Desynchronization of Thermally-Coupled First-Order Systems Using Economic Model Predictive Control

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Abstract—We consider the thermal control of a building, in which multiple rooms are thermally coupled. It is known that in such a case, the temperature control systems for each room may become synchronized, which may deteriorate the performance in terms of power efficiency or fairness. In this paper, economic model predictive control (EMPC) is applied to solve this problem by making each room track a particular temperature trajectory. These trajectories may represent a desired solution of the coupled systems, or may even be infeasible set-points in case the desired solution is infeasible. By solving this economic tracking problem, desynchronization of the units is achieved. Simulations show that by tracking predetermined out-of-phase symmetric trajectories, performance in terms of both power efficiency and fairness can be improved.

I. INTRODUCTION

We study the desynchronization of a large number of weakly coupled and self-sustained thermal oscillators that may represent, e.g. rooms in a building. The phenomenon of synchronization exists in many complex systems including biological, financial, mechanical and electrical systems (see, e.g. [13]). This phenomenon may be used to enhance the performance of systems (e.g. in formation control). However, in some situations, synchronization may mean that the systems converge to an undesired steady-state behavior. In this case, we must pay attention to the elimination of synchronization.

Thermal systems have recently been experimentally demonstrated to exhibit such a behavior (see, e.g. [10]). A simple first-order mathematical model was previously proposed to explain the experimental results (see, e.g. [2], [9], and [14]). In [2], numerical results show that for a number of thermally-coupled rooms arranged in the form of a ring, in which each room’s temperature is controlled by its individual thermostat, there exists a rich array of synchronized dynamics in frequency and phase. In practice, designers usually ignore this synchronization for the sake of simplicity. However, the synchronization in frequency or phase may seriously deteriorate the performance of the system. Temperature synchronization is directly related to the synchronization of power generators. If the temperatures of a large number of rooms reach synchronization through the so-called in-phase solution, cf. [2], those rooms’ thermostats may also be turned on or off during the same time period. This consensus type behavior results in a large amount of power fluctuation. From the power efficiency perspective, this type of synchronization should be avoided. On the other hand, if synchronization leads to the so-called constant solution, cf. [2], the power is consumed by some particular rooms, and not by others, leading to an unfair solution. In this work, we propose a control algorithm to prevent these undesired phenomena.

The control methodology used here is model predictive control (MPC) (see, e.g. [8]). MPC is a particular control law synthesis methodology that has become more popular as the computational capability at controller continues to increase. The main idea of MPC is to transform an infinite horizon optimal control problem into sequential finite horizon subproblems. For each subproblem, by considering the present value of the state to be the initial condition, starting from which an optimization problem is solved. Since the optimization problem is over a finite horizon, state and input constraints can be naturally included. After the subproblem is solved, only the first optimal control input is applied to the plant. Using this input to update the plant’s state, the entire procedure is repeated at the next time step. After more than two decades of development, there exist numerous variations of MPC and a mature body of Lyapunov based stability theories for the method.

Recently, economic model predictive control (EMPC) (see, e.g. [3], [11], and [12]) has drawn attention in the control community yielding improvement in performance as compared to the standard MPC. An economic MPC scheme deals with a general (economic) optimization objective, where the optimal solution of this economic cost function may not satisfy the steady-state constraints, cf. [12]. This economic cost function is in a general form and may not be a quadratic one. In the standard MPC design, the control design usually is decomposed into two steps. In the first step, an optimization problem with the cost function and steady-state constraints is formulated to find the optimal feasible steady-state solution. Then, in the second step, MPC is used to track this optimal feasible target. If the optimal steady-state target is not the optimal solution of the economic cost function, directly using MPC with the economic cost function may improve the performance of the system, cf. [12]. Economic MPC follows the latter idea. There is no necessity of the target that is tracked in an economic MPC to be a feasible steady-state solution. This feature is of particular interest in our problem since all the steady-state solutions in a thermally coupled system may be undesirable.

