'_i) + g(\sigma^*(c'_i, c_i))(w^*_{i*}(c'_i))(c'_i) + B_i(c'_i))
- g(\sigma^*(c'_i, c_i))(\lambda c'_i(T^*_i(c'_i) - \sigma^*(c'_i, c_i))) = 0
\] (13)

3. Siphoning for all dates is never profitable.

This proposition shows that firms who over-state their marginal cost will, like honest firms, work for the entire contract horizon, while firms who under-state their marginal cost will “give up” early and siphon funds for some interval of time near the terminal date.
Figure 1 illustrates the optimal stopping time $\sigma_i^*(c', c_i)$ as a function of the report $c'_i$. When a firm reports honestly, it works continuously from time zero to time $T_i^*(c_i)$ by construction. Deviations to a higher report imply that (1) is still satisfied at all dates, so for $c'_i > c_i$, $\sigma_i^*(c', c_i) = T_i^*(c'_i)$. However, when deviating to a lower report, the payoff of continued effort near the terminal date drops below the value of siphoning, and the firm gives up, so that $\sigma_i^*(c', c_i) < T_i^*(c'_i)$. In Figure 1, the stopping time is monotone in the report $c'_i$, but need not be in general. To ensure incentive compatibility holds, however, monotonicity will be required, which hinges on the form of the bond function.

4.3 Incentive compatible bonds

The only remaining degree of freedom in designing the mechanism is the bond function, $B_i(c)$. To exploit the results of the previous two sections, it must be chosen so that the award function is weakly positive for all true reports at all dates, the conditions in (8), (9) and (10) are satisfied, and the monotonicity condition holds.

Definition Let $m^*(c) = \{P_i^*(c), r_i^*(c_i), w_t^*(c_i), B_i^*(c_i), T_i^*(c_i), s_i^*(c_i), c_i^*(c_i)\}$ be the direct revelation mechanism where $P_i^*(c)$, $r_i^*(c_i)$, $T_i^*(c_i)$ and $s_i^*(c_i)$ are given by Proposition 4.1, the award
function \( w^*_it(c_i) \) satisfies (8) and (9), the effort schedule is given by

\[
\epsilon^*_it(c'_i, c_i) = \begin{cases} 
1 & \text{if } t \leq \sigma^*_i(c'_i, c_i) \lor T^*_i(c'_i) \\
0 & \text{otherwise}
\end{cases}
\]

where \( \sigma^*_i(c'_i, c_i) \) is the unique solution to (13), and the bond is given by

\[
B^*_i(c_i) = G(T^*_i(c_i)) \left\{ \frac{\lambda c^*_i}{h(T^*_i(c_i))} + \frac{(1-\lambda)(c^*_i - c_i)}{h(T^*_i(c'_i))} + \frac{\lambda c^*_i T^*_i(c_i)}{1 - G(T^*_i(c_i))} + \frac{1}{G(T^*_i(c'_i))} \int_0^{T^*_i(c_i)} (1 - G(z))dz(c^*_i - (1-\lambda)c_i) \right\} - \frac{\Lambda_i(c_i)}{p^*_i(c_i)} - \lambda c_i \int_0^{T^*_i(c_i)} g(z)zdz
\]

This mechanism combines the solution to the principal’s relaxed problem in Proposition 4.1, the award function developed in Section 4.1, and the optimal behavior after dishonest reports characterized in Section 4.2 with a particular bond function, \( B^*_i(c_i) \), which is constructed to ensure that the award function is positive at all dates and (2), (10) and (12) are satisfied.

**Theorem 4.5** Suppose \( \psi_i(c_i) \) is increasing and \( h(t) \) is decreasing. Then \( m^*(c) \) is an incentive compatible direct revelation mechanism that implements the solution to the relaxed problem.

This theorem proves the solution of the relaxed program is a solution, and provides one version of the bond function which implements the optimal outcome. The proof of the theorem develops a sufficient set of conditions under which incentive compatibility and obedience obtain, and then constructs the bond appearing above as a particular solution, but there exist many candidate bond functions. While the bond appearing above appears somewhat arbitrary, the indirect implementation in the next section shows how some of its terms can be interpreted as the expectation of the total cost differences between the lowest-cost firm and the next-lowest bidder, providing more economic intuition.

The main practical lesson of Theorem 4.5 is that it is not enough to set the bond at the amount siphonable from the project, \( \lambda c'_i T^*_i(c'_i) \), since this ignores the incentive constraints across types. If a high-cost firm can profitably deviate down and work for some period before giving up and siphoning, these kinds of profitable deviations will lead to inefficiency. In short, the bond plays a non-trivial role in determining incentive compatibility and ensuring that the proper level of effort is exerted, not just preventing spurious bidding.
5 Dynamic Procurement Auctions

This section shows that the direct revelation mechanism developed in the previous section can be implemented through a bidding game that has elements of both a first- and second-price auction.

Consider the following bidding game, the dynamic procurement auction: Let $b = (b_1, ..., b_I)$ be the vector of firm bids, with $b_{(n)}$ the lowest bid and $b_{(n-1)}$ the second-lowest.

1. The principal announces a schedule $T(b_i)$ relating the bid of firm $i$, $b_i$, to a schedule of terminal dates, satisfying

   $$T(b_i) = h^{-1} \left( \frac{\psi_i(b_i)}{v(b_i, b_i)} \right)$$

   and the reserve bid, $b^*$, above which the principal does not award the project, given by

   $$G(T(b^*))v(b^*, b^*) - \psi(b^*) \hat{T}(b^*) 0 = K$$

2. Each participating firm $i$ simultaneously submits a sealed bid $b_i$.

3. The lowest-bidding firm satisfying $b_{(n)} \leq b^*$ wins, and is paid its bid,

   $$r_{it}(b_{(n)}) = b_{(n)}$$

   each moment of the entire contract period $T(b_{(n)})$.

