

# Using Natural Gas Reserves to Mitigate Intermittence of Renewables in the Day Ahead Market

Ashkan Zeinalzadeh<sup>1</sup>, Nayara Aguiar<sup>1</sup>, Stefanos Baros<sup>2</sup>,  
Anuradha M. Annaswamy<sup>2</sup>, Indraneel Chakraborty<sup>3</sup>, and Vijay Gupta<sup>1</sup>

**Abstract**—We formulate and analyze a day-ahead (DA) electricity market in which a thermal power plant and a renewable generator compete with each other in their commitments to the market. The market price of energy is affected by the commitments of the generators. The renewable generator faces a settlement cost if it cannot meet its commitment due to unpredictable generation. As a hedge against this cost, it is equipped with a natural gas reserve that can either be used to compensate generation shortages or be sold in the natural gas market. We model the problem as a Stackelberg game, in which an independent system operator (ISO) sends a price signal to the generators. In response, the generators decide on their commitments to maximize their own profit. The ISO decides on the price such that the total commitment will be equal to the energy demanded by the (estimated) load. We develop sufficient conditions for the uniqueness of Nash equilibrium and obtain a quantitative solution for the Nash equilibrium. It is observed that the market price of energy is lower when the renewable generator is equipped with natural gas reserves. Furthermore, when the renewable generator is equipped with natural gas reserves, the commitments of the generators to the market are less affected by the variance of the renewable energy generation. It is also shown that a larger portion of the natural gas reserves are used for electricity generation when the renewable energy generation has higher uncertainty. Thus, the natural gas reserves act as an effective hedge against the variance in the generation of renewable energy.

## I. INTRODUCTION

Uncertainty and intermittency of renewable energy (RE) generation can increase the electric grid contingency costs if the goal is to provide reliable electricity to the consumer. There is a need for both supply side and demand side solutions to allow RE to be dispatchable, reliable and economically efficient. Recognizing this, there has been increasing research into technological solutions such as demand response management [1], back-up reserves control [2]–[4], and improved forecasting of renewable generation [3], [4]. In addition, there is also a need for new market structures and instruments to better accommodate the intrinsic variability of renewable generation into the electric grid.

In this paper, we consider a market in which the renewable generator (such as a wind power plant) is charged with providing its own insurance against its intermittent electricity production and the consequent externalities imposed on the power grid. To this end, the renewable generator maintains

natural gas reserves that it can optionally use in a natural gas power plant with high ramp up and down rates for complementing the intermittent renewable generation. However, maintaining this reserve to compensate against intermittent renewable generation in the real-time market comes at an opportunity cost of selling this natural gas in the natural gas spot market. We assume that the renewable generator and natural gas power plant belong to a single agent participating in the electricity market against a thermal power plant and develop the optimal bidding and operational strategies for the agent and the thermal plant. We show that the natural gas reserves can be used to decrease the externalities imposed on the non-renewable generators and reduce the impact of renewable fluctuations on the market price of energy. Thus, the reserves act as an effective insurance for the renewable producer.

We would like to mention works such as [5] and [6] that develop an optimal operational strategy for the combination of hydro and wind generation. Other directions such as scheduling consumers to mitigate the intermittency and fluctuations of renewables [1] and optimizing the coupling of the distribution electric system [7] and/or the electricity infrastructure [8] with the natural gas system to manage contingencies in both systems are also related to this work. The most relevant studies to this work are [3] and [4]. In these works, the authors obtain analytical solutions for the profit of wind producers while considering the effect of forecasting errors and energy reserves. Unlike the present work, they assume that the market clearing price of energy is fixed. We assume that the market clearing price is a function of the total commitment in the energy market, which necessitates the formulation of the problem as a Stackelberg game. Further, the focus of our work is different since we do not concentrate on the operational cost of the natural gas power plant and whether that cost justifies the payment of settlement costs in case of shortfalls in renewable production. Rather, we wish to see if the renewable producer can eliminate the shortfalls by using a natural gas reserve even at the expense of an opportunity cost of not selling the natural gas in the natural gas spot market.

