Distributed Energy Management for Networked Microgrids Using Online ADMM with Regret

Wann-Jiun Ma, Member, IEEE, Jianhui Wang, Senior Member, IEEE, Vijay Gupta, Member, IEEE, Chen Chen, Member, IEEE

Abstract—We propose a distributed algorithm for online energy management in networked microgrids with a high penetration of distributed energy resources (DERs). A high penetration of DERs introduces high uncertainty of power generation to the microgrids. In general, the state-of-the-art forecasting for non-dispatchable DERs such as solar energy is not sufficiently accurate, which results in inaccurate energy scheduling. To address the high uncertainty issue in the networked microgrids, we propose an online energy management based on the online alternating direction method of multipliers algorithm with the past power generation information from the DERs. The online algorithm provides less conservative schedule than the robust optimization based approach. The effectiveness of the proposed algorithm is verified by various numerical examples.

Index Terms—Networked microgrids, distributed energy resources, alternating direction method of multipliers, online learning, regret minimization.

NOMENCLATURE

Numbers, and sets

\( M_s \) Set of utility microgrids
\( M \) Number of utility microgrids
\( E \) Set of power lines in the networked microgrids
\( T \) Set of time slots considered
\( T' \) Number of time slots considered

Decision variables

\( p_i^g \) Power generation of micro turbine at microgrid \( i \)
\( p_i^r \) Net power of power transaction and power generation of PV panels at microgrid \( i \)
\( q_i^g \) Reactive power generation at microgrid \( i \)
\( q_i^l \) Active/reactive power load at microgrid \( i \)
\( p_{ij} / q_{ij} \) Active/reactive power flows from microgrid \( i \) to \( j \)
\( V_i \) Voltage at microgrid \( i \)

Uncertain quantities

\( w_i \) Power generation of PV panels at microgrid \( i \)

Parameters

\( r_{ij} \) Line resistance between microgrid \( i \) and \( j \)
\( x_{ij} \) Line reactance between microgrid \( i \) and \( j \)
\( p_i^g \) Minimum active power of micro turbine at microgrid \( i \)
\( p_i^r \) Maximum active power of micro turbine at microgrid \( i \)
\( p_i^l \) Lower bound for \( p_i^r \)
\( p_i^u \) Upper bound for \( p_i^r \)
\( p_i^l \) Minimum active power load at microgrid \( i \)
\( p_i^u \) Maximum active power load at microgrid \( i \)
\( q_i^l \) Minimum reactive power load at microgrid \( i \)
\( q_i^u \) Maximum reactive power load at microgrid \( i \)
\( q_i^u \) Minimum reactive power of micro turbine at microgrid \( i \)

\( q_i^u \) Maximum reactive power of micro turbine at microgrid \( i \)

\( \epsilon \) Maximum allowed voltage deviation
\( a_i \) The second order coefficient in the micro turbine power generation cost function for microgrid \( i \)
\( b_i \) The first order coefficient in the micro turbine power generation cost function for microgrid \( i \)
\( c_i \) The constant coefficient in the micro turbine power generation cost function for microgrid \( i \)
\( \alpha_i \) Price for selling electricity from private microgrid \( i \) to distribution network operator
\( \beta_i \) Price for buying electricity from distribution network operator to private microgrid \( i \)

I. INTRODUCTION

Recent studies have shown that the traditionally isolated microgrids can be interconnected to form a networked system to further improve reliability [1], [2]. Networked microgrids are clusters of distributed energy resources (DERs), electricity loads, and storages in a distribution system where a distribution network operator coordinates the energy scheduling of the microgrids [1]. The ownership of the networked microgrids ranges from utility owned microgrids, non-utility owned microgrids to public purpose microgrids which could be owned by a township but operated by a utility or a third party [3], [4]. The flexible physical structure and ownership of microgrids...
and their capabilities of being islanded when the utility supply fails greatly stimulate their wide deployment in practice.

Implementation of DERs by distributing electric vehicles, solar photovoltaic (PV) panels, and wind energy sources in a microgrid requires different considerations [5], [6]. Contrary to traditionally centralized energy sources, DERs are widely distributed over the underlying power network. The high uncertainty of DERs complicates the design of the energy management in microgrids. For example, the effects of passing clouds on the power generations from the PV panels result in inaccurate power generation predictions for DERs. In a traditional power grid, the unpredictable effects caused by the DERs on the total power generation may be small as the renewable energy sources may only contribute to a small portion of the total power generation. However, as the popularity of the microgrid is continuously growing with a high penetration of DERs, the uncertain and unpredictable features of DERs need to be taken into account to design more accurate energy management systems specifically for microgrids.

