On Auction Design for Crowd Sensing

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Abstract—The recent paradigm of mobile crowd sensing which collects sensed data from pervasive mobile devices enables a broad range of large scale sensing tasks. In this paper, we aim to study one critical challenge in this paradigm, namely, design of compensation or incentive mechanisms for sensors to expend sufficient resources to take high quality measurements and transmit them to the central fusion unit. Inspired by the widespread use of the generalized second-price auction in search engine advertising business, we analyze the potential and performance of a reverse generalized second-price auction being used for this problem. Our main result is that reverse generalized second-price auction has a special subset of Nash equilibria that can achieve a desired level of error covariance at the central fusion unit with less payment than other common mechanisms such as reverse Vickrey-Clark-Groves or reverse generalized first-price auction.

I. INTRODUCTION

Recent years have witnessed explosive growth of smart devices, such as smartphones, tablets, and wearable devices. Most such smart devices are equipped with a rich set of embedded sensors (e.g. GPS, gyroscope, accelerometer, digital compass and camera) that are increasingly being used to generate data for a variety of applications that require these devices to transmit the measurements to a central fusion unit. This general paradigm is often called mobile crowd sensing (MCS). Mobile crowd sensing can be an attractive alternative to traditional wireless sensor networks (WSN), which suffer from high installation and maintenance cost. If properly implemented, MCS may be of great use in applications such as environmental monitoring [1], traffic control [2], social networking [3] and so on.

One basic category of differentiation for MCS implementations is whether the participants are voluntary or compensated. At this point, although there exist many MCS demonstrations [4–6], most of them seem to recruit voluntary participants only. However, it is usually a costly procedure for participants to serve as sensors. Not only does it consume resources such as battery power, and incurs costs such as charges from wireless carriers for data transmission, it also poses a potential privacy threat. For example, sharing location tag for traffic estimation may expose the route that a user typically takes. Thus, we posit that participation rate of such voluntary providers of data may decline once the novelty of the experience has worn off or a privacy-exposing incident happens. Once voluntary participation declines, an incentive mechanism which properly compensates users for their actions and risks will be required.

There have been some attempts to create incentive mechanisms for mobile crowd sensing. Many of these works pose the problem in the framework of a reverse auction in which the central fusion unit (also called the MCS platform) asks the participants to submit bids for the amount of information they can supply and the compensation they are asking for. It then selects enough participants so that its information collection goals are met. Related studies include [7] which proposed a reverse auction with the objective of maximizing social welfare, and [8] which considered maximizing the utility of the central fusion unit (defined as the valuation minus the compensation paid to the users) while ensuring incentive-compatibility for the participants.

In this context, it should be noted that the central fusion unit in most crowd sensing problems is not interested in identifying the true valuation that the sensors place on the collected data or achieving a socially optimal solution. Rather, the problem of more direct interest to the central fusion unit is to minimize the total compensation it should provide to the sensors while ensuring a certain level of sensing or estimation accuracy. In this paper, we model the mobile crowd sensing task as a centralized static estimation problem and propose a mechanism design approach which incentivizes the sensors to share enough data to guarantee a certain error covariance. We formulate an optimization problem to minimize the total compensation that the central fusion unit provides to the participants while ensuring that the error covariance of its estimate satisfies a specified bound. The closest formulation to ours seems to be [9], which formulated the problem using a contract game. However, our solution uses the reverse form of the so-called generalized second-price (GSP) auction that was proposed in the context of search engine advertising [10, 11]. GSP has been a popular auction mechanism used in search engines such as Google and Yahoo! for auctioning advertisement spaces on their webpages. The reason for its popularity is that it guarantees higher revenue for the auctioneer than either the Vickrey-Clark-Groves (VCG) or generalized first-price (GFP) auctions [11]. Our development generalizes the result of [10] to reverse GSP (RGSP). Specifically, we show that in our context RGSP has a special subset of Nash Equilibria that can achieve the desired level of error covariance with less compensation than the reverse VCG (RVCG) and reverse GFP (RGFP). Since the simplicity of RGSP makes it straightforward to implement and participate, it has the potential for widespread use in MCS applications.
The outline of this paper is as follows. In Section II, we formulate the problem of minimizing the compensation paid to sensors to obtain sufficient measurements to guarantee that the error covariance of the minimum mean squared estimate of a random variable is bounded above by a specified value as a reverse auction. We present our main results in Section III and some further discussions in Section IV. In Section V, a numerical simulation is presented to compare the payments under RGSP, RVCG, and RGFP. The work is concluded in section VI, where some directions for future work are also presented.