The contributions of this work are as follows. We consider a DA electricity market, as illustrated in Figure 1, in which a thermal power plant and a renewable generator compete with one another to have a higher share of the market. The focus on only two agents clarifies the issues involved and provides better insights. The price of energy in the market depends on the commitments by the generators, which is the case

<sup>1</sup>Department of Electrical Engineering, University of Notre Dame {azeinalz, ngomesde, vgupta2}@nd.edu

<sup>2</sup>Department of Mechanical Engineering, MIT {sbaros, aanna@mit.edu}

<sup>3</sup>School of Business, University of Miami i.chakraborty@miami.edu

when there are few large players in the energy market [9]. We set up a Stackelberg game in which the ISO adjusts the price function of energy such that when players decide on their commitments, the sum of the commitments is equal to the load. We assume that the market price of energy is a non-increasing concave function of the commitment of the generators and that RE generation in excess of the commitment is curtailed. Since the ISO needs to purchase electricity in the real-time market to meet any load that is unmet, it imposes a penalty on the renewable generator that is proportional to the deviation of the realized energy from its commitment. To insure against the risk of incurring a penalty in the case of shortages, the renewable generator invests in natural gas reserves (and suffers an opportunity cost for not selling these reserves in the natural gas market) to compensate these shortages through a natural gas power plant with sufficiently fast ramp up and down rates. For this non-cooperative Stackelberg game, we identify the bidding strategies for both the generators in the market, and the portion of natural gas that the renewable generator should keep as reserve. We establish sufficient conditions for the uniqueness of the Nash equilibrium at which the generators optimize their operation without communication. It is shown in numerical simulations that more natural gas reserves are needed when renewable energy generation has higher uncertainty. We also study the effect of the natural gas reserves on the commitments by the generators and the market price of energy as the level of renewable penetration increases.

In the Stackelberg game, the ISO is considered as the leader and the generators are considered as the followers. In the first stage of the Stackelberg game, the ISO minimizes the price of energy while taking into account the best response functions of the generators to ensure that the generators' commitments will be sufficient to meet the load. In fact, the pricing strategy is decided by the ISO such that the load will be met if generators follow their Nash Equilibrium policy after receiving the pricing strategy from the ISO. In the second stage of the Stackelberg game, the followers (non-renewable and renewable generators) encounter a competitive game after receiving the pricing strategy. Generators maximize their own revenue by deciding their own commitments. What correlates the operation of the generators is the pricing strategy, which is a function of the generators' commitments. In the competitive game, generators compete to hold a bigger portion of the electricity market supply.

The rest of the paper is organized as follows. In Section II, we describe the problem setup and formulate the expected profit for the generators. In Section III, the Stackelberg game between the ISO and generators is formulated and we establish the Nash equilibrium strategy. In Section IV, the numerical results are presented. Finally, in Section V, some concluding remarks are provided.

*Notations:*

- $I\{\cdot\}$  is the indicator function,
- $\mathbb{R}$ , and  $\mathbb{R}^+$  denote the sets of real and non-negative real numbers respectively,

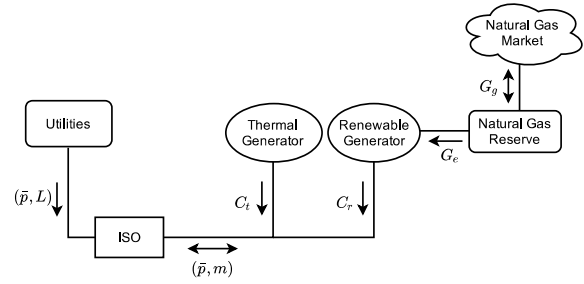


Fig. 1. Natural gas and electricity market model considered in the paper

- $\mathbb{R}^2$  denotes the 2-fold product  $\mathbb{R} \times \mathbb{R}$ ,
- $\lambda$  is the penalty per unit of energy shortage in the real-time market with respect to the commitment,
- $p$  is the price per unit of energy,
- $q$  is the price per unit of natural gas,
- $R$  is a random variable that represents renewable energy generation,
- $C_r$  is the commitment of the renewable generator in the day ahead market,
- $C_t$  is the commitment of the thermal power plant in the day ahead market,
- $G$  is the capacity of the natural gas reserve,
- $G_e$  is the portion of the natural gas reserve used for generating electricity,
- $G_g$  is the portion of the natural gas reserve that is to be sold in the natural gas market,
- $L$  is the amount of the estimated load,
- $F_R(r)$  is the cumulative distribution function of the renewable energy generation and  $f_R(r)$  is the corresponding probability density function, which is assumed to be non-zero on the domain  $[R^{\min} R^{\max}]$ .