Robust optimization is widely used in the design of energy management systems with high penetration of renewable energy sources (see, e.g., [5], [7]). However, to implement the algorithm, it requires prior knowledge about the uncertainties by restricting them within a given set. In order to derive algorithms to take into account all possible outcomes of the uncertainties, robust optimization based energy management algorithms may provide over-conservative results [8].

In this paper, we design an online energy management algorithm for the distribution system operator to manage the energy scheduling of networked microgrids using regret minimization and online alternating direction method of multipliers (ADMM). The proposed online energy management algorithm does not require forecast data to proceed, which can prevent inaccurate forecasting problem.

Regret minimization has recently gained popularity in the online optimization and machine learning community (see e.g., [9], [10] and the references therein). In the regret minimization framework, a player has to make a decision first, and then obtains a loss corresponding to the decision. The environment in which the player makes a decision is assumed to be uncertain and changing over times. Before making a decision, the player is not aware of the loss (or gain) associated to the implementation of that decision. The regret minimization algorithms provide, however, rules to update the decisions such that the average cumulative loss approaches the one of the best fixed decision discovered in hindsight. For our problem, the player is the distribution network operator. The decisions are those to be made on the scheduling of the power generation from the controllable sources (e.g., micro turbines) and the power consumption for various loads (e.g., EVs, batteries). Uncertainty comes from the unpredictable part of DER power generations. Based on this regret minimization framework, the distribution network operator updates the schedules of various components in an online fashion.

For the need of a distributed energy management algorithm, we modify the ADMM algorithm (see e.g., [11]) to coordinate the power scheduling of components in the microgrids to satisfy the underlying power network operation constraints. The ADMM algorithm is well suited for distributed optimization and large-scale distributed computing systems. By using ADMM, we do not need to have a centralized computation center and the computation can be distributed over the local computing devices (the reader is referred to the survey paper [11] for the details of ADMM). ADMM consists of several inner loop subroutines. Each inner loop subroutine is an optimization problem. Unlike the offline energy management algorithms based on ADMM [12], [13], the proposed online ADMM does not require the inner loop subroutines of the algorithm to be fully executed, which results in the speed-up of the runtime and the reduction of the communication demand.

Our contribution is exploring the potential for the use of regret minimization from machine learning and online ADMM for designing energy management algorithm in networked microgrids. The unique features of the proposed algorithm in this paper are twofold. First, the online energy management algorithm does not require any prior knowledge about the unpredictable parts of DERs to proceed. Second, the online energy management algorithm is implemented in a distributed manner and does not require the convergence of the inner loop subroutines of the distributed algorithm leading to the acceleration of the implementation and the reduction of the communication demand substantially.

Some relevant references that apply regret minimization to demand response (DR) are [14], [15]. In [14], the exact demand function of the consumer is assumed to be unknown to the pricing policy maker. The authors design pricing policies for the customers with price responsive loads. In [15], the authors design the real-time electricity pricing strategies for DR using regret minimization. Some recent works have been reported on online energy management for the microgrid [16], [17]. The work [16] studies online algorithms for the microgrid power generation scheduling with intermittent renewable sources. However, rather than the notion of regret, a different metric, competitive ratio, is used to evaluate the performance of algorithms. The two metrics, regret and competitive ratio, are fundamentally incompatible. In general, an algorithm that is designed to minimize the competitive ratio may not minimize the regret [18], namely, an algorithm providing a minimal competitive ratio may have large regret. The reason is that the two metrics are defined in a different way. In particular, the regret is defined using cumulative sum of the costs and the competitive ratio is defined using the ratio of the optimal cost to the suboptimal cost. The two quantities are essentially different. Algorithm that are designed to minimize the regret cannot be directly applied to minimize the competitive ratio. [17] proposes an online optimization algorithm for a single microgrid based on regret minimization. However, the underlying physical power network is ignored in the design of the algorithm. In our work, we take into account the underlying power network and consider networked microgrids in the design of the online energy management.

The remainder of the paper is organized as follows. Section II describes the problem formulation. Section III presents the online energy management algorithm. Section IV provides some numerical results to verify the effectiveness of the
algorithm. Section V concludes the paper.

II. PROBLEM FORMULATION

Consider a scenario in which a distribution network operator owns some of the microgrids (utility microgrids) and sets up real-time power generation and consumption schedules for various components of microgrids in an online fashion. The components include controllable load and generation (e.g., micro turbine). There are uncontrollable power generation sources (e.g., PV panels) in the microgrids. We assume that there are microgrids which are privately owned (private microgrids). The distributed network operator can buy/sell power from/to these private microgrids [1]. Figure 1 describes the interaction between the networked microgrids.