II. PROBLEM STATEMENT

We consider a centralized static estimation problem for a central fusion unit where sensors are selfish and need to be incentivized to participate in the crowd sensing task. Consider a random variable \( X \sim \mathcal{N}(\mu, \sigma_x^2) \) that takes value \( X = x \) in an experiment. The central fusion unit wishes to obtain an estimate \( \hat{x}_g \) (the subscript \( g \) stands for ‘global’) for the random variable by collecting data from some sensors through a reverse auction. There are \( K \) sensors available in total. Each sensor \( n \) can generate multiple measurements of the type \( y_n = H_n x + v_n \), where \( y_n \) is a measurement and the noise \( v_n \sim \mathcal{N}(0, \sigma_n^2) \) is uncorrelated with the variable \( X \). For every measurement taken by sensor \( n \) \((n = 1, 2, 3, \ldots, K)\), it incurs a cost \( c_n \). The cost \( c_n \) incurred by each sensor is private information for the sensor. However, we assume that the error covariance \( \sigma_n^2 \) is known publicly. We consider the estimation to be done in the minimum mean square error (MMSE) sense and consider the case where the constant \( \sigma_x^2 \) is large. In this context, using \( \eta_n \) such measurements, sensor \( n \) can generate an MMSE estimate of \( x \) with error covariance \( \sigma_n^2/\eta_n \).

The communication topology is showed in Fig. 1. The central fusion unit wishes to obtain enough data from the sensors to be able to estimate \( x \) with an error covariance \( \Sigma_{\hat{x}_g} \) that is upper bounded by a specified value \( \Sigma_0 \). To do so, it conducts a reverse auction to purchase data from \( N \) sensors \((N < K)\). Many types of reverse auctions, e.g., reverse generalized first-price auction, reverse VCG, and reverse generalized second-price auction can be used for this purpose. All these reverse auctions require each sensor \( n \) to bid the price per measurement \( b_n \) at which it is willing to supply its data. Note that we assume that each sensor is unconstrained in the number of measurements it can supply and that the bidding price per measurement is constant. Denote the number of measurements purchased from the \( n \)-th sensor by \( \eta_n \) and the compensation per measurement made by the central fusion unit (as specified by the auction mechanism) to the \( n \)-th sensor by \( p_n \). The communication topology is showed in Fig. 1.

The utility for the \( n \)-th sensor is then defined by

\[
U_n = (p_n - c_n)\eta_n, \tag{1}
\]

while the total payment made by the central fusion unit is given by

\[
P = \sum_{n=1}^{N} p_n\eta_n. \tag{2}
\]

To ensure individual rationality in the sense that each sensor obtains a non-negative utility, we include the following constraint.

**Constraint 1.** The central fusion unit informs the sensors about the reverse auction result before they take measurements and incur costs. If the sensors realize their utilities are negative, they are allowed to quit.

Define the vectors \( \eta = [\eta_1, \eta_2, \ldots, \eta_N], \eta = [b_1, b_2, \ldots, b_N], \) and \( p = [p_1, p_2, \ldots, p_N] \). Clearly, each sensor \( n \) wishes to optimize its utility \( U_n \) over the choice of \( b_n \) given the auction rules and the vector \( \eta \). In general, the bidding strategies of sensors would reach a Nash equilibrium [12] with respect to each other for any reverse auction. In this paper, the problem we are interested in is to identify which among these auctions (RGSP, RVCG, and RGSP) leads to the minimum payment from the central fusion unit for the same global error covariance. Our main result is to show analytically that the payment in a special subset of Nash equilibria under RGSP is less than or equal to the payment in the Nash equilibrium under RVCG, and numerically that the payment under RGFP is highest. Note that the vector \( \eta \) is specified by the central fusion unit to satisfy the global error covariance so that

\[
\Sigma_{\hat{x}_g} \leq \Sigma_0, \text{ where} \tag{3}
\]

\[
\Sigma_{\hat{x}_g} = \left( \sum_{n=1}^{N} \frac{\eta_n}{\sigma_n^2} \right)^{-1} \tag{4}
\]

is the global error covariance for the estimate calculated at the central fusion unit [13] and \( \Sigma_0 \) is a pre-specified bound. This is, in general, not a unique specification of \( \eta \) (including the length \( N \)). The central fusion unit should choose \( \eta \) as a function of the bid vector \( b \) to minimize the payment \( P \) while satisfying the global error covariance constraint. In this paper, we assume that \( \eta \) is given and do not concentrate on finding the optimal \( \eta \) beyond making the following natural assumption:

**Assumption 1.** For any auction, if for any two sensors \( i \) and \( j \), \( b_i \leq b_j \), then \( \eta_i \geq \eta_j \).

