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On Distributed Charging Control of Electric Vehicles with Power Network Capacity Constraints

Wann-Jiun Ma, Vijay Gupta, Ufuk Topcu

***Abstract*— We consider the problem of optimally scheduling the charging of electric vehicles to obtain a favorable total load profile. A large collection of our main contribution is to account for the capacity constraints induced by the power grid distribution lines and other components such as transformers in scheduling. We propose three distributed algorithms to minimize the total load variance while satisfying the capacity in the power network. These algorithms provide a trade-off between the convergence speed and the amount of communication required. We study the effectiveness of the algorithms through analysis and simulation.**

I. Introduction

Demand response (DR) is a promising mechanism for improving the efficiency of power grids. It empowers the utility companies and electric vehicles (EVs) to decide collectively, potentially in a distributed manner, the best way to schedule energy usage; see e.g., [8] and references therein. In this paper, we focus on DR for scheduling the charging of EVs at residences.

As the penetration of EVs increases, they will impose a significant burden on the grid. In particular, peak load amplification, creation of new peaks [10], voltage deviations [12], and other effects may occur in various studies. To overcome these problems, many EV charging control methods, both centralized [3] , [16] and distributed [2], [14] have been proposed.

Of particular interest to our work are the algorithms proposed in [4] and [5] which provide analytic guarantees on convergence of the total load profile to a desired one (for instance, a valley-filling profile) even in the presence of heterogeneous EV arrival times and load requirements. Through the exchange of a price-like signal, the EVs and the utility company negotiate suitable charging profiles to optimize the total load profile. There are multiple scheduling profiles of individual EVs that are possible, which lead to the same total load profile. While some of these profiles respect the capacity constraints in terms of the power carried by the various components in the network that supplies power to these EVs, others do not. The algorithms in [4] and [5] do not consider capacity constraints and thus cannot distinguish among such solutions.

In this work, we explicitly include capacity constraints on the power carried by various lines in the network used to supply power to the EVs. We extend the algorithm in [4] to minimize the total load variance while satisfying the capacity constraints in the power network. As in [4], our algorithm is a decentralized negotiation mechanism between the EVs and the utility company. However, we need two different price-like control signals to govern the iterations. One signal is also used as in [4, 5] to mediate the iterations with the utility to optimize the total load profile, while the other one is introduced in iterations with the neighboring components in the power network to satisfy the capacity constraints. The optimal decentralized charging (ODC) algorithm proposed in [4] is adopted to coordinate the charging profiles of EVs to obtain a valley-filling total load profile with the first one of these price-like signals. Here, we adopt an alternating direction method of multipliers (ADMM) algorithm (see e.g., [1]) to coordinate the charging profiles of EVs to satisfy the capacity constraints with the second price-like signal. Specifically, the ODC algorithm, which optimizes the total load profile, requires communication between individual EVs and the utility only. On the other hand, the ADMM algorithm, which enforces the capacity constraints, requires communication between the individual EVs and the components whose load is affected by the charging profile of the EV. Using these two building blocks, we propose three negotiation mechanisms that trade-off between the convergence speed and the amount of message passing. All the three negotiation mechanisms are distributed.

The contributions of this work are twofold: 1) We formulate the problem of optimizing the total load profile with EVs in the presence of the power network capacity constraints. 2) We propose and analyze distributed algorithms to solve this optimization problem. By comparing the optimal load profile with and without capacity constraints, we provide examples where a valley-filling load profile may no longer be feasible or compatible with the capacity constraints.

While the literature on DR with the penetration of EVs is too vast to summarize here, we now point out some recent results closest to ours. We borrow the ODC algorithm proposed in [4] and extended in [5]. However, these references do not consider network capacity constraints. While ADMM and other dual decomposition based methods have been used in the context of demand response in work such as [7], [11], these work focused on obtaining charging profiles that maximize individual EV payoff and minimize the operating costs. Thus, these charging profiles may not lead to total load profiles that are valley filling. We consider explicit optimization of the total load variance while including capacity constraints.

Wann-Jiun Ma and Vijay Gupta are with the Department of Electrical Engineering, University of Notre Dame, Notre Dame, IN 46556, USA. Ufuk Topcu is with the Department of Electrical and Systems Engineering, University of Pennsylvania, Philadelphia, PA 19104, USA. {wma1, vgupta2}@nd.edu, utopcu@seas.upenn.edu. Research supported in part for the first and second authors by NSF grant #1239224 and in part for the third author by NSF grant #1312390.