For planning purposes, we group the utility microgrids and private microgrids (see Figure 1). Although we group the microgrids, the distribution network operator does not directly control the DGs and loads of the private microgrids. Within each group, the utility microgrid buy/sell power from/to the private microgrids in the same group. Also, within each group, we assume that the power transaction can be coordinated and controlled by the utility microgrid controller; thus we do not consider the power network (i.e., power flow constraints) within each group. Each group exchanges power with its adjacent groups based on the power flow constraints (1)-(4). The private microgrids are internal sinks or sources (of power) in each group and each group’s utility microgrid connects to its adjacent groups’ utility microgrids.

We focus on the distribution network operator perspective and design an online energy management algorithm to minimize the power generation costs of the utility microgrids. The scheduling process is over T time slots, and each time slot represents a specific length of time (e.g., 15 minutes).

The networked microgrids are characterized as a connected and undirected graph $G = (\mathcal{M}_s, \mathcal{E})$. For our setup, the set of vertices $\mathcal{M}_s := \{0, 1, ..., M\}$ denotes the set of microgrids and the set of edges $\mathcal{E}$ represents the set of power lines connecting microgrids.

We use the DistFlow model (see e.g., [1]) to model an AC distribution network. Following [1], the DistFlow model is simplified to:

$$p_j = p_{ij} - \sum_{k:(j,k) \in \mathcal{E}, k \in \mathcal{M}_s} P_{jk},$$

(1)

$$q_j = Q_{ij} - \sum_{k:(j,k) \in \mathcal{E}, k \in \mathcal{M}_s} Q_{jk},$$

(2)

$$V_j = V_i - \frac{(r_{ij} P_{ij} + x_{ij} Q_{ij})}{V_0},$$

(3)

$$1 - \epsilon \leq V_j \leq 1 + \epsilon, \forall j \in \mathcal{M}_s,$$

(4)

where $p_j = p^1_j - (p^0_j + p^2_j)$, $q_j = q^1_j - q^2_j$, and $\epsilon$ is a constant and set to 0.05.

In each time slot $t \in \mathcal{T}$, the distribution network operator solves an optimization problem. The objective of the optimization problem is to minimize the power generation costs of the micro turbines and the power transaction costs with the private microgrids. The decision (output) variables are the active/reactive power consumption $p^1_i$, $q^1_i$ of the micro turbines, the net power $p^0_i$ of the power transaction with private microgrids and the power generation of the PV panels, and the voltages $V_i$ of the utility microgrids.

The optimization problem is formulated as follows. For $t \in \mathcal{T}$,

$$\begin{align*}
\text{minimize} & \quad \sum_{i \in \mathcal{M}_s} C_i(p^0_i(t)) + \alpha_i[p^1_i(t) - w_i(t)]^+ - \beta_i[p^1_i(t) - w_i(t)]^- \\
\text{subject to} & \quad p^0_i(t) \leq p_i^0(t) \leq p_i^0(t), \quad t \in \mathcal{T}, \quad i \in \mathcal{M}_s,
\end{align*}$$

(5)

and (1)-(4),

where the cost function $C_i$, $i \in \mathcal{M}_s$ is quadratic in $p_i^0$, which encodes the micro turbine power generation cost [1], i.e.,

$$C_i(p^0_i(t)) := a_i p_i^0(t)^2 + b_i p_i^0(t) + c_i,$$

(11)

where $a_i \in \mathbb{R}_{>0}$, $b_i \in \mathbb{R}_{>0}$, $c_i \in \mathbb{R}_{>0}$ are positive real numbers. The transaction cost is defined as:

$$\alpha_i[p^1_i(t) - w_i(t)]^+ - \beta_i[p^1_i(t) - w_i(t)]^-,$$

(12)

where $[a]^+ := \max\{a, 0\}$, and $[b]^- := \max\{-b, 0\}$. We use $p^0_i$, $i \in \mathcal{M}_s$ to denote the net power of the power transaction with the private microgrids and the onsite PV power generation.
of the utility microgrid. Therefore the power that is bought
from the private microgrids is equal to
\( \alpha_i \left[ p_i^s(t) - w_i(t) \right]^+ \)
and the power that is sold to the private microgrids is equal
to \( \beta_i \left[ p_i^s(t) - w_i(t) \right]^+ \). Following [7], the transaction cost (12)
is convex if the sale price does not exceed the purchase price,
i.e., \( \beta_i \leq \alpha_i \), \( i \in \mathcal{M}_s \). Equations (6)-(10) are the capacity
constraints for the corresponding components. Other convex
cost functions and convex constraints in economic dispatch
and balancing market mechanisms can be included in the
problem formulation (1)-(10). Problem (1)-(10) is repeatedly
solved at the beginning of each time slot \( t \in \mathcal{T} \).