4. If a winning firm succeeds, it receives an award

   $$w_{i0}(b_{(n)}, b_{(n-1)}) = \frac{a(b_{(n)}, b_{(n-1)} \vee b^*) + (1 - G(T_i(b_{(n)})))B_i(b_{(n)}) + \lambda b_{(n)} \int_0^{T(b_{(n)})} g(z)dz}{G(T(b_{(n)}))}$$

   $$w_{it}(b_{(n)}, b_{(n-1)}) = w_{i0}(b_{(n)}, b_{(n-1)} \vee b^*) - \lambda b_{(n)}t$$

   where

   $$a(b_{(n)}, b_{(n-1)}) = (b_{(n-1)} \vee b^*) \int_0^{T(b_{(n-1)} \vee b^*)} (1 - G(z))dz - b_{(n)} \int_0^{T(b_{(n)})} (1 - G(z))dz$$

   $$+ \int_{b_{(n)}}^{b_{(n-1)} \vee b^*} x(-T'(x))(1 - G(x))dx \quad (14)$$

   $\text{Cost Savings}$

   $$\text{Efficiency Bonus}$$

26
If a winning firm fails to complete the project by $T(b^{(n)})$, it forfeits a bond

$$B(b^{(n)}, b^{(n-1)}) = G(T(b^{(n)})) \left\{ \lambda b^{(n)} T(b^{(n)}) + \frac{\lambda b^{*}}{h(T(c))} + \frac{(1 - \lambda)(b^{*} - b^{(n)})}{h(T(b^{*}))} + \frac{\lambda b^{*} T(b^{(n)})}{1 - G(T(c))} \right\}$$

$$+ \frac{1}{G(T(b^{*}))} \int_{0}^{T(b^{(n)})} (1 - G(z)) dz (b^{*} - (1 - \lambda)b^{(n)}) - a(b^{(n)}, b^{(n-1)}) - \lambda b^{(n)} \int_{0}^{T(b^{(n)})} g(z) dz$$

In this indirect mechanism, firms bid competitively, stating a cost-per-unit time. The lowest-bidding firm that is eligible to win receives its bid per unit of time spent working on the project, while its award and the bond are computed using the next highest-bid, similar to a second-price auction. The function $a(b^{(n)}, b^{(n-1)})$ is chosen so that the expectation conditional on $b^{(n-1)}$ being the $n-1$-st lowest draw from a sample of $n$ from the distribution $F(c_i)$, conditional on $b^{(n)}$ being the lowest, equals the informational rent term $\Lambda_i(b^{(n)})$.

**Theorem 5.1** Suppose that $\psi_i(c_i)$ is increasing in $c_i$ and the hazard rate of success $h(t)$ is decreasing in $t$. Then the procurement auction implements the same allocation as the profit-maximizing direct revelation mechanism, so it is an optimal indirect implementation. It is a Bayesian Nash equilibrium to bid $b_i = c_i$.

The theorem shows that the dynamic procurement auction implements the same allocation as the optimal direct revelation mechanism, even though the only place where the distribution of types appears only through the virtual marginal cost in the $T(b_i)$ schedule.

This design combines a number of features common to well-known mechanisms, but is not equivalent to any of them. Similar to a first-price auction, the lowest eligible bidder wins and is paid its bid at each moment to fund the project. Unlike a first-price auction, these bids are honest in equilibrium, and exactly equal the firms’ true costs of operation. Similar to a second-price auction, the winner’s payoff depends on the next highest bid, since it appears in $a(b^{(n)}, b^{(n-1)})$. In particular, through $a(b^{(n)}, b^{(n-1)})$ the mechanism rewards the winner for the lower total cost over the horizon that would have been awarded to the next-best alternative, called the *cost savings* in (14), but the winner also receives a bonus corresponding to the extended terminal date as a result of its lower cost compared to the next best alternative, similar to a Vickrey-Clarke-Groves mechanism, called the *efficiency bonus* in (14). However, it is not a dominant strategy to bid honestly, since the expected utility calculations are carried out assuming that other firms also bid truthfully, and honesty is not
a best response to all potential deviations by rival participants.

A novel feature of the mechanism is that the bond is determined endogenously through the bidding and, indeed, must be determined endogenously. The construction of the bid occurs in the secondary problem along with the award function, and this places restrictions on what kinds of bid functions are compatible with a given award. If the award function is determined through the function $a(b_n, b_{n-1})$, then the bond must satisfy the restrictions imposed by (9) and (10). The only way to guarantee this without literally adopting the optimal direct revelation mechanism is to make bonding endogenous. This emphasizes the fact that while bonding is often assumed only to deter spurious bidders from participation, it also plays a role in maintaining incentive compatibility, and arbitrary bond schedules can undermine incentives more generally.

6 Conclusion

This paper provides a number of useful results about how to design contracts in markets that suffer from both adverse selection and dynamic moral hazard. The ability of firms to engage in siphoning behavior creates a number of channels through which the principal can be harmed, requiring more sophisticated contracts that explicitly incorporate the dynamic nature of the market. This results in “cost-plus” payments, where the firm is compensated for its cost per unit time, and all informational rents are deferred until success is achieved. If the firm fails to succeed, however, it forfeits a bond chosen to satisfy incentive compatibility conditions as well as ensure the firm keeps working late in the contract when it might otherwise prefer to give up and siphon. Lastly, an indirect mechanism is proposed that determines the award and bond functions endogenously through the bidding, and shares features with both first-price auctions and Vickrey-Clarke-Groves mechanisms. This provides a useful benchmark for future research on the design of dynamic mechanisms.