Notation : M^T refers to the transpose of a matrix M. I_p denotes a $p \times p$ identity matrix and 0_p denotes a $p \times p$ zero matrix. The Euclidean norm of a vector is denoted via $||x|| := \sqrt{x^T x}$. The derivative of a scalar function $U : \mathbb{R} \to \mathbb{R}$ is denoted as $U^{'}(\cdot)$. We use x to represent a vector of elements $(x_1, ..., x_N)$, for some $N \in \mathbb{N}$. For any $x_i \in \mathbb{R}^n,\ i \in \mathbb{N}$, we use the notation $(x_i)_j$ to denote the j-th element of x_i, where $j = 1, ..., n$. If A and B are sets, then the set difference of B and A is denoted as $B \setminus A := \{x \in B | x \notin A\}$. If A is a set, its cardinality is denoted as $|A|$.

II. Problem Formulation

Consider a scenario in which a utility company intends to negotiate with N EV customers to set up day-ahead charging profiles for the next day. Let the set $\mathcal{N} := \{1, ..., N\}$ denote the set of EVs. The scheduling is over T time slots, and each time slot may represent some period of time (e.g., 15 minutes). We assume that the day-ahead base load $D(t),\ t = 1, ..., T$ is given. For notational convenience, let $\mathcal{T} := \{1, ..., T\}$ denote the set of time slots. Denote by $\hat{x}_n(t)$ the charging rate of the n-th EV during the t-th time slot. The charging profile of the n-th EV is then given by the vector $\hat{x}_n := [\hat{x}_n(1), \hat{x}_n(2), \cdots \hat{x}_n(T)]$. Let $\hat{x}_n^{up}(t)$ denote the maximum charging rate for the n-th EV during the t-th time slot and $\hat{R}_n$ denote the desired total charge for the n-th EV, which is calculated as the sum of the charging rates for the EV. The total load is given by the sum of the base load and the charging rates for all the EVs.

The power network is characterized as a graph. To be specific, consider a connected and undirected graph $\mathcal{G} = (\mathcal{M}_s, \mathcal{E})$, where $\mathcal{M}_s$ denotes the set of vertices of the graph and $\mathcal{E} \subset \mathcal{M}_s \times \mathcal{M}_s$ denotes the set of edges in the graph. The edge (i, j) connects vertices i and j. For our setup, the set of vertices $\mathcal{M}_s := \{1, ..., M_s\}$ denotes the set of buses in the network. Among these buses, there are M_l buses where EVs connect to, and we denote them as *load buses*. Let the set $\mathcal{M}_l := \{1, ..., M_l\}$ be the set of load buses. Similarly, we assume that there are M_g buses where power injection occurs and we denote them as *injection buses*. Let the set $\mathcal{M}_g := 1, ..., M_g$ be the set of injection buses. We define the set $\mathcal{M}_p := \mathcal{M}_s \setminus \{\mathcal{M}_l \cup \mathcal{M}_g\}$ and let $M_p := |\mathcal{M}_p|$. The edge (i, j) denotes a power line connecting vertices i and j. Each edge in the graph has an associated capacity constraint. We focus on active power capacity constraints in this study. The capacity constraints place upper bounds on the active power flowing through the different edges. Since these edges supply power to a subset of EVs, this may constrain the possible charging profile for these EVs.

It is clear that these constraints can be described by linear equalities that constrain the charging rates of the EVs. For instance, if a bus with capacity $\tilde{c}$ supplies EVs 1, 2, and 3 with charging profiles $\hat{x}_1,\ \hat{x}_2,\ \hat{x}_3$ respectively, the inequality $\hat{x}_1 + \hat{x}_2 + \hat{x}_3 \leq \tilde{c}$ must be satisfied at every time slot. By stacking all such inequalities, we obtain a constraint of the form $\hat{L}\hat{x} \leq \hat{c}$, where $\hat{x} = [\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_N]$ is the vector of charging profiles, $\hat{L}$ is a matrix where elements are either 0 or 1 depending on whether the corresponding EV is part of a particular capacity constraint and $\hat{c}$ is the vector of capacity constraints. Notice that our formulation allows for time-varying capacity constraints for the same line. This may be of use if these capacity constraints were imposed for components such as transformers that have a specified "cool-down" period.