The active power generations of PVs, \( w_i(t), \ t \in \mathcal{T}, \ i \in \mathcal{M}_s \), are uncertain. More precisely, at the beginning of time
slot \( t \in \mathcal{T} \), we assume that the distribution network operator
only has the past information about the power generations
of PVs before time slot \( t, \ w_i(s), \ s = 1, ..., t - 1, \ i \in \mathcal{M}_s \)
and does not know the future power generations of PVs,
\( w_i(s), \ s = t, ..., T, \ i \in \mathcal{M}_s \). In time slot \( t \), the distribution
network operator implements the energy management algo-
rithm. At the end of the time slot \( t \in \mathcal{T} \), the realized power
generations of PVs, \( w_i(t), \ i \in \mathcal{M}_s \), are recorded and can be
used by the operator for scheduling in the next time slot \( t + 1 \).

Alternatively, one can use stochastic or robust programming
techniques to solve problem (1)-(10). In a stochastic program,
the mean values of \( w_i(t) \) for all \( i \in \mathcal{M}_s \) and \( t \in \mathcal{T} \) can
be used in the objective (5). In a robust program (the worst case
formulation), the objective is replaced by

\[
\sum_{i \in \mathcal{M}_s} C_i \left( p_i^+ \right)
+ \max_{w_i(t) \in \mathcal{W}_i(t)} \alpha_i \left[ p_i^s(t) - w_i(t) \right]^+ - \beta_i \left[ p_i^s(t) - w_i(t) \right]^-, \tag{13}
\]

where the sets \( \mathcal{W}_i(t) \) for all \( i \in \mathcal{M}_s \) and \( t \in \mathcal{T} \) are the sets
of all possible realizations of the solar-PV powers.

One can also include the predictions on the power genera-
tions of PVs. However, since the state-of-the-art forecasting
for non-dispatchable DERs is not sufficiently accurate, which
may result in inaccurate energy scheduling. We do not consider
including the predictions into the problem formulation.

The distribution network operator can, in principle, solve
problem (1)-(10) in a centralized manner. In this case, how-
ever, the communication demand between the operator and
the networked microgrids is high [19]. To reduce the
communication demand, it is of interest to design a distributed
algorithm to solve the problem [19]. The mechanism allows
but limits every utility and private microgrid to exchange
information with only its adjacent microgrids.

Remark 2.1: The cost function (5) represents the global
objective, and the proposed algorithm actually aims to achieve
this global objective, but in a distributed manner, i.e., distribute
the calculation to each device. Solving the optimization prob-
lem with the global objective (5) in a distributed manner has
several advantages compared to that in centralized manner,
including lower communication latency, better scalability with
increase of the number of devices, and more resilience
to single point failure.

![Radial electrical network.](image)

Our problem formulation takes into account the underlying
power network described by (1)-(4). The work [17] proposes
an online optimization algorithm for a single microgrid. How-
ever, [17] ignores the underlying power network. The work
[7] considers a robust optimization based energy management
for a single microgrid with high penetration renewables. We
do not use robust optimization to formulate our problem to
prevent conservative results.

III. ON-LINE DISTRIBUTED ENERGY MANAGEMENT
ALGORITHMS

Our problem (1)-(10) involves coupled voltage and power
flow constraints among the microgrids. The power flows
and the voltages in the distribution network form the global
variable denoted by \( z \). For every microgrid \( i \in \mathcal{M}_s \),
the corresponding branch power flows that enter and leave
microgrid \( i \) form the local variable denoted by \( y_i \). Each branch
power flow represents a specific part of the set of the branch
power flows in the microgrids. Thus, the local variable \( y_i \)
is a part of global variable \( z \). We denote this relationship by
the equation \( y_i = \tilde{z}_i \), where \( \tilde{z}_i \) is the specific part of \( z \) that
contains local variable \( y_i \). Similarly, for microgrid \( i \in \mathcal{M}_s \),
the voltages \( V_i \), and \( V_j, j \in \mathcal{M}_i \) form the local variable \( y_i \),
where \( \mathcal{M}_i \) is the set of adjacent microgrids of microgrid \( i \).
For microgrid \( i, i \in \mathcal{M}_s \), the \( i \)-th microgrid’s power consumption
generation form the private variable denoted by \( x_i \). While
private variable \( x_i \) is not shared with other microgrids, it is
related to the local variables \( y_i \). We define the aggregated
variables \( r_i := (x_i, y_i), i \in \mathcal{M}_s \), and \( r := (r_1, ..., r_M) \).
We can also treat local variable \( y_i \) as a copy of the coupled
global variable \( \hat{z}_i \) [11]. We define the constraint violation as
the violation of the consensus for the power flow on the same
power line and the violation of the consensus for the voltage
at the same bus. Thus, the violation of the consensus can be
seen as the quantity \( \| y_i - \hat{z}_i \| \) for all \( i \in \mathcal{M}_s \).