We present one way of identifying a feasible \( \eta \) in section IV. In the sequel, without loss of generality, we index the sensors according to the ascending order of their bids. Thus, \( b_1 \leq b_2 \leq \ldots \leq b_K \) and \( \eta_1 \geq \eta_2 \geq \ldots \geq \eta_N \).
III. MAIN RESULTS

A. RGSP

In RGSP, the payment to the $n$-th sensor is equal to the bid of the $(n+1)$th sensor, i.e., $p_n = b_{n+1}$. The fact that $b_1 \leq b_2 \leq \ldots \leq b_K$ yields $p_1 \leq p_2 \leq \ldots \leq p_N$. In a Nash equilibrium (NE), each sensor obtains higher utility at its current position than any other positions, which is described by the following conditions:

\[
\begin{align*}
(p_n - c_n)\eta_n &\geq (p_t - c_n)\eta_t, \forall t > n \\
(p_n - c_n)\eta_n &\geq (p_{t-1} - c_n)\eta_t, \forall t < n.
\end{align*}
\]  

(5) and (6)

Notice that there are a set of bids satisfying the above conditions and they form a set of Nash equilibria. Within this set, we are interested in a subset of Nash equilibria that are called symmetric Nash equilibria (SNE) in [10]. In this paper, we use NEs and SNEs to denote all the Nash equilibria and all the symmetric Nash equilibria separately. A symmetric Nash equilibrium is a bidding strategy in which the following condition

\[
(p_n - c_n)\eta_n \geq (p_t - c_n)\eta_t, \forall t \text{ and } n
\]  

holds. The condition (7) can be written equivalently as

\[
c_n(\eta_n - \eta_t) \leq p_n\eta_n - p_t\eta_t, \forall t \text{ and } n.
\]  

(8)

Lemma 1. SNEs ⊂ NEs, in the sense that if a bidding strategy leads to (7) being satisfied, then (5) and (6) hold.

Proof. For all $t > n$, condition (7) is the same as condition (5). For all $t < n$, if condition (7) $(p_n - c_n)\eta_n \geq (p_t - c_n)\eta_t$ holds, using the fact that $p_t \geq p_{t-1}$ in RGSP yields $(p_n - c_n)\eta_n \geq (p_{t-1} - c_n)\eta_t$, which implies that (5) holds.

We now provide bounds on the total payment by the central fusion unit in SNEs. We first bound the bids of sensors when each sensor bids according to the strategies that form SNEs.

Lemma 2. If the sensors bid according to the strategies that form SNEs, then the following bounds hold:

\[
b_n^L\eta_{n-1} = \sum_{t \geq n} c_{t-1}(\eta_{t-1} - \eta_t)
\]  

(9)

\[
b_n^U\eta_{n-1} = \sum_{t \geq n} c_t(\eta_{t-1} - \eta_t)
\]  

(10)

where $b_n^L$ denotes the lower bound on the bid by the $n$-th sensor and $b_n^U$ denotes the upper bound on the bid by the $n$-th sensor in SNEs.

Proof. Consider the $(n - 1)$th sensor, using (7) and $t = n$ yields

\[
(p_{n-1} - c_{n-1})\eta_{n-1} \geq (p_n - c_{n-1})\eta_n.
\]  

(11)

Similarly consider the $n$-th sensor and $t = n - 1$, we can obtain

\[
(p_n - c_n)\eta_n \geq (p_{n-1} - c_n)\eta_{n-1}.
\]  

(12)

Combining (11) and (12) we have,

\[
c_{n-1}(\eta_{n-1} - \eta_n) + p_n\eta_n \leq p_{n-1}\eta_{n-1}
\]  

\[
\leq c_n(\eta_n - \eta_{n-1}) + p_n\eta_n.
\]  

(13)

Recall that in RGSP, we have $p_n = b_{n+1}$. Thus, (13) can be rewritten as

\[
c_{n-1}(\eta_{n-1} - \eta_n) + b_{n+1}\eta_n \leq b_n\eta_{n-1}
\]  

\[
\leq c_n(\eta_n - \eta_{n-1}) + b_{n+1}\eta_n.
\]  

(14)