The objective of the system-level optimization problem is to minimize the total load variance while satisfying the constraints induced by the power network and fully charging the EVs up to the desired level. Thus we require a charging profile $\hat{x} = (\hat{x}_1, ..., \hat{x}_N)$ that solves

$$\begin{array}{ll} \text{minimize} & \sum_{t \in \mathcal{T}} (D(t) + \sum_{n \in \mathcal{N}} \hat{x}_n(t))^2 \\ \text{subject to} & 0 \leq \hat{x}_n(t) \leq \hat{x}_n^{up}(t),\ t \in \mathcal{T}, n \in \mathcal{N}, \\ & \sum_{t \in \mathcal{T}} \hat{x}_n(t) = \hat{R}_n,\ n \in \mathcal{N}, \\ & \hat{L}\hat{x} \leq \hat{c}. \end{array} \tag{1}$$

The utility company can, in principle, solve the problem (1) in a centralized manner. In this case, however, the company needs information about the charging profile constraints including timings, and requirements for all the EVs. To avoid communication of such private information, it is of interest to design a distributed algorithm to solve (1). Similar to the algorithms proposed in [4] and [5], we allow the utility company to send price-like control signals to the EVs to guide them to select suitable charging rates. However, the algorithm in [4] and [5] needs to be augmented to ensure that the resulting charging profiles satisfy the capacity constraints. The mechanism for doing so must be such that every EV should exchange information with only its local neighbors (EVs or components).

In this paper, we present such a distributed algorithm. We first consider a homogeneous scenario in which every EV connected to the same load bus has identical feasible sets of charging profiles. More specifically, we assume that for any $i \in \mathcal{M}_l, x_n^{up} = x_m^{up},\ \forall\, n,\ m \in \mathcal{N}_i$ and $\hat{R}_n = \hat{R}_m,\ \forall\, n,\ m \in \mathcal{N}_i$, where the set $\mathcal{N}_i$ is the set of EVs connected to the i-th load bus. In Section III, we develop algorithms for this homogeneous case. For more general setups, the reader is referred to an extended version of the paper [13].

Consider the aggregate charging profiles at each load bus. Let $x_i := \sum_{n \in \mathcal{N}_i} x_n$, $\bar{x}_i := |\mathcal{N}_i| x_n^{up}$, $R_i := |\mathcal{N}_i| \hat{R}_n,\ n \in \mathcal{N}_i$ be the aggregate charging profile, aggregate charging upper limit, and aggregate charging sum, respectively for the EVs connected to the i-th load bus. In addition, define the average charging profile of x_i to be $\frac{1}{|\mathcal{N}_i|} x_i$.

The problem in (1) can be modified for this homogeneous case to

$$\begin{array}{ll} \text{minimize} & \sum_{t \in \mathcal{T}} (D(t) + \sum_{i \in \mathcal{M}_l} x_i(t))^2 \\ \text{subject to} & 0 \leq x_i(t) \leq \bar{x}_i(t),\ t \in \mathcal{T},\ i \in \mathcal{M}_l, \\ & \sum_{t \in \mathcal{T}} x_i(t) = R_i,\ i \in \mathcal{M}_l, \\ & Lx \leq c. \end{array} \tag{2}$$

We now present distributed algorithms to solve the problem (2) with the aggregate charging profiles as our optimization variables.

III. Distributed Algorithms

From the structure of (2), it is evident that coupling among the EV charging rates occurs due to both the cost function and the capacity constraints. We propose three distributed algorithms to solve the problem. The first two algorithms use ODC to optimize the total load and ADMM to enforce the capacity constraints. The two algorithms differ in the time-scale at which these two sub-routines are executed. Specifically, in the first algorithm, ODC is run in the outer loop and ADMM in the inner loop. The second algorithm reverses the order of the execution of these sub-routines. The third algorithm uses only one sub-routine (ADMM) in both the loops. These different algorithms result in different message passing structures and exhibit different tradeoffs between the feasibility and optimality of the solutions of the problem (2), as explored more thoroughly in Section III-D.

Here we introduce some notation that will be used in the description of the algorithms. Our problem involves coupled constraints and hence the framework of general form consensus optimization [1] applies. The set of branch power flows for all branches in the network form the global variable in the optimization problem that is denoted by z. For every bus $i \in \mathcal{M}_s$, the corresponding branch power flows form the local variable y_i. The local variable is thus a linear function of the global variable z. We denote this relation by the equation $y_i = \tilde{z}_i$, where $\tilde{z}_i$ is the specific part of z that contains this local variable. Finally, the charging profiles of the EVs connected at the i-th bus form the private variable x_i for this bus. While the private variables are not shared with other buses, they are related to the local variables. If we define $r_i = (x_i, y_i)$, then these relations are modeled as equality constraints $H_i r_i = 0$ for suitably defined matrices H_i.