We define three types of sets. For each microgrid \( i \in \mathcal{M}_s \),
the set \( \mathcal{L}_i \) denotes the set of feasible branch power flows
that go through the \( i \)-th microgrid and the voltages \( V_i, V_j, j \in \mathcal{M}_i \) that satisfy the underlying power network constraints for

\[
\begin{aligned}
i - 1 & & i + 1 \\
V_{i-1} & & V_i & & V_{i+1} \\
P_{i-1} & & P_i & & P_{i+1} \\
Q_{i-1} & & Q_i & & Q_{i+1} \\
p_{i-1} + jq_{i-1} & & p_i + jq_i & & p_{i+1} + jq_{i+1}
\end{aligned}
\]
the corresponding power lines. For each microgrid $i \in \mathcal{M}_s$, the set $\mathcal{F}_i$ denotes the set of feasible power generation and consumption. For each microgrid $i \in \mathcal{M}_s$, the set $\mathcal{H}_i$ encodes the set of constraints for the private variable $x_i$ and the local variable $y_i$. Denote the set of feasible power schedules by $\mathcal{O} := \{y_i \in \mathcal{L}_i, x_i \in \mathcal{F}_i, r_i \in \mathcal{H}_i, i \in \mathcal{M}_s\}$.

A. ADMM

Following the previous discussion, we are able to construct the corresponding set $\mathcal{O} := \{y_i \in \mathcal{L}_i, x_i \in \mathcal{F}_i, r_i \in \mathcal{H}_i, i \in \mathcal{M}_s\}$ for our problems (1)-(10). Thus, our problem is transformed to the following optimization problem with the suitable defined sets $\mathcal{L}_i$, $\mathcal{F}_i$, and $\mathcal{H}_i$, $i \in \mathcal{M}_s$. Consider the optimization problem:

$$\begin{align*}
\text{minimize} & \quad f^t(r) + g(r), \tag{14} \\
\text{subject to} & \quad \lambda \in \mathcal{H}_i, \quad i \in \mathcal{M}_s
\end{align*}$$

where the equality $r + Bz = 0$ with a suitable defined matrix $B$ encodes the relationship between the local variables $y_i$, $i \in \mathcal{M}_s$ and the global variable $z$. In (14), $f^t(r)$ represents the objective function in (5) and $g(r)$ is the indicator function of the feasible set $\mathcal{O} = \{y_i \in \mathcal{L}_i, x_i \in \mathcal{F}_i, r_i \in \mathcal{H}_i, i \in \mathcal{M}_s\}$, i.e.,

$$g(r) = \begin{cases} 
\text{constant,} & \forall r \in \mathcal{O}, \\
\infty, & \text{otherwise.}
\end{cases} \tag{15}$$

We formulate the augmented Lagrangian of (14):

$$L = f^t(r) + g(r) + \lambda^T(r + Bz) + \frac{\rho}{2} \|r + Bz\|^2, \tag{16}$$

where $\lambda$ is the corresponding Lagrangian multiplier [20] and the positive real number $\rho$ is a given parameter.

The ADMM iteration (with iteration index $k = 1, ..., K$) uses two steps to update the primal variables $r, z$ and one step to update the dual variable $\lambda$:

$$\begin{align*}
r^{k+1} &= \arg\min_r f^t(r) + g(r) + \lambda^T(r + Bz^k) + \frac{\rho}{2} \|r + Bz^k\|^2, \\
z^{k+1} &= \arg\min_z \lambda^T(r^{k+1} + Bz) + \frac{\rho}{2} \|r^{k+1} + Bz\|^2, \\
\lambda^{k+1} &= \lambda^t + \rho(r^{k+1} + Bz^{k+1}). \tag{17}
\end{align*}$$

The ADMM iteration (17) is in a distributed manner. Given $\tilde{z}_i$, the $r$ update can be implemented by each microgrid. Similarly, given $r_i^{k+1}$ and $\tilde{z}_i^{k+1}$, the $\lambda$ update can be implemented by each microgrid as well. In the $z$ update, the neighboring microgrids are coupled through the matrix $B$ and the update can be implemented among neighboring microgrids.

At time $t + 1$, the current objective function $f^{t+1}(t)$ is not known to the decision maker due to the presence of the uncertainty $w^{t+1}$. A standard ADMM iteration (17) thus requires the forecast of the uncertainty to proceed. An inaccurate forecast may lead to high generation cost. A standard ADMM can also adopt the robust optimization formulation, i.e., incorporating the robust objective (13) [7]. However, it may lead to conservative result. These constraints motivate us to consider an approach that does not depend on the forecast of the uncertainty. The online ADMM which is based on regret minimization is suitable in this case.