The most significant drawback of the present analysis is that it assumes that the amount which a firm can post as bond is unlimited. In practice, firms will be constrained by the value of their real, pledgeable assets, and this limited liability will have to be considered as a constraint on the principal’s design of the mechanism. In situations where intellectual property such as human capital or patents make up the majority of the firm’s value, it would be difficult or impossible — both practically and legally — to construct agreements where high bond values are at stake. Since the bond essentially plays the role of a “side bet” between the principal and firm that allows them
to gamble over the project’s value and the informational rents in a way that is incentive compatible, there are a number of potential solutions. One option is for the principal to restrict attention to only those firms who can afford the risk of a high bond, thereby reducing competition. Another approach is for the principal to solve the problem in which the limited liability constraint binds at the optimum, and perhaps use a negative bond or pooling in the secondary mechanism design problem. That analysis would be very different from the one in this paper, but could provide useful and more general results.

The issue of negative award functions arises in Proposition 4.2, but the prevalence of this practice in the real world makes it worthy of further study in light of Corollary 4.3: Whenever high cost firms are given a task they are considered unlikely to complete at the allocation stage, negative award functions necessarily result in inefficiency through delays in revelation of success or inefficient effort provision. The main advantage of picking an award that penalizes the firm near the deadline is that it allows a lower default penalty, since some of the punishment for failure is shifted into the contract. If a high-quality firm optimally works such a long time that its limited liability constraint binds, switching from a high bond and strictly positive award to a lower bond and weakly negative award near the terminal date might relax these constraints. However, it is unclear which firms have the most expensive contracts, since $c_iT^*_i(c_i)$ is not necessarily a monotone or single-peaked function.

In risky industries in which projects end in default, the principal may take steps to “check in” with the firm before the deadline or better track the firm’s efforts over the course of the project. If the project is awarded to the most efficient firm, the principal will want to renegotiate ex post after a failure, and potentially forgive the bond. A model that incorporates noisy signals of a firm’s progress (or lack thereof) might provide a basis for contracts in which renegotiation or monitoring is endogenous. As in this paper, this potential for renegotiation will likely impact the optimal bond structure, and understanding the interplay of bonding and renegotiation may provide some guidance for procurement in large scale projects where failures or missed deadlines are common.

An even more comprehensive theory of industry management might be developed by allowing firms to keep their own “buffer-stocks” of funds, where the likelihood and survival of the firm depend on these hidden assets. As Doepke and Townsend [13] show, the problem of hidden actions and hidden savings can be formalized in a general framework, incorporating default issues similar to Calveras et al. [5]. Understanding how bidding becomes a signal of a firm’s health in a dynamic
model, and how to mitigate firms’ tendencies to hide their weaknesses, could provide useful insights into how competition can become destructive.

Finally, this paper is an independent private values framework, in which each firm’s cost draw is independent of the others. In situations where each firm receives information that is useful to the others in deciding the value of the project, the optimal mechanisms will likely be different. Alternatively, uncertainty could be modelled about the hazard rate, \( h[E|s_1, ..., s_n] \), where \( s_i \) is the signal received by firm \( i \) about the likelihood of success. Better and more efficient markets might be designed where information is aggregated to incorporate winner’s curse phenomena.
References


7 Appendix: Proofs

Proof of Proposition 3.1

Proof A firm $i$ who holds the rights to the project receives a value from exerting effort for a small period of time $\Delta$ of

$$J_i(t, E_{it}, c', c) = e^{-\rho \Delta} h(E_{it}) \Delta \hat{w}_{it}(c', c_{-i}) + (1 - h(E_{it}) \Delta) J_i(t + \Delta, E_{it} + \Delta, c', c) + \Delta (r_{it}(c', c_{-i}) - c_i)$$

Re-arranging and dividing by $\Delta$ yields

$$- \frac{J_i(t + \Delta, E_{it} + \Delta, c', c) - J_i(t, E_{it}, c', c)}{\Delta} + \left( \frac{1 - e^{-\rho \Delta}}{\Delta} + h(E_{it}) \right) J_i(t + \Delta, E_{it} + \Delta, c', c)$$

$$= h(E_{it}) e^{-\rho \Delta} \hat{w}_{it}(c', c_{-i}) + r_{it}(c', c_{-i}) - c_i$$
and taking the limit as $\Delta \to 0$ gives

$$-\frac{dJ_i(t, E_{it}, c_i', c)}{dt} + (\rho + h(E_{it})) J_i(t, E_{it}, c_i', c) = h(E_{it})\bar{w}_{it}(c_i', c_{-i}) + r_{it}(c_i', c_{-i}) - c_i$$

Solving this first-order differential equation in $t$ yields an expression for the value function on an interval of time $[t_a, t_b]$ during which effort is exerted,

$$(1 - G(E_{ita}(c_i', c)))e^{-\rho t_a}J_i(t_a, E_{ita}(c_i', c), c_i', c) =$$

$$\int_{t_a}^{t_b} (1 - G(E_{iz}(c_i', c)))e^{-\rho z} \left\{ h(E_{iz}(c_i', c))\bar{w}_{it}(c_i', c_{-i}) + r_{it}(c_i', c_{-i}) - c_i \right\} dz$$

$$+ (1 - G(E_{itb}(c_i', c)))e^{-\rho t_b}J_i(t_b, E_{itb}(c_i', c), c_i', c)$$

Then as $\rho \to 0$, the value of working for an interval $[t_a, t_b]$ equals

$$\begin{align*}
(1 - G(E_{ita}(c_i', c)))J_i(t_a, E_{ita}(c_i', c), c_i', c) &= \\
&= \int_{t_a}^{t_b} (1 - G(E_{iz}(c_i', c))) \left\{ h(E_{iz}(c_i', c))\bar{w}_{it}(c_i', c_{-i}) + r_{it}(c_i', c_{-i}) - c_i \right\} dz \\
&\quad + (1 - G(E_{itb}(c_i', c)))J_i(t_b, E_{itb}(c_i', c), c_i', c)
\end{align*}$$

and the value of siphoning for an interval $[t_a, t_b]$ equals, noting that $E_{ita}(c_i', c) = E_{itb}(c_i', c)$,

$$\begin{align*}
(1 - G(E_{ita}(c_i', c)))J_i(t_a, E_{ita}(c_i', c), c_i', c) &= \\
&= \int_{t_a}^{t_b} \lambda(1 - G(E_{ita}(c_i', c)))r_{iz}(c_i', c_{-i})dz + (1 - G(E_{ita}(c_i', c)))J_i(t_b, E_{ita}(c_i', c), c_i', c)
\end{align*}$$