## II. NATURAL GAS RESERVE, MARKET PRICE OF ENERGY AND PROFIT FUNCTIONS

Consider the set-up in Figure 1. We consider a day-ahead electricity market in which a thermal power plant and a renewable generator decide on their respective commitments to the electricity market. If the renewable generator cannot supply its day-ahead commitment in real time, it incurs a penalty proportional to the shortage. The RE unit owns a natural gas reserve with a fixed capacity. In the day-ahead market, if shortages in renewable production are expected, the producer must evaluate whether it will be more profitable to use natural gas to cover possible shortages or sell the gas in the natural gas market and face the penalty. The natural gas capacity is divided into two portions: i) the first portion is used for the compensation of RE shortages, ii) the second portion is sold in the natural gas market. Let  $G$  be the total capacity of the natural gas reserve,  $G_g$  be the amount of natural gas to be sold in the natural gas market at unit price  $q$ , and  $G_e = G - G_g$  be the amount of natural gas to be kept as a reserve for possible shortages in renewable production. We make the following assumptions for simplicity:

**Assumption 1.** Both the total capacity of natural gas ( $G$ ) and the natural gas market price ( $q$ ) are fixed. Further, all  $G_g$  units are sold in the natural gas market.

**Assumption 2.** The density function of the renewable energy generation,  $f_R(r)$ , is a continuous and single modal function. Further  $f_R(r) > 0$  for all  $R^{\min} \leq r \leq R^{\max}$  and  $f_R(r) = 0$  otherwise.

**Assumption 3.** Renewable production in excess of the commitment is curtailed.

To encourage the renewable generator to provide its own insurance against its intermittent generation, a penalty is imposed that is equal to  $\lambda$  per unit of energy shortage. The energy shortage is realized in real-time as the renewable generation is compared to its commitment in the DA market. The renewable generator decides on its commitment to the electricity market,  $C_r$ , and the portion of the natural gas capacity that is used for compensation of renewable shortages,  $G_e$ . The thermal power plant decides on its commitment  $C_t$  to the electricity market.

**Assumption 4.** The thermal power plant is always able to meet its commitment.

#### A. Market Price of Energy

Let  $p(C_r, C_t; m)$  denote the market price of energy. This price is a function of the renewable generator commitment  $C_r \geq 0$ , thermal power plant commitment  $C_t \geq 0$ , and the parameter  $m$  that is selected by the ISO. While variation of the price as a function of the commitments is common in markets with few generating companies [9], analytic models for variation of the price as a function of the commitments are difficult to obtain. The following assumption is a natural one.

**Assumption 5.**  $p(C_r, C_t; m)$  is a concave and non-increasing function in  $(C_r, C_t)$  and strictly decreasing in  $m$ , where  $C_r \geq 0$ ,  $C_t \geq 0$  and  $m \geq 0$ .

#### B. Renewable Generator

The utility function  $g_r(C_r, G_e, C_t; m)$  of the renewable generator is its expected profit, which is given as

$$g_r(C_r, G_e, C_t; m) = p(C_r, C_t; m) C_r + q(G - G_e) - E_R \left[ I(C_r - R - G_e) \lambda (C_r - R - G_e) \right], \quad (1)$$

where  $E_R$  denotes the expectation over the renewable energy generation. The first term in (1) is the profit from meeting the commitment. The second term is the profit from selling the natural gas reserve  $G_g$  in the natural gas market. The third term is the penalty paid in the case that the aggregate of the RE and natural gas capacity is unable to meet the commitment. More elaborated models which include terms such as a time-varying price of natural gas, the operating cost of the natural gas power plant, or the storage cost of the natural gas reserve can also be considered.

#### C. Thermal Power Plant

The utility function of the thermal power plant  $g_t(C_r, C_t; m)$  is its expected profit which is given as

$$g_t(C_r, C_t; m) = p(C_r, C_t; m) C_t - (\alpha + \beta C_t + \gamma C_t^2), \quad (2)$$

where the first term is the revenue from meeting the commitment and the second term, in parentheses, is the cost of generation.