B. Online ADMM

We use online ADMM to solve (14) [21], which iteratively solve (with feasible initial conditions, i.e., $r^1 \in \mathcal{O}$ and $\lambda^1 = 0$), at time $t + 1$,

$$\begin{align*}
r^{t+1} &= \arg\min_r f^t(r) + g(r) + \lambda^T(r + Bz^t) + \frac{\rho}{2} \|r + Bz^t\|^2, \\
z^{t+1} &= \arg\min_z \lambda^T(r^{t+1} + Bz) + \frac{\rho}{2} \|r^{t+1} + Bz\|^2, \\
\lambda^{t+1} &= \lambda^t + \rho(r^{t+1} + Bz^{t+1}), \tag{18}
\end{align*}$$

where the iteration (18) is executed in each time slot $t = 1, ..., T - 1$.

In (18), in every time slot $t + 1$, there is only one iteration to update the variables $(r^{t+1}, z^{t+1}$, $\lambda^{t+1})$, i.e., maximal iteration index $K = 1$, which is a standard formulation in the online convex optimization framework (see, e.g., [9], [10], [21]). Also, in every time slot $t + 1$, we update the primal variables $(r^{t+1}, z^{t+1})$ based on the past cost function $f^t$ to perform such updates.

We characterize the performance of the online ADMM (18) in the regret minimization framework [9], [10], [21]. The regret $R(T)$ after $T$ time slots is defined as:

$$R(T) := \sum_{t=1}^{T} f^t(r^t) + g(r^t) - \min_{r^0, Bz^0} \sum_{t=1}^{T} f^t(r^t) + g(r). \tag{19}$$

The regret $R(T)$ measures the difference between the cumulative cost of decisions $r^t$, $t = 1, ..., T$ and that of the best fixed decision in hindsight [9]. A fixed decision remains the same for all time slots $t = 1, ..., T$. The best fixed decision can only be computed in hindsight (i.e., at the end of the $T$-th time slot). The goal of the online algorithm is to generate decisions making the regret $R(T)$ small. If the regret is bounded by a sublinear function of $T$, e.g., $R(T) = O(\sqrt{T})$ [9], then the average regret converges to zero as $T \to \infty$, which indicates that the sequence of the decisions converges to the best fixed decision in hindsight. For our problem, this is equivalent to the online power system schedule converges to the best fixed schedule asymptotically. In our problem, the decisions are variables $r^t$, $t = 1, ..., T$ which consist of private variables $x^t_i$, $t = 1, ..., T$ and local variables $y^t_i$, $t = 1, ..., T$ for all microgrids $i \in \mathcal{M}_s$.

In general, the ADMM iteration (18) does not provide feasible solutions before convergence, i.e., $r^t + Bz^t \neq 0$ for all $t = 1, ..., T$ [11]. We define the regret for the corresponding equality constraint violation [21]. The regret $R^c(T)$ for the constraint violation is defined as:

$$R^c(T) := \sum_{t=1}^{T} \|r^{t+1} + Bz^{t+1}\| + \|Bz^{t+1} - Bz^t\|. \tag{20}$$

The results in [21] show that both regrets $R(T)$ and $R^c(T)$ are in the order $O(\sqrt{T})$, i.e., $R(T)/T \to 0$, $R^c(T)/T \to 0$ as...
The results indicate that asymptotically, the average cumulative generation cost obtained from the online ADMM iteration converges to the one with the optimal fixed decision in hindsight. That $R(T)/T$ converges to zero indicates that the generation and consumption schedules converge to the best fixed schedules. That $R^c(T)/T$ converges to zero indicates that the online schedule becomes feasible asymptotically. In the simulations, we observe that as the maximal iteration index $K$ is increased, $r^t + B z^t \to 0$ for all $t = 1, ..., T$, which ensures that the system operates within the feasible state.

Note that in (19), the best decision is time-invariant, which remains the same for all $t = 1, ..., T$. To capture the variability of solar-PV generation, we relax the time-invariant nature of the decision and compare the performance of the solutions generated by the algorithm with the one generated by an optimal sequence of decisions which may vary between time slots $t \in T$. Thus, a different notion of regret, tracking regret [22] is defined as follows. Consider $r^t$ such that $r^t + B z^t = 0$, for all $t = 1, ..., T$. The tracking regret for $\bar{r}^t$, $t = 1, ..., T$ is defined as:

$$R_{\text{tracking}}(T) := \sum_{t=1}^{T} f^t(\bar{r}^t) + g(\bar{r}^t) - \min_{r^t + B z^t = 0, t \in T} \sum_{t=1}^{T} f^t(r^t) + g(r). \quad (21)$$