Using the boundary condition that failure gives a terminal payoff of $-(1 - G(E_{iT}(c_i', c_{-i})))B_t(c_i', c_{-i})$, we can work backwards to generate the linear optimal control problem,

$$\max_{\epsilon_i(c_i', c)} J_i(c_i', c, \epsilon_i(c_i', c)) =$$

$$\max_{\epsilon_i(c_i', c)} \int_0^{T_i(c_i', c_{-i})} \epsilon_i(t, c_i') \left\{ g(E_{it}(c_i', c))\bar{w}_{it}(c_i', c_{-i}) + (1 - G(E_{it}(c_i', c))(r_{it}(c_i', c_{-i}) - c_i) \right\}$$

$$\underbrace{y_{it}(c_i', c, E_{it}(c_i', c))}_{y_{it}^0(c_i', c, E_{it}(c_i', c))}$$

$$+ (1 - \epsilon_i(t, c_i'))(1 - G(E_{it}(c_i', c))) \{ \lambda r_{it}(c_i', c_{-i}) \} \, dt - (1 - G_{iT}(c_i', c_{-i}))(E_{it}(c_i', c)))B_t(c_i', c_{-i})$$

subject to $\epsilon_i(t, c_i') \in \{0, 1\}$ and $\dot{E}_{it}(c_i', c) = \epsilon_i(t, c_i')$.

The Hamiltonian of the optimal control problem is

$$H(\epsilon_{it}, E_{it}; t, c_i', c) = \epsilon_{it}y_{it}^1(c_i', c, E_{it}) + (1 - \epsilon_{it})y_{it}^0(c_i', c, E_{it}) + \mu_{it}(c_i', c)\epsilon_{it}$$

Due to the linearity of the Hamiltonian, by Pontryagin’s necessary conditions, the optimal control
takes a bang-bang form:

\[
\epsilon_{it}(c'_i, c) = \begin{cases} 
1 & y_{it}^1(c'_i, c, E_{it}(c'_i, c)) - y_{it}^0(c'_i, c, E_{it}(c'_i, c)) + \mu_{it}(c'_i, c) \geq 0 \\
0 & \text{otherwise}
\end{cases}
\]

and the co-state variable evolves as

\[
- \frac{\partial H}{\partial E_{it}} = \hat{\mu}_{it}(c'_i, c) = \epsilon_{it}(c'_i, c) \left\{ -g'(E_{it}(c'_i, c))\hat{w}_{it}(c'_i, c_{-i}) + g(E_{it})(r_{it}(c'_i, c_{-i}) - c_i) \right\}
\]

\[
+ (1 - \epsilon_{it}(c'_i, c))g(E_{it}(c'_i, c))\lambda r_{it}(c'_i, c_{-i})
\]

The transversality condition for \( E_{it}(c'_i, c) \) is \( \mu_{iT_i(c'_i, c_{-i})}(c'_i, c) = g(E_{iT_i(c'_i, c_{-i})}(c'_i, c))B_i(c'_i, c_{-i}) \), since the marginal benefit of having exerted another instant of effort is the probability of avoiding the bond payment times the loss. □

**Proof of Proposition 3.2**

**Proof** Properties of the derivative or subgradient of the value function and upper semi-continuity in parameters are proved in Seierstad and Sydsaeter [29] (p. 220) or Clarke et al. [10] (p. 105). The characterization of incentive compatibility is proved in Milgrom and Segal, Proposition 4.1. The monotonicity condition is a consequence of rearranging the direct and indirect utility functions. □

**Proof of Proposition 4.1**

**Proof** Since delay after success is always costly for the principal, the unconstrained solution is to set \( s_{it}(c_i, c_{-i}) = 0 \) for all \( c_i \).

Whenever the coefficient of \( \epsilon_{it}(c_i, c) \) is positive, it should be set to one, and zero otherwise. Whenever \( \epsilon_{it}(c_i, c) \) is one, \( r_{it}(c_i, c_{-i}) \) should be set equal to \( c_i \) to avoid incurring the payoff reduction through \( v(r_{it}(c_i, c_{-i}), c_i) \).

When \( \epsilon_{it}(c_i, c) \) is set equal to zero and \( r_{it}(c'_i, c_{-i}) \geq 0 \), the principal incurs a loss through siphoning. Since the agents have no time preference, it can be arranged so that the firm works continuously, since otherwise the principal could adjust the contract so that all productive dates occur from time zero to time \( T' \), and set \( T' \) to \( T_i(c'_i, c_{-i}) \). This implies that \( G(E_{it}(c_i, c)) = G(t) \).

Then the principal’s payoff becomes

\[
\max_{m(c)} \mathbb{E}_c \left[ \sum_{i} P_i(c) \left\{ \int_{T_i(c)}^{T_i(c)} (1 - G(t)) (h(t)v(r_{it}(c), c_i) - \psi(c_i)) dt - K \right\} \right]
\]

Maximizing over \( T_i(c) \) for each firm \( i \) yields the first-order necessary condition \( h(T_i(c_i, c_{-i}))v(c_i, c_i) - \psi(c_i) = 0 \) and second-order sufficient condition \( h'(T_i(c_i, c_{-i}))v(c_i, c_i) < 0 \). So that there is an internal solution that depends only on the winner’s marginal cost, \( c_i \), if the hazard rate is decreasing at the optimum. If \( h'(t) > 0 \) for all \( t \), the problem becomes convex and any critical point is a local
minimum, so that there is a corner solution where a winning firm is contracted to work until success
occurs. Note that since $c_{-i}$ appears nowhere in $h(T_i(c_i, c_{-i}))v(c_i, c_i) - \psi_i(c_i) = 0$, $T_i(c_i, c_{-i})$ is not
a function of $c_{-i}$.