We also impose the constraint, that the solution be distributed to be scalable to a large building. In the paper, a distributed economic model predictive control (DEMPC) is thus also presented. Also notice that while for simplicity, we present an idea in the context of a circular building, the
method is also applicable to other configurations.

A similar idea of desynchronization is discussed in [7], where a hybrid model predictive control is applied to a supermarket refrigeration system to increase efficiency and reduce wear. Although the objectives of the work in [7] and here are both aimed to eliminate the synchronization of coupled systems based on MPC technology, the formulation of economic tracking in this paper for thermally-coupled oscillators is nevertheless new. Notice that hybrid MPC is not applicable to our problem since as noted before, the target to be tracked is not necessarily feasible.

The paper is organized as follows. In Section II, a simple mathematical model is presented to model the thermally-coupled first order systems. In Section III, the economic model predictive control is presented and in Section IV, a distributed implementation is discussed. The stability analysis is given in Section V and simulation examples are presented in Section VI. Finally, the conclusion can be found in Section VII.

II. THERMALLY-COUPLED FIRST ORDER MODEL

For simplicity, consider a building with N rooms arranged in the form of a ring. The non-dimensional temperature \( T^i(t) \) of the \( i \)-th room is thermostatically controlled by a heater, which provides a non-dimensional heat value \( Q^i(t) \) at time \( t \). The \( i \)-th heater is turned on, i.e. \( Q^i(t) = 1 \) when the temperature \( T^i(t) \) falls below the lower bound \( T_L \). As the temperature \( T^i(t) \) rises above the upper bound \( T_U \), the heat source is turned off, i.e. \( Q^i(t) = 0 \). Each room exchanges heat with adjacent ones and with the exterior. Assuming lumped capacitances, after normalization, the dimensionless first-order heat balance equation is given by

\[
\frac{dT^i(t)}{dt} = -(2r + 1)T^i(t) + r[T^{i+1}(t) + T^{i-1}(t)] + Q^i(t),
\]

where \( r \in \mathbb{R}_{\geq 0} \) is the non-dimensional thermal coupling parameter, which couples two adjacent rooms. As \( r \) increases, the coupling effect becomes higher. Room \( i \) is adjacent to room \( i + 1 \) and \( i - 1 \). The index \( i \) is to be read cyclically. The non-dimensional heat source is the control input for the systems (1) and is described by

\[
Q^i(t) = \begin{cases} 
1 & \text{if the heater is on at time } t \\
0 & \text{if the heater is off at time } t.
\end{cases}
\]

Since the heat source \( Q^i \) depends on the non-dimensional temperature \( T^i \), equation (1) defines a coupled nonlinear system of equations. However, between two switching events, the system is a set of coupled first-order ordinary differential equations. In [2], numerical simulations reveal a rich array of synchronization dynamics in frequency and phase for this system of equations based on the values of the parameters \( r, T_U, T_L \), etc. The work [9] analyzes the equations in a three-room setup. It reveals three basic types of solutions: (a) An in-phase periodic solution, in which all room temperatures oscillate in phase and frequency; (b) An out-of-phase solution, in which the temperatures are periodic with

the same frequencies, but the phases are equally separated from each other; and (c) A constant solution, in which room temperatures remain constant. Which of the periodic solution is realized depends on the value of thermal coupling parameter \( r \), temperature upper bound \( T_U \), lower bound \( T_L \), initial condition, and the symmetric structure of the rooms.

If the regulation of temperatures relies only on thermostats, because of the coupling effect of these self-sustained thermal oscillators, the synchronization of the states may not be avoidable. This leads to the synchronization of generators which may be an undesirable phenomenon. In particular, the in-phase solution can lead to synchronization of the power consumption and large fluctuations in power demand. On the other hand, a constant solution may not be desirable for fairness. In this solution, some of the rooms may always turn on the heaters and the other rooms turn off the heaters all the time, leading to a constant temperature for every room. We look to avoid such undesirable steady state solutions through the use of economic model predictive control.