Using these variables, we define three sets. The set $\mathcal{L}_i = \{y_i \in \mathbb{R}^{n_i} | \bar{L}_i y_i \leq \bar{c}_i\}$ denotes the set of branch power flows at the buses that satisfy the capacity constraints in the network. The set $\mathcal{F}_i = \{x_i | 0 \leq x_i(t) \leq \bar{x}_i(t), \forall t \in \mathcal{T}, \sum_{t\in\mathcal{T}} x_i(t) = R_i, \forall i \in \mathcal{M}_l\}$ denotes the set of feasible charging profiles without considering capacity constraints. Finally, the set $\mathcal{H}_i = \{r_i | H_i r_i = 0\}$ simply encodes the set of values for charging profiles that can result from specified values of the power flows at the buses.

A. Algorithm 1

As noted above, Algorithm 1 uses ODC to optimize the total load profile and ADMM to enforce the capacity constraints. ODC is executed in the outer loop, while ADMM is executed in the inner loop. The iteration indices j and k are used for the outer loop and inner loop, respectively. The iterations continue for J and K times respectively. In this paper, we perform a fixed number of iterations and better stopping criteria are part of future work. We assume that the derivative of the objective function $U^{'}$ is Lipschitz with Lipschitz constant $\beta > 0$.

For Algorithm 1, the utility company sends the price-like signal p^j to all load buses and every load bus $i \in \mathcal{M}_l$ sends back the charging profile $x_i^{j,K}$ to the utility company in the j-th outer iteration. On the other hand, every bus $i \in \mathcal{M}_s$ exchanges local variable $y_i^{j,k+1}$ and the scaled dual variable $u_i^{j,k}$ with its neighbors in the j-th outer and k-th inner iteration, where the neighbors of the i-th bus, $i \in \mathcal{M}_s$ is defined as the set $\{j \in \mathcal{M}_s | (i,j) \neq 0, j \in \mathcal{M}_s\}$.

Given: Scheduling horizon $\mathcal{T}$. The utility company knows the base load profile D and the number M_l of load buses. Each load bus $i \in \mathcal{M}_l$ knows the set $\mathcal{F}_i$ of its feasible charging profiles, and each injection bus $i \in \mathcal{M}_g$ knows the set $\mathcal{F}_i$ of its feasible power injection profiles. Each bus $i \in \mathcal{M}_s$ knows the set $\mathcal{L}_i$ of feasible capacity constraints and the set $\mathcal{H}_i$. Select γ such that $0 < \gamma < 1/(M_l \beta)$.

Output: Charging profiles $x_i, \ i \in \mathcal{M}_l$.

Set $j \leftarrow 0, \ x_i^j(t) \leftarrow 0, \ i \in \mathcal{M}_l, \ t \in \mathcal{T}$.

Repeat while $j < J$

The utility company computes and broadcasts

$$p^j(t) \leftarrow \gamma U^{'}(D(t) + \sum_{i \in \mathcal{M}_l} x_i^j(t)), \ t \in \mathcal{T}.$$

Set $k \leftarrow 0, \ p_i^{j,k}(t) \leftarrow p_i^j(t), \ x_i^{j,k}(t) \leftarrow x_i^j(t), \ r_i^{j,k}(t) \leftarrow 0, \ u_i^{j,k}(t) \leftarrow 0, \ \tilde{z}_i^{j,k}(t) \leftarrow 0, \ i \in \mathcal{M}_s, \ t \in \mathcal{T}$.

Repeat while $k < K$

Perform r-Update:

$$\begin{aligned} &\text{For } i \in \mathcal{M}_l, \\ &r_i^{j,k+1} \leftarrow \underset{x_i, y_i}{\arg\min} \ (p^{j,k})^T x_i + 1/2 ||x_i - x_i^{j,k}||^2 \\ &\qquad\qquad + (\rho/2) ||y_i - \tilde{z}_i^{j,k} + u_i^{j,k}||^2 \\ &\qquad \text{subject to} \ \ x_i \in \mathcal{F}_i, \ y_i \in \mathcal{L}_i, \\ &\qquad\qquad r_i := (x_i, y_i) \in \mathcal{H}_i. \end{aligned} \tag{3}$$

$$\begin{aligned} &\text{For } i \in \mathcal{M}_g, \\ &r_i^{j,k+1} \leftarrow \underset{x_i, y_i}{\arg\min} \ (\rho/2) ||y_i - \tilde{z}_i^{j,k} + u_i^{j,k}||^2 \\ &\qquad \text{subject to} \ \ x_i \in \mathcal{F}_i, \ y_i \in \mathcal{L}_i, \ r_i \in \mathcal{H}_i. \end{aligned}$$

$$\begin{aligned} &\text{For } i \in \mathcal{M}_p, \\ &y_i^{j,k+1} \leftarrow \underset{y_i}{\arg\min} \ (\rho/2) ||y_i - \tilde{z}_i^{j,k} + u_i^{j,k}||^2 \\ &\qquad \text{subject to} \ \ y_i \in \mathcal{L}_i, \ r_i \in \mathcal{H}_i. \end{aligned}$$