Below we study the Stackelberg game, in which the ISO is the leader and the renewable and non-renewable generators are the followers. The leader (ISO) decides on the pricing strategy to meet the load. The generators decide on their commitments to maximize their own objective functions. In the next Section, we establish the Nash equilibrium strategy for the Stackelberg game.

### III. MAIN RESULTS

#### A. Second Stage of the Stackelberg Game

We begin by showing that a pure Nash equilibrium exists for the competitive game between the generators. Equations (1) and (2) imply that the commitment of each producer affects the profit of the other producer. Let  $\Gamma$  denote the non-cooperative competitive game where the players are the renewable generator and thermal power plant. The generators try to maximize their own profit functions given by (1) and (2) respectively by deciding their strategies in the sets  $C_r \in \mathbb{R}^+$ ,  $G_e \in [0, G]$ , and  $C_t \in \mathbb{R}^+$  respectively. Thus, the renewable generator decides on  $C_r(m)$  and  $G_e(m)$  to maximize its own profit function

$$\max_{C_r \geq 0, 0 \leq G_e \leq G} g_r(C_r, G_e, C_t; m), \quad (3)$$

while the thermal power plant decides on  $C_t(m)$  to maximize its own profit function

$$\max_{C_t \geq 0} g_t(C_r, C_t; m). \quad (4)$$

Note that the generators' profits are a function of the pricing function which includes the parameter  $m$  chosen by the ISO. Since the ISO acts first in terms of sending the pricing function to the generators, we model the game as a Stackelberg game with the ISO as the leader, and the two generators as the followers that act simultaneously.

**Assumption 6.** If  $L$  is the estimated load, the following upper-bounds on the commitments of the generators hold

$$\begin{aligned} 0 &\leq C_r \leq L, \\ 0 &\leq C_t \leq L. \end{aligned} \quad (5)$$

The support for Assumption 6 is provided in Section III-C. The problem considered in this Section is to determine the Nash strategies for the generators and the choice of the pricing signal by the ISO in this equilibrium. It is evident that the thermal power plant objective function (2) is concave in  $C_t$ . In part a) of Theorem 1, it is shown that the objective function of the renewable generator (1) is concave. In part b)

of Theorem 1, we conclude the existence of the pure Nash equilibrium strategy for the game between the generators. It is assumed that the pricing strategy is a general function of generators' commitments that satisfies Assumption 5.

**Theorem 1.** *Consider the game  $\Gamma$ . If Assumptions 1-6, hold then:*

- a) the function  $g_r(C_r, G_e, C_t; m)$  is differentiable and concave in  $(C_r, G_e)$  for  $R^{\min} \leq C_r - G_e \leq R^{\max}$ .
- b) the game  $\Gamma$  has a pure Nash equilibrium strategy.

*Proof.* Proof will be provided in the journal version.  $\square$

### B. Cournot-like model

We now consider a more specific model of the pricing function and derive crisper results specifying the Nash equilibrium in this case. Specifically, since the products of the two generators are perfect substitutes of each other, we take inspiration from the famous Cournot model of pricing [10] and assume that the price of energy is a linear, decreasing function of the total energy commitment in the market. Thus, the market price of energy is given as

$$p(C_r, C_t; m) = \bar{p} - m(C_r + C_t), \quad (6)$$

where  $\bar{p}$  is the maximum price of energy provided to the ISO by consumers, and  $m \geq 0$  is the decrease in the price of energy for a one-unit increment in the generators' commitments. The parameter of the market price of energy  $m$  is set by the ISO. We consider the following assumption to ensure a feasible solution to the optimization (3) and (4). This assumption ensures that the price of electricity is high enough so that the generators commit a positive quantity.

**Assumption 7.** *It is assumed that  $\bar{p} + q > 2\beta$  and  $\bar{p} > 2q$ .*

For notational ease, we define the following quantities:

- $a := \frac{(m+2\gamma)\bar{p}+m\beta}{2(m+\gamma)\lambda}$ ,
- $b := \frac{m(3m+4\gamma)}{2(m+\gamma)\lambda}$ ,
- $C_r^{(1)} := \frac{\bar{p}-q}{2m} - \frac{\bar{p}-2\beta+q}{2(3m+4\gamma)}$ ,
- $C_r^{(2)}$  satisfies  $F_{R+G}(C_r^{(2)}) = a - bC_r^{(2)}$ ,
- $C_r^{(3)}$  satisfies  $F_R(C_r^{(3)}) = a - bC_r^{(3)}$ .