(14) provides a recursion that lower and upper bounds the bids by each sensor $n$. We can define

\[
b_n^L\eta_{n-1} = c_{n-1}(\eta_{n-1} - \eta_n) + b_{n+1}\eta_n
\]  

(15)

\[
b_n^U\eta_{n-1} = c_n(\eta_n - \eta_{n-1}) + b_{n+1}\eta_n,
\]  

(16)

where $b_n^L$ denotes the lower bound on the bid by the $n$-th sensor and $b_n^U$ denotes the upper bound on the bid by the $n$-th sensor in SNEs. Solving the recursion, we have,

\[
b_n^L\eta_{n-1} = \sum_{t \geq n} c_{t-1}(\eta_{t-1} - \eta_t)
\]  

(17)

\[
b_n^U\eta_{n-1} = \sum_{t \geq n} c_t(\eta_{t-1} - \eta_t).
\]  

(18)

Notice that condition (7) is stricter than (11) and (12). Since the lower and upper bounds on bids are derived from (11) and (12), they are respectively larger and smaller than the minimum and maximum values of bids in SNEs. However, the fact that (11) and (12) hold for all SNEs ensures the derived bounds are valid.

Using Lemma 4, we can obtain the lower and upper bounds on the payments $P$ in SNEs as follows.

Theorem 1. For the problem formulated in Section II, the lower and upper bounds on the payments in all the symmetric Nash equilibria of RGSP are given by

\[
P_{\text{RGSP}}^L = \sum_{n=2}^{N+1} \sum_{t \geq n} c_{t-1}(\eta_{t-1} - \eta_t)
\]  

(19)

\[
P_{\text{RGSP}}^U = \sum_{n=2}^{N+1} \sum_{t \geq n} c_t(\eta_{t-1} - \eta_t).
\]  

(20)

Proof. Recall that the total payment by the central fusion unit is calculated as $P = \sum_{n=1}^{N} p_n\eta_n = \sum_{n=2}^{N+1} b_n\eta_{n-1}$. Summing (9) and (10) over $n = 2, \ldots, N + 1$ yields the lower bound and upper bound on the total payments in SNEs.
B. RVCG

As stated in [10], the payment to the \((n - 1)\)th sensor in RVCG is given by

\[
\text{Payment to the } (n - 1)\text{th sensor} = \sum_{t \geq n} b_t(\eta_{t-1} - \eta_t).
\]  

(21)

It is well known that in the dominated strategy Nash equilibrium of RVCG mechanism, each sensor bids its true cost. Thus, the payment to the \((n - 1)\)th sensor is given by

\[
\text{Payment to the } (n - 1)\text{th sensor} = \sum_{t \geq n} c_t(\eta_{t-1} - \eta_t).
\]  

(22)

We derive the total payment in the NE of RVCG as follows.

**Theorem 2.** For the problem formulated in Section II, the payment in the Nash equilibrium of RVCG is given by

\[
P_{\text{RVCG}} = \sum_{n=2}^{N+1} \sum_{t \geq n} c_t(\eta_{t-1} - \eta_t).
\]  

(23)

**Proof.** Recall that the total payment by the central fusion unit is calculated as \(P = \sum_{n=1}^{N} p_n \eta_n\). Summing (22) over \(n = 2, ..., N + 1\) yields the total payment in the NE of RVCG. \(\square\)

C. Comparison of Total Payments

We can now compare the total payments under RVCG and RGSP.

**Theorem 3.** The total payment made by the central fusion unit for any vector \(\eta\) when the sensors bid in NE strategy of RVCG and SNE strategies of RGSP satisfy the relation: \(P_{\text{RGSP}} \leq P_{\text{RVCG}}\).

**Proof.** Comparing (20) and (23), the payment made by the central fusion unit when sensors bid according to NE strategy of RVCG is exactly the same as the upper bound on the payment when sensors bid according to SNE strategies of RGSP, which is shown in Fig. 2. \(\square\)

IV. FURTHER DISCUSSIONS

A. Payment in NEs versus payment in SNEs under RGSP

Since SNEs are a subset of NEs, it’s intuitive to speculate that the lower bound and upper bound on the payments in SNEs are respectively, larger and smaller than, those in NEs. It’s easy to construct examples where the upper bound on the payments in SNEs is smaller than those in NEs. However, it turns out that the lower bound on the payments in SNEs and the lower bound on the payments in NEs are the same, as stated in the following theorem.