Perform $\tilde{z}$-Update:

$$\tilde{z}^{j,k+1} \leftarrow \underset{\tilde{z}_i}{\arg\min} \ (\rho/2) \sum_{i=1}^{M_s} ||\tilde{z}_i - u_i^{j,k} - y_i^{j,k+1}||^2.$$

Perform u-Update:

$$\text{For } i \in \mathcal{M}_s, \ u_i^{j,k+1} \leftarrow u_i^{j,k} + y_i^{j,k+1} - \tilde{z}_i^{j,k+1}.$$

Set $k \leftarrow k + 1$.

Set $j \leftarrow j + 1$.

Each load bus $i \in \mathcal{M}_l$ sends charging profile $x_i^{j-1,K}$ to the utility company.

Set $x_i^j \leftarrow x_i^{j-1,K}$.

Return $x_i^J, \ i \in \mathcal{M}_l$.

Proposition 1: Suppose that x_i^j is an exact minimizer of the inner loop optimization, then the charging profile x^j converges to the set $\mathcal{O}$ of optimal charging profiles, as $j \rightarrow \infty$, where j is the outer iteration index.

The proof can be found in [13].

B. Algorithm 2

In Algorithm 2, we switch the order of the building blocks in the loops of Algorithm 1. More specifically, the outer loop now executes ADMM, whereas the inner loop is based on the ODC algorithm.

For Algorithm 2, every bus $i \in \mathcal{M}_s$ exchanges local variable y_i^{j+1} and the scaled dual variable u_i^j with its neighbors in the j-th outer iteration. On the other hand, the utility company sends the price-like signal $p^{j,k}$ to all buses $i \in \mathcal{M}_s$ in the j-th outer and k-th inner iteration. Every bus $i \in \mathcal{M}_s$ sends back the signals $r_i^{j,k+1}$ to the utility company in the k-th inner iteration and sends the control signals u^{j+1} and $\tilde{z}^{j+1}$ to the utility in the j-th outer iteration.

Given: Scheduling horizon $\mathcal{T}$. The utility company knows the base load profile D and the number M_l and types of load buses. Each load bus $i \in \mathcal{M}_l$ knows the set $\mathcal{F}_i$ of its feasible charging profiles, and each injection bus $i \in \mathcal{M}_g$ knows the set $\mathcal{F}_i$ of its feasible power injection profiles. Each bus $i \in \mathcal{M}_s$ knows the feasible set of capacity constraints $\mathcal{L}_i$, and the set $\mathcal{H}_i$. Select γ such that $0 < \gamma < 1/(M_s\beta)$.

Output: Charging profiles $x_i,\ i \in \mathcal{M}_l$.

Set $j \leftarrow 0,\ u_i^j(t) \leftarrow 0,\ \tilde{z}_i^j(t) \leftarrow 0,\ i \in \mathcal{M}_s,\ t \in \mathcal{T}$.

Repeat while $j < J$

Set $k \leftarrow 0,\ \tilde{z}_i^{j,k} \leftarrow \tilde{z}_i^j,\ u_i^{j,k} \leftarrow u_i^j,\ x_i^{j,k}(t) \leftarrow 0,\ y_i^{j,k}(t) \leftarrow 0,\ i \in \mathcal{M}_s,\ t \in \mathcal{T}$.

Repeat while $k < K$

The utility company computes and broadcasts

For $i \in \mathcal{M}_l$,

$$p_i^{j,k}(t) \leftarrow \gamma \begin{bmatrix} U'(D(t) + \sum_{i \in \mathcal{M}_l} x_i^{j,k}(t)) \\ \rho(y_i^{j,k} - \tilde{z}_i^{j,k} + u_i^{j,k}) \end{bmatrix}.$$

For $i \in \mathcal{M} \setminus \mathcal{M}_l$,

$$p_i^{j,k}(t) \leftarrow \gamma\rho(y_i^{j,k} - \tilde{z}_i^{j,k} + u_i^{j,k}).$$

Perform r-Update:

For bus $i \in \mathcal{M}_s \setminus \mathcal{M}_p$ computes

$$\begin{aligned} &r_i^{j,k+1} \\ &\leftarrow \arg\min_{r_i}\ (p_i^{j,k})^T r_i + 1/2\|r_i - r_i^{j,k}\|^2 \\ &\quad \text{subject to}\ \ x_i \in \mathcal{F}_i,\ y_i \in \mathcal{L}_i,\ r_i \in \mathcal{H}_i. \end{aligned} \tag{4}$$

For bus $i \in \mathcal{M}_p$ computes

$$\begin{aligned} &r_i^{j,k+1} \\ &\leftarrow \arg\min_{r_i}\ (p_i^{j,k})^T r_i + 1/2\|r_i - r_i^{j,k}\|^2 \\ &\quad \text{subject to}\ \ y_i \in \mathcal{L}_i,\ r_i \in \mathcal{H}_i. \end{aligned} \tag{5}$$

Set $k \leftarrow k+1$.