The results in [23] show that if the optimal solution is time invariant, i.e., $z^{t+1} = z^t$ for all $t = 1, ..., T$, then both $R_{\text{tracking}}(T)$ and $R^c(T)/T$ in the order $O(\sqrt{T})$. If this is the case, then the average regrets $R_{\text{tracking}}/T$ and $R^c(T)/T$ converge to zeros as $T \to \infty$. That $R_{\text{tracking}}/T$ converges to zero indicates that the generation and consumption schedules converge to the best schedules. That $R^c(T)/T$ converges to zero indicates that the online schedule becomes feasible asymptotically. If the optimal solution is time-varying, i.e., $z^{t+1} \neq z^t$ for all $t = 1, ..., T$, then the regrets are not in the order $O(\sqrt{T})$. The performance however depends on the variation of the optimal solution. In the simulations, as we increase the maximal iteration index $K$, $r^t + B z^t \to 0$ for all $t = 1, ..., T$, which ensures that the system operates within the feasible state. Also the regret maintains the same low level as well.

### IV. Numerical Results

We consider an IEEE 37-bus\(^2\) ($M = 37$) test system [25]. Each bus represents a group of microgrids as noted in Section II (Figure 1). We consider 96 time slots ($T = 96$). Each time slot represents 15 minutes time duration. We set the starting time at 00:00. Therefore, the first time slot starts at 00:00 and ends at 00:15; the last time slot starts at 23:45 and ends at 00:00 (the next day). The base power is set to 10 MVA. Table I shows the parameters of the microgrids and Table II shows the parameters of the cost function (5). We use the solar-PV power generation profile in Belgium [26] as shown in Figure 3. It is clear that the real-time measurements deviate substantially from the forecast data. Figure 4 shows the cost of the optimal slot-varying power schedule. The optimal slot-varying cost can only be computed at the end of the last time slot (after all the power generations of the PV energy sources are realized). Figure 4 shows that the optimal slot-varying cost varies between the time slots. The utility microgrids sell the surplus power generation to the privately owned microgrids and obtain earnings.

We use the online ADMM algorithm (18) to generate the power schedule and to track the optimal costs shown in Figure 4. In each time slot $t \in T$, we use CVX/CVXGEN [27], [28] to solve the optimization problem (1)-(10). Each optimization problem is a convex program. Figure 5 shows the power schedules of the PV energy sources and the power bought/sold from/to the privately owned microgrids. Figure 5 shows that the power generation decreases as the power generation of PV energy sources increases. Notice that when the online ADMM algorithm generates the power schedule in each time slot, we do not include the forecast of the PV power generation (since the forecast may not be accurate). However, the generated power schedule solutions have the ability to adjust according to the variation of the PV power generation. In Figure 6, we reduce the power generation of the micro turbines. The generation price is changed from $b_i = 0.2$ $$/\text{kWh}$$ to $b_i = 0.1$ $$/\text{kWh}$.

In order to verify the effectiveness of the online ADMM algorithm proposed in this work, we compare our online algorithm with the standard ADMM algorithm which applies the PV forecast (see, e.g., [12], [13]). The online ADMM algorithm does not require any PV forecast data to proceed. Since the forecast data is not accurate and the online ADMM

\(^2\)ADMM algorithm has been tested for instances including a few thousand buses and the solve times are within a few hundred seconds.
does not use it to update the schedules, it may yield a lower power generation cost than the standard ADMM algorithm. We use the solar-PV power generation profiles in Belgium from 12/01/2014 to 12/05/2015 [26] as shown in Figure 7. Figure 8 shows the convergence results of the average regret of the online ADMM and standard ADMM with forecast. Figure 8 shows that in this case, at the end of the evaluation phase, the average cumulative cost generated by the online ADMM algorithm is about $38.8/15 minutes ($3,725 daily) less than the cost generated by the standard ADMM algorithm with the PV forecast data.

We compare online ADMM algorithm (18) with robust optimization approach. In a robust optimization problem, the objective is replaced by (13). We use the solar-PV power generation profiles in Belgium from 11/24/2014 to 11/28/2015 [26] to construct the sets $\mathcal{W}_i(t)$ for all $i \in \mathcal{M}_s$ and $t \in \mathcal{T}$, which are the sets of all possible realizations of the solar-PV powers during the time period. The performance is evaluated on 11/29/2015. Figure 9 shows the convergence results of the average regrets. Figure 9 shows that in this case, at the end of the evaluation phase (11/29/2015), the average cumulative cost generated by the online ADMM algorithm is about $184.6/15 minutes ($17,722 daily) less than the cost generated by the ADMM with the robust objective. It shows that the proposed approach provides less conservative schedule than the robust optimization based approach.