In order to solve the problem numerically, the dynamics is discretized in time to get the following discrete-time model,

\[
x^i(k+1) = -[(2r + 1)T_s + 1]x^i(k) + rT_s[x^{i-1}(k) + x^{i+1}(k)] + T_su^i(k),
\]

where the temperature is the state variable and the heat value is the control input. \( T_s \) is the sampling time period and \( r \) is the same coupling parameter as in the continuous-time model.

III. ECONOMIC MODEL PREDICTIVE CONTROL

The control objective is to eliminate the undesired in-phase periodic and constant solution. In this paper, model predictive control (MPC) is used to achieve this goal. The advantage of using MPC is that the control problem can be formulated into a series of optimization subproblems and the constraints can be easily included. The main idea is to formulate the cost function to track a predetermined symmetric out-of-phase periodic target. If the desired trajectories can be tracked well, then synchronization can be eliminated. However, notice that outside of this symmetric simple model considered in Section II, the predetermined reference trajectories might not form a feasible steady-state solution. If we directly include this infeasible set of reference trajectories into the cost function, the optimal cost function value might not be finite and the state trajectories might not converge to the desired reference trajectories.

In process control, standard MPC decomposes the optimization problem into two steps. In the first step, the optimal steady-state solution is computed based on optimizing an economic cost function (e.g. it might be a linear cost function related to power consumption). Then in the second step, receding horizon controller is used to track this optimal steady-state solution. Since the optimal solutions of the economic cost function may not be feasible (that is, the optimal feasible steady-state solutions may not be the optimal solutions of the economic cost function), direct optimization of the economic cost function using receding horizon controller might instead
improve the system performance (see, e.g. [12]). In economic model predictive control (EMPC), such direct optimization of economic cost function is considered. For our problem, under the assumptions: (a) The structures of the building are not symmetric; or (b) The material of walls does not allow the symmetric out-of-phase solution to exist, then the use of MPC to track this out-of-phase target becomes an EMPC problem.

The economic model predictive control is derived from the following optimization problem

\[
\begin{align*}
\text{minimize} & \sum_{n=0}^{M-1} l(x_{n,k} - \hat{x}_s(n,k), u_{n,k} - \hat{u}_s(n,k)) \\
\text{subject to} & x_{n+1,k} = Ax_{n,k} + Bu_{n,k}, \\
& x_{n,k} \in X, n = 0, 1, ..., M, \\
& u_{n,k} \in U, n = 0, 1, ..., M - 1, \\
& x_{0,k} = x^i(k),
\end{align*}
\]

where the linear dynamics is discretized in time and can be calculated by (3). The temperature is the state variable and the heat value is the control input. \(x^i(k) = [x^i_1(k), x^i_2(k), ..., x^i_N(k)]^T \in \mathbb{R}^N\) is the initial state for the \(k\)-th subproblem. Given the initial condition \(x^i(k), x_{n,k} \in \mathbb{R}^N\) and \(u_{n,k} \in \{0, 1\}^N\) are the predicted state and control input, respectively, at time instant \(n\). Here, the predicted states and inputs are equivalent to the predicted temperatures and heat values, respectively in our heating problem. \(i\) is the economic cost function, e.g. a cost function of the form \(x^i_1(n,k)^T P_x^i (x^i_1(n,k) - \hat{x}_s(n,k))^T P_x^i (u_{n,k} - \hat{u}_s(n,k))^T P_u^i (u_{n,k} - \hat{u}_s(n,k))^T\) can be used, where \(P_x^i > 0\) and \(P_u^i > 0\) are state and control input weighting matrices, respectively. The set \(X^i\) and \(U^i\) are state and input constraints, respectively. Specifically, \(X^i = \{x^i_1, x^i_2, ..., x^i_N\}^T \in \mathbb{R}^N[T_L \leq x^i \leq T_U, i = 1, 2, ..., N]\), \(U^i = \{0, 1\}^N\). For the sake of simplicity, we assume that all rooms have the same temperature upper and lower bounds, and input constraint is equivalent to the control space. \(M\) is the prediction horizon. \(\hat{x}_s\) is the desired target. It might not be feasible and is changing with time. In our heating problem, \(\hat{x}_s\) is the desired temperature target to track. \(\hat{u}_s\) is the corresponding steady-state control input. Notice that the control inputs are integers, 0 or 1, therefore the above optimization problem turns then into a Mixed-Integer Program (see, e.g. [1]). If the 2 or \(\infty\)-norm are used, this minimization is a Mixed-Integer Quadratic Program (MIQP) or Mixed-Integer Linear Program (MILP), respectively. For the details about solving MIP, the reader is referred to [1].