Lastly, the inner integral $\phi_i(c_i) = \int_0^{T_i(c_i)} (1 - G(t)) (h(t)v(c_i, c_i) - \psi(c_i)) dt - K$ is decreasing in
$c_i$ if $h(t)$ is monotone, so that the optimal decision is

$$P_i(c_i, c_{-i}) = \begin{cases} 1 & \text{if } c_i = \min_k c_k \text{ and } \phi_i(c_i) \geq K \\ 0 & \text{otherwise} \end{cases}$$

□

Proof of Proposition 4.2

Proof Since the firm’s payoff once it succeeds does not depend on its private information, the
delay is independent of the firm’s private information and relies only on the report. Consider the
maximization problem

$$\tilde{w}_{it}(c'_i) = \max_{0 \leq s \leq T_i(c'_i)} w_{is}(c'_i) + \lambda sc'_i$$

A necessary condition at any optimal $s^*_i(c'_i) < T_i(c'_i)$ is

$$\tilde{w}_{is}(c'_i) + \lambda c'_i = 0$$

Solving the equation for $w_{is}(c'_i)$ yields (8),

$$w_{it}(c'_i) = w_{i0}(c'_i) - \lambda t c'_i$$

where the arbitrary constant $w_{i0}(c_i)$ will be chosen to satisfy the ex ante individual rationality
constraint. Substituting the proposed award function into the maximization problem yields

$$\max_{0 \leq s \leq T_i(c'_i)} w_{i0}(c'_i)$$

which is independent of $s$. Consequently, reporting immediately is a solution for all $c'_i$. If this
indifference is a cause for concern, adding an arbitrarily small but positive and decreasing function
will break the indifference in favor of immediate reporting.

To ensure that types who report honestly receive the correct expected payoff, substituting the
award function into the ex ante individual rationality constraint yields

$$p_1(c_i) \left\{ \int_0^{T_i(c_i)} f(z) \left\{ w_{i0}(c_i) - \lambda c_i z \right\} dz - (1 - G(T_i(c_i)))B_i(c_i) \right\} = \int_{c_i}^{c^*_i} p_i(x) \int_0^{T_i(x)} (1 - G(z))dz dx$$
from which (9) is derived,

$$w_{i0}(c_i) = \frac{\Lambda_i(c_i) / pi(c_i) + (1 - G(T_i(c_i))B_i(c_i) + \lambda c_i \int_0^{T_i(c_i)} zg(z)dz}{G(T_i(c_i))}$$

Returning to (1) with $E_{it}(c_i, c_i) = t$, effort will be implemented by the $c$ type at all dates $t$ if,

$$y_{i1}^1(c_i, c_i, t) - y_{i1}^0(c_i, c_i, t) + \mu_{it}(c_i, c_i) \geq 0$$

For $E_{it}(c_i, c_i) = t$, this equals

$$g(t)w_{it}(c_i) + \int_t^{T_i(c_i)} -g'(z)w_{iz}(c_i)dz + g(T_i(c_i))B_i(c_i) - (1 - G(t))\lambda c_i \geq 0$$

To ensure that this inequality holds for all $t \in [0, T_i(c_i)]$, we look for a minimum to the left-hand side in $t$. The first-order necessary condition is

$$2g'(t^*)w_{it^*}(c_i) \leq 0$$

Since $w_{it}(c_i)$ is monotone decreasing in time and $g'(t) \neq 0$, the first-order necessary condition has at most one interior critical point,

$$t^* = \frac{w_{i0}(c_i)}{\lambda c_i}$$

Depending on the sign of $w_{iT_i(c_i)}(c_i)$, there are two candidate solutions: $t^*$ and $T_i(c_i)$. If $w_{iT_i(c_i)}(c_i) > 0$, the solution is $T_i(c_i)$ because the first-order condition has no zeros and is monotone decreasing, while if $w_{iT_i(c_i)}(c_i) < 0$, then $t^*$ is the solution because the first-order necessary condition has a unique zero and the second-order sufficient condition is satisfied. This generates two cases.

In the first case where $w_{it}(c_i)$ is positive at all dates, evaluating (1) at $T_i(c_i)$ and substituting in $w_{it}(c_i)$, the inequality becomes

$$h(T_i(c_i))(w_{i0}(c_i) - \lambda c_i T_i(c_i) + B_i(c_i)) - \lambda c_i \geq 0$$

because

$$w_{i0}(c_i) + B_i(c_i) = \frac{\Lambda_i(c_i) + B_i(c_i) + \lambda c_i \int_0^{T_i(c_i)} zg(z)dz}{G(T_i(c_i))}.$$ 

Consequently, implying that exerting effort at the final date of the contract gives a better payoff than siphoning at $T_i(c_i)$. Re-arranging the inequality to solve for $B_i(c_i)$ provides the inequality in (10),

$$B_i(c_i) \geq G(T_i(c_i)) \left( \lambda c_i T_i(c_i) + \frac{\lambda c_i}{h(T_i(c_i))} \right) - \frac{\Lambda_i(c_i)}{pi(c_i)} - \lambda c_i \int_0^{T_i(c_i)} zg(z)dz$$

In the second case where $w_{it}(c_i)$ becomes negative for some dates near $T_i(c_i)$, evaluating (1) at
\( t^* = \frac{w_{i0}(c_i)}{\lambda c_i} \) yields
\[
\int_{T_i(c_i)}^{T_i(c_i)} -g'(z)w_{iz}(c_i)dz + g(T_i(c_i))B_i(c_i) - (1 - (G(t^*))\lambda c_i \geq 0
\]
Integrating the first term by parts yields
\[
g(T_i(c_i))(B_i(c_i) - w_{iT_i(c_i)}(c_i)) + \lambda c_i(G(T_i(c_i)) - G(t^*)) - (1 - (G(t^*))\lambda c_i \geq 0
\]
and substituting in the award function and re-arranging yields the condition in (11). \( \square \)