The standard formulation of online convex optimization framework is to set the iteration number in each time slot to 1 (i.e., $K = 1$), which causes the constraint violation issues. As noted in Section III, our problem (1)-(10) involves coupled voltage and power flow constraints among the microgrids. The violation of the consensus can be seen as the quantity $\|y_i - \bar{z}_i\|$ for all $i \in \mathcal{M}_s$. Essentially, every microgrid $i$ makes a copy $y_i$ of the coupled global variable $\bar{z}_i$. As the term $\|y_i - \bar{z}_i\| \to 0$, the consensus for the power flow on the same power line and the consensus for the voltage at the same bus are reached.

To address the constraint violation issue, we modify the standard formulation to increase the maximal iteration number $K$ in each time slot. This idea comes from the ADMM iteration in (17) that by increasing the number of iterations for decision update in each time slot, the term $\|r + Bz\|^2$, which serves as penalty cost of constraint violation in the augmented Lagrangian in (16), will approach to zero; thus addressing the constraints violation problem. This engineering method can treat the constraint violation issue properly. In Figure 9, we evaluate the performances of online ADMM with different
numbers of maximal iterations. It is interesting to see how the number of iterations affects the performance. Figure 9 shows that increasing maximal number of iterations of the algorithm does not result in a better performance. A better performance means a smaller regret. It is expected since the cost function is slot-varying and the current cost function is not revealed to the scheduler when the decision is made. However, as the maximal number of iterations increases, the regret maintains the same low level. Average cumulative cost generated by the online ADMM algorithm with $K = 15$ is about $151.8/15 minutes less than the cost generated by the ADMM with the robust objective. Also, as the maximal number of iterations increases, the constraint violation decreases. Figure 10 shows the results of the constraint violations of online ADMM with different numbers of iterations. The maximum constraint violation is defined as $\max_{i \in M} \| y_i - \tilde{y}_i \|^2$ for all $i \in M$. The figure shows that the constraint violations are very close to zero. In order to further examine the effects of the violations, we also calculate the percentage value of the average constraint violations for active power, reactive power flow, and voltages, respectively. The percentage value of the constraint violations is defined as $\frac{\| y_i - \tilde{y}_i \|^2}{\| x_i \|}$ for all $i \in M$. The results shows that the extent of the constraint violation using the proposed method is acceptable in practice, and the system or measurement errors. In addition, since the energy constraint violation.

To better validate that the extent of the constraint violation using the proposed method is acceptable in practice, Figure 11 shows the trajectories of the constraint violations in a day with the maximal iteration number $K = 100$. The figure shows that the constraint violations are very close to zero. In order to further examine the effects of the violations, we also calculate the percentage value of the average constraint violations for active power, reactive power flow, and voltages, respectively. The percentage value of the constraint violations is defined as $\frac{\| y_i - \tilde{y}_i \|^2}{\| x_i \|}$ for all $i \in M$. The results shows that as the number of the iterations increases, the percentage value of the average constraint violations for active power, reactive power, and voltage are $0.25\%$, $0.1\%$, and $0.71\%$ respectively. Because the power grid model itself also has errors due to inaccurate grid parameters and ambient weather effects, and the measurement data (e.g., data from SCADA system) may also have errors due to various reasons, we could not get an absolutely exact solution for the actual power systems. If we assume $1\%$ error of the power system model or measurements, the errors from constraints violation are at the same scale of the system or measurement errors. In addition, since the energy
management for networked microgrids serves as the tertiary-level control, these minor errors can be compensated/adjusted in the primary-level and secondary-level control systems. In this sense, we can claim that the constraint violation using our approach is acceptable.

![Figure 11. Average constraint violations for active power, reactive power, and voltage](image)

Regarding the actual implementation, because the energy management is the tertiary-level control which has a larger time scale, i.e., in this paper each time slot represents 15-minute time duration, increase of the iteration number within each time slot is not an issue. Each iteration in the distributed computation just has low computation and communication complexity, so a 15-minute duration is long enough to accommodate the required calculations and communication latency.

Based on the above discussion, by increasing the number of iterations in each time slot, the constraint violation issue can be properly addressed and is acceptable to the actual system.

V. CONCLUSION

We propose a distributed algorithm for online energy management in networked microgrids based on online ADMM. The proposed algorithm does not require any forecast for DERs to proceed and is implemented in a distributed manner. Also the algorithm does not require the convergence of the inner loop subroutines of the distributed algorithm, which leads to the acceleration of the implementation and the reduction of the communication complexity. The simulation results show that the proposed online algorithm performs better than the offline algorithm with inaccurate predictions and provides less conservative schedule than the robust optimization based approach.

REFERENCES