IV. DISTRIBUTED CONTROLLER DESIGN

Consider the optimization problem (4). It is reasonable to use a quadratic function with a diagonal weighting matrix to decouple the cost function into problems that can be solved by the controller in each room. However, the dynamics constraints are still coupled among the rooms. One standard distributed controller design is to let those rooms to share the predicted states to their neighbors. Using this information of the adjacent rooms’ states, each individual controller is able to update the dynamics and synthesize the control law in a distributed way. We also propose this modification to obtain a distributed EMPC solution. Specifically, each room’s temperature controller solves the following optimization problem

\[
\begin{align*}
\text{minimize} & \sum_{n=0}^{M-1} l(x_{n,k}^i - \hat{x}_s^i(n,k), u_{n,k}^i - \hat{u}_s^i(n,k)) \\
\text{subject to} & x_{n+1,k}^i = -(2r + 1)T_s + 1 x_{n,k}^i + \\
& + rT_s [x_{n,k}^i + x_{n,k}^i + 1] + T_u u_{n,k}^i, \\
& x_{n,k}^i \in X^i, n = 0, 1, ..., M, \\
& u_{n,k}^i \in U^i, n = 0, 1, ..., M - 1, \\
& x_{0,k}^i = x^i(k),
\end{align*}
\]

where the temperature is the state variable and the heat value is the control input. \(x^i(k) \in \mathbb{R}\) is the initial state for the \(k\)-th subproblem. Given the initial condition \(x^i(k), x_{n,k}^i \in \mathbb{R}\) and \(u_{n,k}^i \in \{0, 1\}\) are the predicted state and control input, respectively, at time instant \(n\). Here, the predicted states and inputs are equivalent to the predicted temperatures and heat values, respectively in our heating problem. \(l\) is the economic cost function, e.g. a cost function of the form \(x^i_1(n,k)^T P_x^i (x^i_1(n,k) - \hat{x}_s^i(n,k))^T P_x^i (u_{n,k} - \hat{u}_s^i(n,k))^T P_u^i (u_{n,k} - \hat{u}_s^i(n,k))^T\) can be used, where \(P_x^i > 0\) and \(P_u^i > 0\) are state and control input weighting matrices, respectively. The set \(X^i\) and \(U^i\) are the \(i\)-th room’s state and input constraints, respectively. Specifically, \(X^i = \{x^i_1, x^i_2, ..., x^i_N\}^T \in \mathbb{R}^N[T_L \leq x^i \leq T_U]\), \(U^i = \{0, 1\}^N\). \(M\) is the prediction horizon. \(\hat{x}_s^i\) is the desired target. It might not be feasible and is changing with time. In our heating problem, \(\hat{x}_s^i\) is the \(i\)-th room’s desired temperature target to track. \(\hat{u}_s^i\) is the corresponding steady-state control input. For the \(i\)-th room, the predicted states and inputs of the \((i-1)\)-th and \((i+1)\)-th rooms are assumed to be constant variables. Before each room starts to solve the next optimization subproblem, those rooms communicate with their adjacent neighbors to share the predicted state and input information. In the distributed implementation, every controller \(i, i = 1, ..., N\) proceeds as follows:

(a) At time \(k\), given \(x^i(k)\), the predicted state trajectory \((x_0^i, ..., x_{M-1}^i)\) and predicted control input \((u_0^i, ..., u_{M-1}^i)\), \(i\)-th room’s controller solves problem in (5).