**Proof of Proposition 4.4**

**Proof** Consider deviations \( c'_i > c_i \). Evaluating the left-hand side of (1) at \( E_{it}(c'_i, c_i) = t \) yields
\[
y_{it}^1(c'_i, c_i, E_{it}(c'_i, c_i)) - y_{it}^0(c'_i, c_i, E_{it}(c'_i, c_i)) + \mu_{it}(c'_i, c_i)
\]
which equals
\[
g(t)w_{it}(c'_i) - \lambda c'_i - g'(t)w_{it}(c'_i) + (g(t) + (1 - G(t))(c'_i - c_i)
\]
The first three terms equal the right-hand side of (1) when the \( c'_i \) type reports \( c'_i \) honestly, so by construction must be positive. The last term is positive because \( c'_i > c_i \), and \( g(t) \) and \( 1 - G(t) \) are positive. Therefore, (1) holds at \( E_{it}(c'_i, c_i) = t \) for all \( c'_i > c_i \), and it is a profit-maximizing policy for firm that overbid to work continuously.

Lying downward implies that (1) becomes
\[
y_{it}^1(c'_i, c_i, E_{it}(c'_i, c_i-1)) + (c'_i - c_i)(1 - G(E_{it}(c'_i, c_i))) - y_{it}^0(c'_i, c_i, E_{it}(c'_i, c_i)) + \mu_{it}(c'_i, c_i) \geq 0
\]
The \( (g(t) + (1 - G(t))(c'_i - c_i) \) term that was positive at \( E_{it}(c'_i, c_i) = t \) for upward deviations is now negative, so that it is no longer guaranteed that the deviator works the entire horizon.

Note that the Hamiltonian of the optimal control problem is strictly decreasing in \( t \), since \( w_{it}(c'_i) \) is strictly decreasing in \( t \), so that if the firm ever works, it works at the beginning of the contract.

Let \( \sigma_i(c'_i, c_i) \) be the stopping time at which the \( c'_i \) type gives up on the project given true type \( c_i \). Then the optimal stopping time \( \sigma(c'_i, c_i) \) with \( c'_i < c_i \) necessarily satisfies
\[
\max_{\sigma(c'_i, c_i)} \int_0^{\sigma} (1 - G(z))(h(z)w_{iz}(c'_i) + c'_i - c_i) dz + \int_{T_i(c'_i)}^{T_i(c'_i)} (1 - G(\sigma))\lambda c'_i dz - (1 - G(\sigma))B_i(c'_i)
\]
whose critical points are characterized by (8), which is a version of (1). \( \square \)

**Proof of Theorem 4.5**

**Proof** For types \( c'_i > c_i \), incentive compatibility is satisfied, because
\[
\frac{\partial^2 U_i(c'_i, c_i)}{\partial c'_i \partial c_i} = -p_i'(c'_i) \int_0^{T_i(c'_i)} (1 - G(z)) dz - p_i(c'_i)(1 - G(T_i(c'_i)))T_i(c'_i) \geq 0
\]
so that supermodularity holds. By standard mechanism design arguments, this implies that deviations above one’s true type are unprofitable.

For deviations \( c'_i < c_i \), the argument is more complicated because the firm gives up at some date near \( T_i(c'_i) \), but \( T_i(c'_i) \) is greater than \( T_i(c_i) \), so the firm might gain by deviating and giving up early, leading to a profitable deviation. To deter such deviations, consider bond functions of the form

\[
B_i(c_i) = G(T_i(c_i)) \{ \lambda c_i T_i(c_i) + \xi_i(c_i) \} - \frac{\Lambda_i(c_i)}{p_i(c_i)} - \lambda c_i \int_0^{T_i(c_i)} zg(z)dz
\]

where \( \xi_i(c_i) \) is an as-yet undetermined function. By developing criteria for which incentive compatibility is ensured in terms of \( \xi_i(c_i) \), any \( \xi_i(c_i) \) that also satisfies (10) and (12) will then solve the optimal contracting problem.

Note that if the no-arbitrage condition holds, any type who fails to work receives a weakly negative payoff regardless of its policy, which is worse than reporting honestly. Therefore, we need only consider deviations in which the firm finds it profitable to work for some strictly positive period of time before giving up. As a slight abuse of notation, define the payoff function in \((c'_i, c_i)\) for a \( c'_i < c_i \) type-report pair in which \( \sigma^*(c'_i, c_i) > 0 \) as

\[
U_i(c'_i, c_i) = p_i(c'_i) J_i^*(\sigma^*(c'_i, c_i), c'_i, c_i)
\]

Note that if \( U_i(c'_i, c_i) \) is supermodular, then by standard arguments incentive compatibility follows (see Fudenberg and Tirole (1991) or Mas-Colell et al. (1993)). This function is supermodular if

\[
p'_i(c'_i) \frac{\partial J_i^*(\sigma^*_i(c'_i, c_i), c'_i, c_i)}{\partial c'_i} + p_i(c'_i) \left\{ \frac{\partial^2 J_i^*(\sigma^*_i(c'_i, c_i), c'_i, c_i)}{\partial \sigma \partial c'_i} \frac{\partial \sigma^*_i(c'_i, c_i)}{\partial c_i} + \frac{\partial^2 J_i^*(\sigma^*_i(c'_i, c_i), c'_i, c_i)}{\partial c_i \partial c'_i} \right\} \geq 0
\]