Each bus $i \in \mathcal{M}_s$ sends $r_i^{j,k}$ to the utility company.

Set $y_i^{j+1} \leftarrow y_i^{j,K}$.

$\tilde{z}$-Update:

$$\tilde{z}^{j+1} \leftarrow \arg\min_{\tilde{z}_i} \sum_{i=1}^{M_s} (\rho/2)\|y_i^{j+1} - \tilde{z}_i + u_i^j\|^2.$$

u-Update:

$$\text{For } i \in \mathcal{M}_s,\ u_i^{j+1} \leftarrow u_i^j + y_i^{j+1} - \tilde{z}_i^{j+1}.$$

Set $j \leftarrow j+1$.

Buses send u^j and $\tilde{z}^j$ to the utility company.

Return $x_i^J,\ i \in \mathcal{M}_l$.

Proposition 2: Suppose that r_i^j is an exact minimizer of the inner loop optimization, then the charging profile x^j converges to the set $\mathcal{O}$ of optimal charging profiles, as $j \to \infty$, where j is the outer loop iteration index.

The proof is presented in [13].

C. Algorithm 3

Algorithm 3 consists of only one ADMM loop. Denote by k the iteration index. The stopping criterion in the inner loop is $k = K$, where K is a specified number of iterations.

For Algorithm 3, every bus $i \in \mathcal{M}_s$ exchanges local variable y_i^{k+1} and the scaled dual variable u_i^k with its neighbors in the k-th iteration. Every load bus $i \in \mathcal{M}_l$ sends the charging profile x_i^{k+1} and the scaled dual variable u_i^k to the utility company and the utility company sends back the control signal $\hat{z}_i^{k+1}$ to all load bus $i \in \mathcal{M}_l$ in the k-th iteration.

We reformulate the problem in (2) to

$$\begin{array}{ll} \text{minimize} & \sum_{t=1}^{T}(\sum_{i=1}^{M_l} \hat{z}_i(t) + D(t))^2 \\ \text{subject to} & r_i := (x_i, y_i),\ \bar{z}_i := (\hat{z}_i, \tilde{z}_i), \\ & x_i \in \mathcal{F}_i,\ y_i \in \mathcal{L}_i,\ r_i \in \mathcal{H}_i,\ i \in \mathcal{M}_s, \\ & x_i = \hat{z}_i,\ y_i = \tilde{z}_i,\ i \in \mathcal{M}_s. \end{array} \tag{6}$$

ADMM is used to solve the problem (6).

Given: Scheduling horizon $\mathcal{T}$. The utility company needs to know the base load profile D, the number M_s of buses, and the types of buses. Each load bus knows its feasible set of charging profiles, and each injection bus knows its feasible set of power injection profiles. Each bus $i \in \mathcal{M}_s$ knows the feasible set of capacity constraints $\mathcal{L}_i$, and the set $\mathcal{H}_i$.

Output: Charging profiles $x_i,\ i \in \mathcal{M}_l$.

Set $k \leftarrow 0,\ r_i^k(t) \leftarrow 0,\ u_i^k(t) \leftarrow 0,\ \bar{z}_i^k(t) \leftarrow 0,$
$i \in \mathcal{M}_s,\ t \in \mathcal{T}$.

Repeat while $k < K$

r-Update:

For $i \in \mathcal{M}_s$,

$$\begin{aligned} r_i^{k+1} &\leftarrow \arg\min_{r_i}\ (\rho/2)\|r_i - \bar{z}_i^k + u_i^k\|^2, \\ &\quad \text{subject to}\ x_i \in \mathcal{F}_i,\ y_i \in \mathcal{L}_i,\ r_i \in \mathcal{H}_i, \end{aligned}$$

$\bar{z}$-Update:

$$\begin{aligned} \bar{z}^{k+1} \leftarrow \arg\min_{\bar{z}} &\ \sum_{t=1}^{T}(\sum_{i=1}^{M_l} \hat{z}_i(t) + D(t))^2 \\ &+ (\rho/2)\sum_{i=1}^{M_s} \|\bar{z}_i - u_i^k - r_i^{k+1}\|^2, \end{aligned}$$

u-Update:

$$u_i^{k+1} \leftarrow u_i^k + r_i^{k+1} - \bar{z}_i^{k+1}.$$

Set $k \leftarrow k+1$.