(b) The \(i\)-th room updates its state \(x^i(k+1) + 1\) using the dynamics (3), where \(u^i(k) = u_0^i\).

(c) The \(i\)-th room transmits the predicted state trajectory \((x_0^i, ..., x_{M}^i)\) and predicted control input \((u_0^i, ..., u_{M-1}^i)\) to the \((i+1)\)-th and \((i-1)\)-th room, and receives the predicted state trajectory \((x_0^i, ..., x_{M}^i)\) and predicted control input \((u_0^i, ..., u_{M-1}^i)\), \(j = i + 1, i - 1\), from its neighbors.

(d) Return to the step (a) and reiterate.

V. STABILITY ANALYSIS

The main difficulties of stability analysis come from the time varying nature and the infeasible property of the target. For the standard MPC, it is known that the optimal value function can be used as the Lyapunov function (see, e.g. [8]). The stability then can be inferred by the decreasing property of the optimal value function. However, in EMPC, the target may no longer be feasible, the optimal value function is not
decreasing and it cannot be used as the Lyapunov function anymore. Recently, in [3], an alternative Lyapunov function approach was identified for the EMPC setup.

The second difficulty of stability analysis comes from the fact that the target is time-varying. In [4] and [5], Lyapunov stability of EMPC for the so-called cyclic processes is discussed. For completeness, now we state the main results in [4], [5] and show how the proof can be modified for application to our problem.

Consider the following modified EMPC,

\[ \min \sum_{n=0}^{\infty} l(x_{n,k}^\ast, \dot{x}_{n,k}^\ast, u_{n,k}^\ast - u_s(n,k)) \]

subject to

\[ x_{n+1,k} = Ax_{n,k} + Bu_{n,k}, \]

\[ x_{n,k} \in X, n = 0, 1, \ldots, \infty, \]

\[ u_{n,k} \in U, n = 0, 1, \ldots, \infty, \]

\[ x_{0,k} = x(k), \]

(6)

where the interpretations of state, control input, cost function, and constraints are the same as those in (4).

Several assumptions are made as follows,

**Assumption 5.1:** There exists a class \( K_\infty \) function \( g(\cdot) \) such that for every feasible initial condition \( x(0) \), there exists a feasible control input vector \((u_{0,k}, u_{1,k}, \ldots)\) and

\[ \sum_{n=0}^{\infty} \sum_{k=0}^{K-1} \| \eta_k x_{n,k} + u_{n,k}^\ast \| \leq \sum_{n=0}^{\infty} \gamma(\| x_{n,k} - x_{n,k}^\ast \|), \]

(7)

where \( x_{n,k}^\ast \) and \( u_{n,k}^\ast \) are the optimal feasible steady-state target and control input, respectively. Let the constant \( c = M/K \). Where \( M \) is a positive constant and \( K \) is the period of the target.

This assumption is referred to as weak controllability at the cyclic steady-state. It simply means that starting from any feasible initial condition, the system can drive the state to the periodic target in \( M \) steps with bounded feasible control input. For our heating problem, this condition assumes that each room’s temperature trajectory converges to the optimal feasible steady-state trajectory in finite time. Notice that for our heating problem, \( x_{n,k}^\ast \) is the optimal feasible steady-state temperature and \( u_{n,k}^\ast \) is the corresponding steady-state heat value.