Since \( p'_i(c'_i) < 0 \) and

\[
\frac{\partial J_i^*(\sigma^*_i(c'_i, c_i), c'_i, c_i)}{\partial c_i} = -\int_0^{\sigma^*_i(c'_i, c_i)} (1 - G(z))dz < 0,
\]

the first term is positive by (2). The second term is positive as long as

\[
\frac{\partial^2 J_i^*(\sigma^*_i(c'_i, c_i), c'_i, c_i)}{\partial \sigma \partial c'_i} \frac{\partial \sigma^*_i(c'_i, c_i)}{\partial c_i} \geq 0
\]

Using (1), Proposition 4.4, and the necessary condition for maximization in \( \sigma_i \) from (9), \( \partial \sigma^*_i(c'_i, c_i)/\partial c_i \) has the same sign as \( -(1 - G(\sigma^*_i(c'_i, c_i))) < 0 \), so that term is unambiguously negative. Since

\[
\frac{\partial J_i(\sigma_i, c'_i, c_i)}{\partial \sigma_i} = (1 - G(\sigma_i)(c'_i - c_i - \lambda c'_i + g(\sigma_i)(w_{i\sigma}(c'_i) + B_i(c'_i)) - g(\sigma_i)(\lambda c'_i(T_i(c'_i) - \sigma_i),
\]

and

\[
w_{i\sigma}(c'_i) + B_i(c'_i) = \xi_i(c'_i) + \lambda c'_i(T_i(c'_i) - \sigma_i),
\]

38
we have
\[
\frac{\partial J_i(\sigma_i, c'_i, c_i)}{\partial \sigma_i} = (1 - G(\sigma_i))(c'_i - c_i - \lambda c'_i) + g(\sigma_i)\xi_i(c'_i)
\]

Then computing the cross-partial derivative of \(J_i(\sigma_i, c'_i, c_i)\) with respect to \(c'_i\) and \(\sigma_i\) yields the condition for \(U_i(c'_i, c_i)\) to exhibit supermodularity,
\[
h(\sigma_i)\xi'_i(c_i) + (1 - \lambda) \leq 0. \tag{15}
\]

Exploiting the functional form assumption on \(B_i(c_i)\) yields four conditions that \(\xi_i(c'_i)\) must satisfy: The award function must be positive,
\[
\xi_i(c_i) \geq \frac{\lambda c_i}{h(T_i(c_i))} \tag{16}
\]
the condition in (10),
\[
\xi_i(c_i) \geq \frac{\lambda c_i}{h(T_i(c_i))} \tag{17}
\]
the no arbitrage condition in (12),
\[
\xi_i(c_i) \geq \frac{\lambda c_i}{h(T_i(c_i))} \tag{18}
\]
and incentive compatibility in (15),
\[
\frac{\partial \xi_i(c_i)}{\partial c_i} \leq \frac{-(1 - \lambda)}{h(\sigma_i(c'_i, c_i))} \tag{19}
\]

Such functions \(\xi_i(c_i)\) exist, since the inequalities above collectively imply that \(\xi_i(c_i)\) must be “sufficiently large” and decrease “sufficiently quickly”. Any function \(\xi_i(c_i)\) that satisfies all four inequalities will achieve the desired outcome. It is straightforward to verify that the proposed bond does so, with
\[
\xi_i(c_i) = \frac{\lambda c^*_i}{h(T_i(c_i))} + \frac{(1 - \lambda)(c^*_i - c_i)}{h(T_i(c^*_i))} + \frac{\lambda c^*_i T_i(c_i)}{1 - G(T_i(c_i))} + \frac{\int_{0}^{T_i(c_i)}(1 - G(z))dz(c^*_i - (1 - \lambda)c_i)}{G(T_i(c^*_i))} \tag{20}
\]
which is shown in the extended appendix. □

**Proof of Theorem 5.1**

**Proof** Let \(f_{(n-1)}(c)\) be the density function of the \(n - 1\)-st order statistic drawn from \(F(c)\), given that \(c_{(n-1)} > b(n)\). First, note that
\[
\mathbb{E}_{b_{(n-1)}}[a(b(n), b_{(n-1)})|b_{(n-1)} > b(n)] = \frac{\int_{b(n)}^{b^*} a(b(n), x)f_{(n-1)}(x)dx + \int_{b(n)}^{b^*} a(b(n), b^*)f_{(n-1)}(x)dx}{1 - F_{(n-1)}(b(n))}
\]

39
and after an integration by parts

\[
- \frac{[a(b_n), x)(1 - F_{n-1}(x))]_{b_n}^{b^*} + \int_{b_n}^{b^*} \frac{\partial a(b_n, x)}{\partial x} (1 - F_{n}(x))dx + a(b_n, b^*)(1 - F_{n-1}(b^*))}{1 - F_{n-1}(b_n)}
\]

\[
= \frac{\left[ a(b_n), x)(1 - F_{n-1}(x))]_{b_n}^{b^*} + \int_{b_n}^{b^*} \left( \int_T^0 (1 - G(z))dz \right) (1 - F_{n}(x))dx + a(b_n, b^*)(1 - F_{n-1}(b^*))}{1 - F_{n-1}(b_n)}
\]

\[
= \int_{b_n}^{b^*} \int_T^0 (1 - G(z))dz(1 - F_{n}(x))dx
\]

implying

\[
\mathbb{E}[a(b_n), b_{n-1})|b_{n-1}> b_n] = \frac{\int_{b_n}^{b^*} \int_T^0 (1 - G(z))dzdx}{p_i(b_n)} = \frac{A(b_n)}{p_i(b_n)}
\]

So the expectation of \(a(b_n, b_{n-1})\) conditional on winning is equal to the informational rent conditional on winning.