Note that since  $m$ ,  $\lambda$ ,  $a$ , and  $b$  are all non-negative quantities, solutions for  $C_r^{(2)}$  and  $C_r^{(3)}$ , that satisfy the above equations, exist. Below we obtain a quantitative solution for the Nash equilibrium strategy in this case.

**Theorem 2.** *At the unique Nash equilibrium of the game  $\Gamma$  with the pricing function (6), the strategies of the two generators denoted by the tuple  $C^*(m) = (C_r^*(m), G_e^*(m), C_t^*(m))$  are given as follows:*

- If  $0 \leq C_r^{(1)} - F_R^{-1}(\frac{q}{\lambda}) \leq G$ , then

$$C^*(m) = \left( C_r^{(1)}, C_r^{(1)} - F_R^{-1}\left(\frac{q}{\lambda}\right), \frac{\bar{p} - mC_r^{(1)} - \beta}{2(m+\gamma)} \right). \quad (7)$$

- If  $F_R(C_r^{(2)}) - G > \frac{q}{\lambda} > 0$ , then

$$C^*(m) = \left( C_r^{(2)}, G, \frac{\bar{p} - mC_r^{(2)} - \beta}{2(m+\gamma)} \right). \quad (8)$$

- If  $F_R(C_r^{(3)}) < \frac{q}{\lambda}$ , then

$$C^*(m) = \left( C_r^{(3)}, 0, \frac{\bar{p} - mC_r^{(3)} - \beta}{2(m+\gamma)} \right). \quad (9)$$

*Proof.* Proof will be provided in the journal version.  $\square$

In Theorem 2,  $F_R^{-1}(\frac{q}{\lambda})$  acts as a threshold value. In the first case, the difference between the commitment of the renewable generator and  $F_R^{-1}(\frac{q}{\lambda})$  is non-negative and less or equal to the total capacity of the natural gas reserves. This implies that the capacity of the natural gas reserve is large enough for compensation of the renewable shortages. In the second case, the difference between the commitment of the renewable generator and  $G$  is bigger than  $F_R^{-1}(\frac{q}{\lambda})$ , meaning that the entire natural gas capacity is used to compensate shortages in the renewables, but a yet higher capacity of natural gas is needed to compensate the shortages fully. In the third case, the commitment of the renewable generator is smaller than  $F_R^{-1}(\frac{q}{\lambda})$ . Therefore, the renewable generator unit will not use the natural gas reserve to compensate the shortages. This can happen when the renewable energy is generated within the desired confidence level to meet the commitment. Note that if  $q > \lambda$  then  $\frac{\partial g_r(C_r, G_e, C_t; m)}{\partial G_e} < 0$  and the renewable generator does not use the natural gas reserves for compensation of the shortage,  $G_e^* = 0$ , and  $C^*$  is given as (9).

### C. First Stage of the Stackelberg Game

The ISO acts as the leader and decides on the parameter of the pricing functions such that the price of energy is minimized and the resulting commitments from the generators are sufficient to meet the (estimated) load  $L$ . For the Cournot-like model (6), this translates to adjusting the value of  $m$  so that

$$\max m \quad (10)$$

$$m \geq 0, C_r^*(m) + C_t^*(m) = L. \quad (11)$$

Due to (11), Assumption 6 holds.

Note that the value of  $\bar{p}$  is the maximum price that consumers are willing to pay for energy. This is assumed to be given to the ISO and is not changed by the ISO.

$C_t^*(m)$  can be written as a function of  $C_r^*(m)$  by taking the derivative of (2) with respect to  $C_t$  to obtain

$$C_t^*(m) = \frac{\bar{p} - mC_r^*(m) - \beta}{2(m+\gamma)}. \quad (12)$$

By substituting  $C_t^*(m)$  as a function of  $C_r^*(m)$  in (11), we can write the desired condition as

$$(m+2\gamma)C_r^*(m) + (\bar{p} - \beta) = 2L(m+\gamma). \quad (13)$$

Using Theorem 2, the right mapping for the  $C_r^*(m)$  is substituted in (13) and the equation is solved to obtain the

parameter  $m$  that will ensure that (11) holds. It is obvious that any non-negative value of  $m$  that satisfies (13) satisfies the balance between load and generation. The ISO can find the largest  $m \geq 0$  that satisfies equation (13) and transmit the value of  $\bar{p}$  and  $m$  to the generators. It should be noted that the maximum  $m$  corresponds to the lowest price of energy for consumers. In this manner, the ISO minimizes the price of energy on behalf of the consumers and ensures the existence of sufficient generators' commitments to meet the expected load  $L$  by adjusting the price signal.