**Assumption 5.2:** The optimization problem in (6) is well-posed and has unique local solution.

Define \( z_{n,k} = x_{n,k} - x_{n,k}^\ast \) and \( v_{n,k} = u_{n,k} - u_{n,k}^\ast \). The modified cost function is defined as follows,

\[ \tilde{l}(z_{n,k}, v_{n,k}) := l(z_{n,k} + x_{n,k}^\ast, v_{n,k} + u_{n,k}^\ast) - l(x_{n,k}^\ast, u_{n,k}^\ast). \]

(8)

**Assumption 5.3:** There exists a \( K_\infty \) function \( \alpha(\cdot) \) such that the stage cost satisfies

\[ \tilde{l}(z_{n,k}, v_{n,k}) \geq \alpha(\| z_{n,k} \|). \]

Notice that this assumption is satisfied if the quadratic cost function is used.

The main theorem is as follows:

**Theorem 5.1:** If Assumptions 5.1-5.3 are satisfied, then the transformed system \((z, v)\) is asymptotically stable at \((0,0)\). Consequently, the original system is asymptotically stable at the limit cycle \((x^\ast(k), u^\ast(k))\).

Proof: Follow the proof of [5, Theorem 1].

Notice that the Lipschitz conditions of the dynamics and stage cost function in the proof of [5, Theorem 1] are satisfied since the dynamics in (6) is linear, and if the quadratic cost function is used. Assumption 5.1 and 5.2 are verified based on the simulations in Section VI. For our heating problem, this theorem ensures that these rooms’ temperature trajectories converge to the optimal feasible steady-state temperature trajectories if the assumptions are satisfied.

In [5], the above nominal stability result is extended to the robust case using input-to-state stability (ISS) technique, cf. [6]. Notice that in the distributed implementation, each room’s controller receives the predicted state and control input trajectories from its adjacent rooms to solve the optimization problem. Since the \( i \)-th room treats its adjacent rooms states and inputs as constants, each room’s temperature should be modeled as a dynamical system with disturbances. Specifically, in the distributed implementation, the dynamics of temperatures is modeled as

\[ x(k+1) = Ax(k) + Bu(k) + g(x(k), u(k), w(k)), \]

(10)

where \( w(k) \in W \) is the disturbance term, \( W \) is the disturbance constraint set, and the function \( g \) represents the uncertain part of the system. We assume that the disturbance is bounded, i.e. \( \max_{0 \leq \tau \leq k} \| w(\tau) \| \leq w \), where \( w \) is a positive constant.

For the robust stability analysis, ISS technique is used. The definition of ISS stability is given by (see, e.g. [6]).

**Definition 5.1:** The system (10) is said to be ISS stable if there exists a class \( KL \) function \( \beta \) and a class \( K \) function \( \delta \) such that for any feasible initial state \( x(0) \) and any bounded disturbance \( w(k) \), the trajectory \( x(k) \) satisfies

\[ \| x(k) \| \leq \beta(x(k), k) + \delta(w), \forall k \geq 0. \]

(11)

**Assumption 4.5:** The uncertain part \( g \) is bounded such that

\[ \| g_n(x, u, 0) \| \leq \eta \bar{\alpha}(\| x \|), n = 1, \ldots, K - 1 \]

(12)

where \( \eta > 0 \) is a positive constant, and \( \bar{\alpha} \) is a class \( K_\infty \) function. \( g_n(x, u, w) := g(x + x_{n,k}^\ast, u + u_{n,k}^\ast, w). \)

The main result for the robust stability is as follows:

**Theorem 5.2:** If Assumptions 5.1-5.4 are satisfied, the system (10) is ISS stable at the optimal cyclic steady-state solution.

Proof: Follow the proof of [5, Theorem 4].

Notice that in our problem, the uncertain part \( g \) is a linear function of the disturbance \( w \), and the Lipschitz conditions of the uncertain part \( g \) in the proof of [5, Theorem 4] is satisfied.