Return $x_i^K,\ i \in \mathcal{M}_l$.

Proposition 3: Charging profile x^k converges to the set $\mathcal{O}$ of optimal charging profiles, where k is the iteration index.

The proof can be proven using standard techniques [1].

D. Comparisons of the Algorithms

In Algorithm 1, the utility company and load buses negotiate suitable charging profiles in every iteration of the outer loop, whereas the load buses exchange charging profiles with their neighbors in every iteration of the inner loop. In Algorithm 2, load buses exchange charging profiles with their neighbors in every iteration of the outer loop, whereas the utility company and load buses negotiate suitable charging profiles in every iteration of the inner loop and in the transition of the inner loop to the outer loop. In Algorithm 3, the utility company and load buses negotiate charging profiles, and the load buses exchange charging profiles with their neighbors in the same iteration.

Notice that for Algorithm 1, the primal solutions (charging profiles) x^j remain feasible for every iteration of the outer loop $j = 1, ..., J$. At the end of every iteration of the outer loop, the utility company and the customers will obtain feasible charging profiles but they may be sub-optimal. On the other hand, for Algorithm 3, since there is only one ADMM loop, the primal solutions (charging profiles) may not be feasible during every iteration. In other words, we cannot obtain feasible charging profiles in Algorithm 3 before the primal solutions reach the set of the optimal charging profiles. In this sense, Algorithm 1 is advantageous in terms of the trade-off between the feasibility and the optimality of the solutions. For Algorithm 2, since the outer loop consists of ADMM, it can only provide feasible charging profiles after the solutions reach the set of optimal charging profiles. However, it is well known that the r-update in equations (4), (5) of ADMM does not needs to be carried out exactly [1]. This r-update procedure in Algorithm 2 is implemented through the ODC algorithm. In other words, it is possible to allow ODC in Algorithm 2 to be solved only approximately to reduce the amount of communication required.

IV. Numerical Examples

We consider the 47-bus power distribution network from the service area of Southern California as described in [6]. The peak capacity constraints of the load buses and the Photovoltaic (PV) generators are chosen according to [6, Table I]. We focus on active power capacity constraints in this study, therefore, the unit adopted for the capacity constraint is MW instead of MVA[1]. We provide two numerical examples. In the first example, we compare the optimal load profile with and without capacity constraints. The desired pattern is a modification of the valley-filling load profile subject to the capacity constraints of the load buses. In the second example, we compare the performance of the proposed three distributed algorithms. In addition to verifying that the charging solutions generated by the algorithms converge to the optimal one in terms of the cost function value for this 47-bus distribution network, we also study the tradeoffs between the communication requirements and the performances of the algorithms. Notice that in the simulation, we consider only the homogeneous case discussed in Section III. CVX/CVXGEN [9], [15] solvers are used to solve the optimization problems. For both examples, the starting and the end time for charging are at 20:00 pm and 9:00 am (the next day), respectively, and the based loads are chosen artificially to have valleys during off peak hours.

In the first example, we impose the aggregate charging sum $R_i = 1$ MW and the aggregate upper limit $\bar{x}_i = 0.2$ MW for each load bus. For the details of choosing charging requirements $\hat{R}_n$ and $\hat{x}_n^{up}$, the reader is referred to [4]. The time slot used for charging is one half hour The charging profiles as the solution of the optimization in (2) with and without the capacity constraints are shown in Fig. 1. We observe that the solution is not feasible subject to the peak capacity constraints of PV generators listed in [6, Table I]. Therefore, we enlarge the capacity constraints of the PV generators to three times the original values. One can see in Fig. 1 that the valley-filling profiles are modified to satisfy the capacity constraints of the load buses. Furthermore, these modifications result in higher variability of the total load. By comparing the optimal load profile with and without capacity constraints, it shows that a valley-filling load profile may no longer be feasible or compatible with the capacity constraints.