**Assumption 8.** It is assumed that  $0 < L < \frac{\bar{p}-\beta}{2\gamma} + F_R^{-1}(\frac{\bar{p}}{\lambda})$ .

We now show that for any value of load  $0 < L < \frac{\bar{p}-\beta}{2\gamma} + F_R^{-1}(\frac{\bar{p}}{\lambda})$ , there exists an  $m$  that satisfies equality (11).

**Theorem 3.** If Assumptions 1-8 hold, then there exists an  $m \geq 0$  such that equality (11) holds.

*Proof.* Proof will be provided in the journal version.  $\square$

**Remark 1.** It is quite clear that the reasoning behind the above proof, and hence Theorem 3, can be extended to a more general pricing function that satisfies Assumption 5, beyond the Cournot-like model.

**Remark 2.** If the price of energy is fixed at  $\bar{p}$  and  $\lambda < \bar{p}$  then the renewable generator profit function is increasing in  $C_r^*$ . Therefore, the renewable generator's commitment can be equal to any value of  $L > 0$ .

**Theorem 4.** If the Assumptions 1-8 hold, then the Stackelberg game has a unique pure Nash equilibrium.

*Proof.* Proof will be provided in the journal version.  $\square$

#### IV. SIMULATIONS

In this section, we study the above results numerically. We will compare the commitments of the generators with and without the presence of natural gas reserves for a range of renewable penetration. Additionally, we are interested in how natural gas reserves can affect the market price of energy.

We consider a thermal power plant with parameters  $\alpha = 841.75$  (\$/h),  $\beta = 50.431$  (\$/MWh) and  $\gamma = 0.1987$  (\$/MWh<sup>2</sup>) ([11]). The penalty for shortages is  $\lambda = 200$  (\$/MWh) and the price of natural gas is  $q = 10$  (\$/MWh). The total capacity of the natural gas is  $G = 9$  (MWh). The parameters of the market price of energy are given as  $\bar{p} = 1000$  (\$/MWh) and  $m = 10$  (\$/MWh<sup>2</sup>). It is assumed that the renewable energy has a Gaussian distribution with mean  $\mu_r$  (MW) and standard deviation  $\sigma_r$  (MW) as given in Table I.

In Figure 2 and 3, the commitment of the renewable generator and thermal power plant both with and without natural gas reserves is plotted. It is shown that the commitments of the generators vary more without the natural gas reserves. In the presence of the natural gas reserves, both generators can agree on an almost fixed commitment. In Figure 4, the portion of the natural gas reserve used for generating electricity versus different scenarios of RE penetration is shown. If  $F_R^{-1}(\frac{q}{\lambda})$  is big enough to meet the commitment

TABLE I  
RENEWABLE ENERGY SCENARIOS

|                              |     |      |     |     |     |      |
|------------------------------|-----|------|-----|-----|-----|------|
| Ren.                         | 1   | 2    | 3   | 4   | 5   | 6    |
| $\mu_r^t$                    | 34  | 35   | 38  | 38  | 42  | 44   |
| $\sigma_r^t$                 | 2   | 2    | 5   | 2   | 6   | 3    |
| $\frac{\mu_r^t}{\sigma_r^t}$ | 17  | 17.5 | 7.6 | 19  | 7   | 14.7 |
| Ren.                         | 7   | 8    | 9   | 10  | 11  | 12   |
| $\mu_r^t$                    | 45  | 46   | 48  | 49  | 50  | 51   |
| $\sigma_r^t$                 | 11  | 11   | 11  | 8   | 12  | 10   |
| $\frac{\mu_r^t}{\sigma_r^t}$ | 4.1 | 4.2  | 4.4 | 6.1 | 4.2 | 5.1  |