VI. SIMULATION EXAMPLES

In this section, several numerical examples are presented to illustrate the effectiveness of the EMPC controller. For all the simulations, the temperature upper and lower limits are set to be 0.8 and 0.2, respectively. The sampling time is 0.1 (normalized time unit). The prediction horizon in the MPC
algorithm is 2. First, consider a system with 3 thermally-coupled rooms. Room $i$ is adjacent to room $i+1$ and $i-1$. The index $i$ is to be read cyclically. The configuration of this thermally coupled systems is shown in Fig. 1. In Fig. 2, no control is applied, and the coupling parameter is set to be 0.2. The in-phase synchronization periodic solution is seen to be the steady-state solution of the uncontrolled system (without MPC). In Fig. 3, no control is applied, and the coupling parameter is set to be 0.2. An out-of-phase periodic solution is seen to be the steady-state solution of the uncontrolled system. Comparing Fig. 2 and Fig. 3, we can see that if temperatures reach synchronization (Fig. 2), the variation of the total control inputs is larger than that of the desynchronization case (Fig. 3). Notice that if the regulation of temperatures merely relies on thermostats, due to the coupling effect, this synchronization may not be avoidable. From the power efficiency perspective, the amount of variation should be reduced. In Fig. 4, the coupling parameter is set to be 0.5, the constant solution is seen to be the steady-state solution of the uncontrolled system. Since for this type of solution, some of the rooms always turn their heaters on or off, the fairness cannot be held for this type of solution. In Fig. 4, one of the rooms always turn off its heater, and the other two rooms on the contrary always turn on their heaters. One of the rooms’s temperature keeps at 0.4, and the other two rooms keep at the upper bound 0.8 in the steady-state.

The corresponding out-of-phase tracking result is shown in Fig. 5. In the tracking simulations, the rooms share the predicted trajectories with their neighbors to compute the optimal solutions of the optimization subproblems. One can tell from the results that by tracking an out-of-phase periodic target, the variation of the total power is reduced. In Fig. 6, the coupling parameter changes from 0.2 to 0.5, and the system is still tracking the same out-of-phase periodic target. Since the predetermined out-of-phase target is generated according to a smaller (0.2) coupling parameter, the tracking solution performs not so well as the previous one. However, the variation of the total power can still be further reduced and the temperatures are controlled within the constraints. In Fig. 7, the coupling parameter raised to 1. According to [9], the only possible solutions are constant and in-phase solutions for this value of coupling parameter. Out-of-phase solution exists only for the coupling parameter less than one ($\nu < 1$). In this case, since the out-of-phase target is not feasible, the tracking performance becomes even worse. However, the variation of the total power is reduced and the temperatures are within the constraints. The objective of desynchronization is achieved. Moreover, in the out-of-phase tracking, since rooms here are not kept in the same constant temperatures, they will not be forced to turn heaters on or off accordingly all the time. The fairness can thus be achieved.

As noted before that while for simplicity, we present an idea in the context of a circular building, the method is also applicable to other configurations. In general, under the assumptions: (a) The structures of the building lack symmetry; or (b) The coupling effect does not allow the symmetric out-of-phase solution to exist, then the use of MPC to track this out-of-phase target becomes an EMPC problem. The EMPC method proposed in this paper can be applied to those cases as well.

Fig. 1. The thermally-coupled systems.

Fig. 2. The in-phase periodic solution.

Fig. 3. The out-of-phase periodic solution.

VII. CONCLUSIONS

In this work, desynchronization of thermally-coupled first-order systems is discussed. Depending on the coupling
parameters, the synchronization of temperatures can lead to undesirable effects on power efficiency and fairness. We have proposed using EMPC to track potentially infeasible trajectories of temperatures to prevent such effects. By tracking predetermined out-of-phase reference trajectories, the system’s performance can be improved in terms of power efficiency and fairness. Future work could include experimental verification of the algorithm, further theoretical study of the distributed EMPC, and robust EMPC.

REFERENCES