**Theorem 4.** The lower bound on the payments in SNEs is equal to the lower bound on the payments in NEs for RGSP.

**Proof.** We prove this theorem in the appendix A for a better flow of the paper. \(\square\)

At this point, we can summarize the comparisons among the payments in NEs of RGSP, SNEs of RGSP and the NE of RVCG in Fig. 3.
Denote the lowest bid of the \(n\)-th sensor that would not induce the \((n - 1)\)-th sensor to move down by \(b'_n\). Then,
\[
(b'_n - c_{n-1})\eta_{n-1} = (b_{n+1} - c_{n-1})\eta_n.
\]
(26)

Thus,
\[
b'_n\eta_{n-1} = c_{n-1}(\eta_{n-1} - \eta_n) + b_{n+1} + \eta_n,
\]
(27)
which is the lower bound on bids in SNEs (15).

Therefore, if sensors consider about robustness, their bids will form SNEs rather than the other NEs.

C. How to Choose \([\eta_1, \eta_2, ..., \eta_N]\)?

We have assumed that the central fusion unit selects the \(N\) sensors with lower bids among the total \(K\) sensors and \([\eta_1, \eta_2, ..., \eta_N]\) is a given vector. We also mentioned that \([\eta_1, \eta_2, ..., \eta_N]\) is not unique to satisfy the global error covariance constraint. In this paper, we use the following algorithm to identify a feasible \([\eta_1, \eta_2, ..., \eta_N]\).

Step 1. The central fusion unit specifies an arbitrary sequence \(\eta_n = N - n + 1\) it wishes to purchase with the stipulation in Assumption 1.

Step 2. The sensors bid \([b_n]\) to maximize their utilities.

Step 3. The central fusion unit sorts the bids to identify the sequence of increasing bids.

Step 4. Since the error covariance per measurement is public information, the central fusion unit can now calculate \(\eta_n\) such that Assumption 1 and (4) hold.

Step 5. The sensors update their bids in response to these updated \(\eta_n\)’s.

Later on, \(\eta\) does not need to be updated after Step 5, since we can show that the order of bids by sensors after Step 2 and Step 5 remains the same in SNEs. Thus, the constraint on global error covariance (4) holds by construction.

**Theorem 5.** In all the symmetric Nash equilibria, if for any two sensors \(i\) and \(j\), \(c_i \leq c_j\), then \(\eta_i \geq \eta_j\).

**Proof.** Use (8) and switch \(t\) and \(n\),
\[
c_n(\eta_n - \eta_t) \leq p_n\eta_m - p_t\eta_t, \forall t \text{ and } n,
\]
(28)
\[
c_t(\eta_t - \eta_n) \leq p_t\eta_t - p_n\eta_n, \forall t \text{ and } n.
\]
(29)

Combining (28) and (29), we have \((c_t - c_n)(\eta_t - \eta_n) \leq 0\), which shows the sensors with lower costs will always bids lower prices and sell more measurements in SNEs of RGSP. Thus, the order of bids remains the same for different \([\eta_1, \eta_2, ..., \eta_N]\) vectors.

Notice this is also true for the NE of RVCG where \(b_n = c_n\) and for RGFP which will be presented in the next section. Therefore, the same \([\eta_1, \eta_2, ..., \eta_N]\) yields the same global error covariance under these mechanisms.

V. Numerical Simulation

Consider the case where there are \(N = 3\) sensors available and the central fusion unit selects two sensors. The costs are given by \(c_1 = 2, c_2 = 4, c_3 = 10\). Consider the vector \(\eta = [200, 100, 0]\). We now calculate the payment \(P\) under various reverse auction mechanisms.

A. Payments in all the symmetric Nash equilibria of RGSP

According to Theorem 1, the payments in SNEs of RGSP have the following lower and upper bound:
\[
P_{\text{RGSP}}^L = \sum_{n=2}^{3} \sum_{t \geq n} c_t(\eta_{t-1} - \eta_t)
\]
(30)
\[
= c_1(\eta_1 - \eta_2) + c_2(\eta_2 - \eta_3) + c_2(\eta_2 - \eta_3) = 1000.
\]
\[
P_{\text{RGSP}}^U = \sum_{n=2}^{3} \sum_{t \geq n} c_t(\eta_{t-1} - \eta_t)
\]
(31)
\[
= c_2(\eta_1 - \eta_2) + c_3(\eta_2 - \eta_3) + c_3(\eta_2 - \eta_3) = 2400.
\]

B. Payment in the Nash equilibrium of RVCG

According to Theorem 2, the payment in the NE of RVCG is given by
\[
P_{\text{RVCG}} = \sum_{n=2}^{3} \sum_{t \geq n} c_t(\eta_{t-1} - \eta_t)
\]
(32)
\[
= c_2(\eta_1 - \eta_2) + c_3(\eta_2 - \eta_3) + c_3(\eta_2 - \eta_3) = 2400.
\]