In the second example, we impose the aggregate charging sum $R_i = 0.8$ MW and the aggregate upper limit $\bar{x}_i = 0.066$ MW for each load bus. The time slot used for charging is 15 minutes. The initial conditions of the three algorithms are initialized to be zero. For Algorithm 1, the parameter $\gamma = 0.0106$ satisfies the condition $0 < \gamma < 1/(\mathcal{M}_l \beta)$ in Algorithm 1, where $\mathcal{M}_l = 26$ and $\beta = 2$. For Algorithm 2, the parameter $\gamma = 0.0104$ satisfies the condition $0 < \gamma < 1/(\mathcal{M}_s \beta)$ in Algorithm 2, where $\mathcal{M}_l = 47$ and $\beta = 2$. We set the step size $\rho = 1$ appeared in the ADMM updates for all three algorithms. To obtain a better performance by optimizing the values of the system parameters γ and ρ is beyond the scope of this study and left for future work. Figure 2 provides the convergence results of the cost function values generated by the proposed algorithms.

To study the tradeoffs between the communication requirements and the performances of the proposed algorithms, we run the outer loop iteration once, i.e. $J = 1$ and inner loop 100 times, i.e. $K = 100$. The total numbers of the iterations in the outer and inner loops have different meanings for the three algorithms. For Algorithm 1, the utility company only needs to communicate once with all the load buses, which on the other hand communicate with their neighbors 100 times. For Algorithm 2, all the buses communicate with their neighbors only once, but the utility company communicate with all the buses 100 times. For algorithm 3, since there is only one loop, the utility company needs to communicate 100 times with all the buses, which likewise need to communicate with their neighbors 100 times. Figure 3 and 4 show the total load profiles generated by the proposed algorithms and the convergence results of the primal residuals of the algorithms, respectively. The primal residual is measured by the norm $\sum_{i,j \in \mathcal{M}_s} ||(\tilde{z}_i)_{(i,j)} - (\tilde{z}_j)_{(i,j)}||$, where $(\tilde{z}_i)_{(i,j)}$ denotes the part of $\tilde{z}_i$ on the edge (i, j), $i, \ j \in \mathcal{M}_s$. As we defined

[1]The unit for the peak capacity constraint is MVA in [6, Table I].

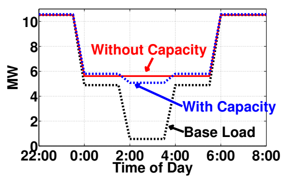

Fig. 1. The total load profiles with and without capacity constraints.

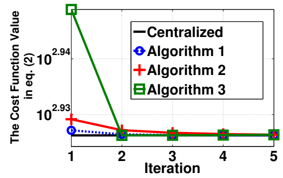

Fig. 2. The convergence results of the cost function values. The horizontal axis represents the total number of times that the load buses communicate with the utility company, and the vertical axis represents the cost function value in (2).

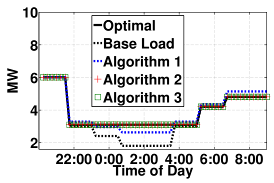

Fig. 3. The total load profiles generated by the algorithms proposed in this work, where $J = 1$, $K = 100$.

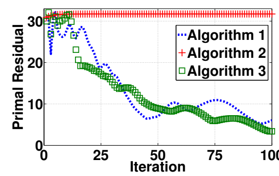

Fig. 4. The convergence results of the primal residuals, where $J = 1$, $K = 100$. The horizontal axis represents the total number of iterations in the inner loop, and the vertical axis represents the value of the primal residual.

in Section III, $\tilde{z}_i$ is the part of the global variable z that contains the local variable which is the corresponding branch power flows at each bus. This primal residual measures the difference between the power flows through the same power line originated from the adjacent buses. If the primal residual is zero, it indicates that the adjacent buses set up the same power flowing through the corresponding power line. Essentially, in order to implement the power distribution solutions generated from the algorithms, the primal residual must be zero or approach zero for every power line since there is only one power flow through the corresponding power line. We can see from Fig. 3 that Algorithm 2 and 3 perform better in the sense of the convergence of the total load profiles to the optimal one (valley-filling profile). Algorithm 1 trades this optimality to reduce the burden of the communication among the utility company and the load buses. On the other hand, we can see from Fig. 4 that Algorithm 1 and 3 have better performance in terms of the convergence of the primal residuals than Algorithm 2 which trades the primal feasibility to reduce the burden of the communication among the buses. These different message passing structures required to satisfy the desired performances in terms of the shape of the total load profile and the primal feasibility may distinguish the distributed algorithms proposed in this work.

V. Conclusion

We have incorporated the capacity constraints in the power network aiming at optimizing the total load profile with the penetration of EVs. In comparing the optimal load profile either with or without capacity constraints, the simulation result shows that a valley-filling profile may no longer be feasible with the incorporation of the capacity constraints. We have further proposed three distributed charging algorithms composed of different message passing structures with the trade-off between the feasibility and the optimality of the charging profiles.

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