then,  $G_e = 0$ . In scenario 7, the ratio of the mean to the standard deviation of RE generation ( $\frac{\mu_r}{\sigma_r}$ ) is at the minimum and more of the natural gas reserve is used for generating the electricity (Figure 4). In Figure 5, the market price of energy is plotted. It is observed that with the natural gas reserves, the price of energy is lower and experiences fewer fluctuations. This implies that reserving natural gas through a contract between the renewable energy generator and a natural gas power plant is an effective insurance strategy against the intermittence of the renewable production. In a scenario in which the natural gas price ( $q$ ) is greater than the penalty imposed for the shortages ( $\lambda$ ), the renewable generator will choose to sell the natural gas in the natural gas market and face the penalty, instead of using it to compensate possible shortages. The scenario in which  $G_e^* = 0$  is denoted by a red line in Figures 2-5.

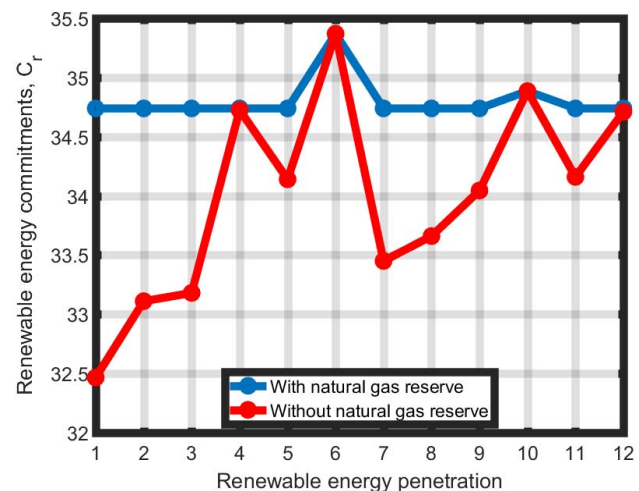


Fig. 2. Commitments of the renewable generator with and without the natural gas reserves versus the penetration levels.

#### V. CONCLUSIONS

We consider a situation in which a renewable generator and a thermal power plant compete in a day-ahead market. The renewable generator is penalized if there is a shortage

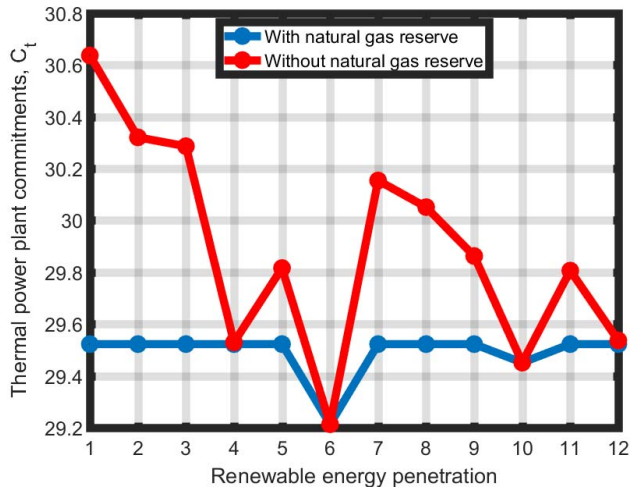


Fig. 3. Commitments of the thermal power plant with and without the natural gas reserves versus the penetration levels.

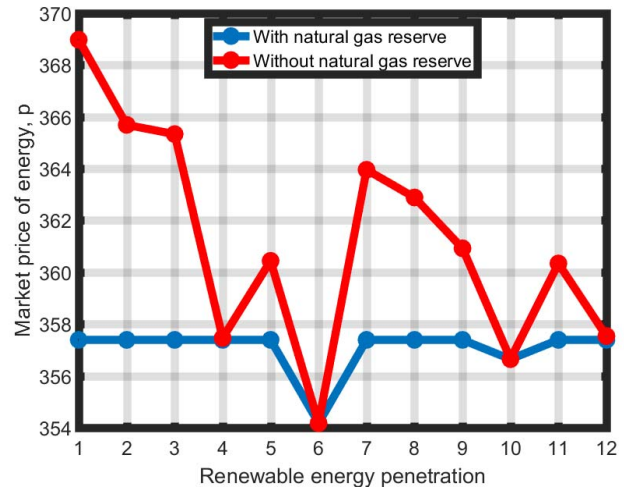


Fig. 5. Market price of energy ( $p$ ) with and without natural gas reserves.