C. Payment under RGFP

Recall in RGFP, the payment to the \(n\)-th sensor is equal to its own bid, i.e., \(p_n = b_n\). We follow the argument about the GFP example in [11] in a reverse sense. As the GFP auction repeats, sensors will want to revise their bids to best respond to each other. Suppose the minimum step size of bids is 0.1. Sensor 2 finds out that sensor 3 never bids less than 10, so it will bid 9.9 to guarantee it obtains at least \(\eta_2\). Then sensor 1 will bid 9.8 to obtain \(\eta_1\). Sensor 2 will then bid 9.7 to obtain \(\eta_1\), so on and so forth. Finally, sensor 2 knows it can’t beat sensor 1 to obtain \(\eta_1\), then it will bid 9.9 and stays for \(\eta_2\). Sensor 1 will bid 9.8 to obtain \(\eta_1\). The total payment \(P_{\text{RGFP}} = 200\times 9.8 + 100\times 9.9 = 2950\).

VI. Conclusions and Future Work

In this paper, we examined the total payments to participants in crowd sensing under various reverse auctions. Specifically, we modeled the problem as a centralized static estimation problem in which sensors of various accuracies and costs can supply measurements to the central fusion unit. We computed the lower and upper bounds on the payments in SNEs of RGSP and the exact payment in the NE of RVCG when the two mechanisms yield the same global error covariance. We showed analytically that the total payment in the NE of RVCG is equal to the upper bound on the payments in SNEs of RGSP. Further, we showed numerically that the total payment under RGFP could be larger than the upper bound on the payments in SNEs of RGSP. The result is summarized in Fig. 4.

There are several directions for future work. An important question remains open is how to ensure that the bidding strategies converge to SNEs rather than the other NEs. Future work can also include extending our setup to a vector estimation problem.
Hence, for the \( \eta \) to the right side,

\[
(p_n^N - c_n)\eta_n \geq (p_n^N - c_n)\eta_t, \text{ for } t > n.
\]

Replace \( t \) with \( n+1 \) and move \( c_n\eta_n \) to the right side,

\[
p_n^N\eta_n \geq c_n(\eta_n - \eta_{n+1}) + p_{n+1}^N\eta_{n+1}.
\]

Let \( n = N \), and recall that \( \eta_{N+1} = 0 \),

\[
p_N^N\eta_N \geq c_N(\eta_N - 0) + 0 = c_N\eta_N.
\]

Thus,

\[
p_N^N \geq c_N.
\]

On the other hand, let \( p_n^L \) denote the lower bound of payment to the \( n \)-th sensor in SNEs, it should satisfy (15). Replace \( n \) with \( n+1 \),

\[
b_{n+1}^L\eta_n = c_n(\eta_n - \eta_{n+1}) + b_{n+2}\eta_{n+1}.
\]

Recall that \( p_n = b_{n+1} \),

\[
p_n^L\eta_n = c_n(\eta_n - \eta_{n+1}) + p_{n+1}^L\eta_{n+1}.
\]

Let \( n = N \), using the fact that \( \eta_{N+1} = 0 \) yields

\[
p_N^L\eta_N = c_N(\eta_N - 0) + 0 = c_N\eta_N.
\]

Thus,

\[
p_N^L = c_N.
\]

Combining (36) and (40) yields

\[
p_N^N \geq c_N = p_N^L,
\]

which means for the \( N \)-th sensor, lower bound of its payment in NEs is greater than or equal to the lower bound of its payment in SNEs.

Furthermore, combining (34) and (38) yields

\[
p_n^N\eta_n \geq p_n^L\eta_n + (p_n^N - p_{n+1}^L)\eta_{n+1}.
\]

For the \( (N-1) \)-th sensor,

\[
p_{N-1}^N\eta_{N-1} \geq p_{N-1}^L\eta_{N-1} + (p_N^N - p_{N-1}^L)\eta_N \\
\geq p_{N-1}^L\eta_{N-1}
\]

Hence,

\[
p_{N-1}^N \geq p_{N-1}^L
\]

Similarly, we can prove \( p_N^N \geq p_N^L \) to be true for all \( n = N-2, N-1, \ldots, 1 \). Therefore, the lower bound of total payment in SNEs is equal to the lower bound of total payment in NEs for RGSP.

REFERENCES