Substituting this into the direct utility function where it appears in the bond and award function generates the same objective for a firm as in the direct revelation mechanism in Section 4. Since truth-telling is an equilibrium strategy there, it is here as well. Since the allocation of the project and the payoffs are all equivalent in expectation, this is a profit-maximizing indirect implementation. 

\[\square\]
8 Extended Appendix

Verification of (16)-(19):
To see that (19) holds, note that
\[
 w_{i0}(c_i') = \frac{\Lambda_i(c_i')/p_i(c_i') + (1 - G(T_i(c_i')))B_i(c_i') + \lambda c_i' \int_0^{T_i(c_i')} zg(z)dz}{G(T_i(c_i'))} - \lambda c_i' \sigma_i(c_i', c_i) + B_i(c_i') - \lambda c_i' \sigma_i(c_i', c_i) = \frac{\Lambda_i(c_i')/p_i(c_i') + (1 - G(T_i(c_i')))B_i(c_i') + \lambda c_i' \int_0^{T_i(c_i')} zg(z)dz}{G(T_i(c_i'))} - \lambda c_i' \sigma_i(c_i', c_i)
\]
Substituting this into
\[
 \frac{\partial J_i(c_i, c_i')}{\partial \sigma_i} = (1 - G(\sigma_i))(c_i' - c_i - \lambda c_i') + g(\sigma_i)(w_{i0}(c_i') + B_i(c_i')) - g(\sigma_i)(\lambda c_i'(T_i(c_i') - \sigma))
\]
yields
\[
 \frac{\partial J_i(c_i, c_i', c_i)}{\partial \sigma_i} = g(\sigma_i'(c_i', c_i))\xi_i(c_i') + (1 - G(\sigma_i'(c_i', c_i)))(c_i' - \lambda c_i' - c_i)
\]
and taking the partial derivative with respect to \(c_i'\) yields (19).
To see where the other three conditions come from, note that
\[
 w_{i0}(c_i') = \frac{\Lambda_i(c_i')/p_i(c_i') + (1 - G(T_i(c_i')))B_i(c_i') + \lambda c_i' \int_0^{T_i(c_i')} zg(z)dz}{G(T_i(c_i'))}
\]
and substituting in the bond
\[
 B_i(c_i) = G(T_i(c_i)) \{\lambda c_i T_i(c_i) + \xi_i(c_i)\} - \frac{\Lambda_i(c_i)}{p_i(c_i)} - \lambda c_i \int_0^{T_i(c_i)} zg(z)dz
\]
yields
\[
 w_{i0}(c_i') = (1 - G(T_i(c_i'))) \{\lambda c_i T_i(c_i) + \xi_i(c_i)\} + \frac{\Lambda_i(c_i')}{p_i(c_i')} + \lambda c_i' \int_0^{T_i(c_i')} zg(z)dz
\]
Substituting this into (10) and (12) and re-arranging yields the inequalities
\[
 \xi_i(c_i, c_{-i}) \geq \frac{\lambda c_i \int_0^{T_i(c_i)} G(z)dz - \Lambda_i(c_i)/p_i(c_i)}{1 - G(T_i(c_i))}
\]
\[
 \xi_i(c_i) \geq \frac{\lambda c_i}{h(T_i(c_i))}
\]
\[
 \xi_i(c_i) \geq \frac{\lambda c_i \int_0^{T_i(c_i)} G(z)dz + \Lambda_i(c_i)/p_i(c_i)}{G(T_i(c_i))}
\]
Verification that (20) satisfies (16)-(19):
To verify that the proposed \(\xi_i(c_i)\) function satisfies the sufficient conditions, note first that
\[
 \frac{\partial \xi_i(c_i)}{\partial c_i} = - \frac{h'(T_i(c_i))T_i'(c_i)}{h(T_i(c_i))^2} T \lambda c_i - \frac{(1 - \lambda)}{h(T_i(c_i)^*)} + \frac{\lambda c_i^*}{1 - G(T_i(c_i)^*)} T_i'(c_i) - (1 - \lambda) \int_0^{T_i(c_i)} (1 - G(z))dz + T_i'(c_i)(c_i^* - (1 - \lambda)c_i) < - \frac{(1 - \lambda)}{h(T_i(c_i)^*)}
\]
and since the last inequality will hold for any $\sigma_i^*(c'_i, c_i) > 0$, since $c_i \geq c_i^*$ if the contract was awarded, so that no type will ever produce less total effort than $T_i(c_i^*)$. If $\sigma_i^*(c'_i, c_i) = 0$, of course, the firm is siphoning at all dates, and this is unprofitable due to the no arbitrage condition. This implies that (19) holds.

Verifying the other three conditions relies on the string of inequalities

$$
\frac{\Lambda_i(c'_i)}{p_i(c'_i)} = \frac{\int_{c_i^*}^{c_i} p_i(x) \int_{0}^{T_i(x)} (1 - G(z)) dz dx}{p_i(c_i)} \\
\leq \frac{\int_{c_i^*}^{c_i} p_i(x) dx \int_{0}^{T_i(c_i)} (1 - G(z)) dz}{p_i(c_i)} \\
\leq \frac{\int_{c_i^*}^{c_i} dx p_i(c_i) \int_{0}^{T_i(c_i)} (1 - G(z)) dz}{p_i(c_i)} \\
= (c_i^* - c_i) \int_{0}^{T_i(c_i)} (1 - G(z)) dz
$$

Using this to bound the informational rent, and recalling that

$$
\xi_i(c_i) = \frac{\lambda c_i^*}{h(T_i(c_i))} + \frac{(1 - \lambda)(c_i^* - c_i)}{h(T_i(c_i^*))} + \frac{\lambda c_i^* T_i(c_i)}{1 - G(T_i(c_i))} + \int_{0}^{T_i(c_i)} (1 - G(z)) dz (c_i^* - (1 - \lambda)c_i)
$$

it is clear that the third and fourth terms of $\xi_i(c_i, c_{-i})$ imply that (16) and (18) are satisfied. The first term ensures that (17) is satisfied. Consequently, the proposed bond satisfies all the sufficient conditions.