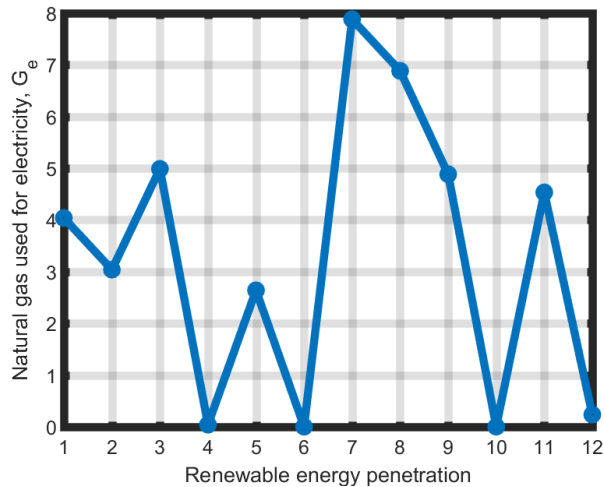


Fig. 4. The portion of the natural gas reserve used for generating electricity.

in the generated energy as compared to its commitment. To insure against this penalty, the generator sets a reserve of natural gas for use in a natural gas power plant to compensate any shortage in the renewable production; however, maintaining this reserve comes at an opportunity cost of being able to sell the natural gas in the natural gas market. We model the competition between the generators as a game and establish a Nash equilibrium strategy for the model. The ratio of the natural gas price ( $q$ ) to the penalty ( $\lambda$ ) for the shortage is shown to be a threshold in the Nash equilibrium strategy. The renewable generator decides the optimal usage of the natural gas reserve and the commitment based on the distribution of the renewable energy generation. It is also shown numerically that if renewable generators use natural gas reserves to compensate for their shortages, then the market price of energy and the commitments of the generators have less variation.

**Acknowledgement:** This work was supported by the Center for Sustainable Energy at Notre Dame and NSF grants ECCS 1550016, CNS 1544724 and EFRI 1441301.

#### REFERENCES

- [1] C. Wu, H. Mohsenian-Rad, J. Huang, Wind power integration via aggregator-consumer coordination: A game theoretic approach. 2012 IEEE PES Innovative Smart Grid Technologies (ISGT), 1-6.
- [2] A Castillo, DF Gayme. Grid-scale energy storage applications in renewable energy integration: A survey. *Energy Conversion and Management* 87, 885-894, 2014.
- [3] E.Y. Bitar, R. Rajagopal, P.P. Khargonekar, K. Poolla, P. Varaiya. Bringing wind energy to market. *IEEE Transactions on Power Systems*, 1-11, 2012.
- [4] E.Y. Bitar et al. Optimal contracts for wind power producers in electricity markets. 49th IEEE Conference on Decision and Control (CDC), 1919-1926, 2010.
- [5] J. L. Angarita, J. Usaola, J. Martinez-Crespo. Combined hydro-wind generation bids in a pool-based electricity market, *Electric Power Systems Research*, vol. 79, pp. 1038-1046, 2009.
- [6] E.D. Castronuovo, J.A. Peças Lopes. On the optimization of the daily operation of a wind-hydro power plant. *IEEE Transactions on Power Systems*. 19 (3):1599-1606, 2004.
- [7] C. A. Saldarriaga, R. A. Hincapie, and H. Salazar, A holistic approach for planning natural gas and electricity distribution network, *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 4052-4063, Nov. 2013.
- [8] M. Shahidehpour, Y. Fu, and T. Wiedman, Impact of natural gas infrastructure on electric power systems, *Proc. IEEE*, vol. 93, no. 5, pp. 1042-1056, May 2005.
- [9] Kirschen, Daniel S., and Goran Strbac. *Fundamentals of power system economics*. John Wiley & Sons, 2004.
- [10] Friedman, James. *Oligopoly theory*. *Handbook of mathematical economics* 2 (1982): 491-534.
- [11] Allen J. Wood and Bruce F. Wollenberg, *Power Generation, Operation, and Control*, 2nd ed., John Wiley and Sons, 2005.