Trade Patterns, Income Differences and Gains from Trade*

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Abstract

Quantifying the gains from international trade is an area of research that has been widely studied using a variety of trade models. At the same time, it has been shown that non-homotheticities are useful for matching the systematic patterns of trade present in disaggregated trade data. We bring these two literatures together to ask how non-homotheticities affect our predictions for gains from trade. To do so, we develop a N-country trade model that exactly matches bilateral trade, population, GDP per capita and within country income inequality for many countries. We include non-homotheticities to match patterns of trade between rich and poor countries that we observe in highly disaggregated trade data. We then make use of the results from Arkolakis, Costinot, and Rodriguez-Clare (2012), which gives a simple formula for gains from trade in a large class of homothetic models, including a version of our model with the non-homotheticity removed. Our main finding is that homothetic models underestimate gains from trade in countries with small populations and low productivities, and overestimate gains in countries with large populations and high productivities. The homothetic model overestimates the gains from being open to trade in the U.S. and Japan by 14% and 22%, and underestimates them in Spain and Italy by 24% and 14%.

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1 Introduction

An important area of research in the study of international trade is the measurement of the welfare gains from trade. This question is particularly important for assessing the gains from potential liberalizations of trade policies in countries around the world, and in predicting the effects of bilateral trade agreements. This issue has been widely studied using a variety of models, each of which emphasizes different margins of adjustment when undergoing trade reform. At the same time, another strand of the international trade literature has demonstrated the usefulness of models with non-homotheticities for matching patterns of trade between countries. Non-homothetic preferences allow models to match a variety of facts observed in disaggregated trade data, such as the systematic difference in the volume and composition of goods traded between rich and poor countries.

The goal of this paper is to combine the findings of these two literatures, and see if non-homotheticities generate qualitatively new predictions about the gains from trade liberalization. To do this, we analyze highly disaggregated bilateral trade data and construct a model consistent with the patterns of trade that we observe between countries of different populations and income levels. We construct an N-country model, each calibrated to match the characteristics of countries in the data. Our model exactly matches pair-wise volumes of bilateral trade, population, GDP per capita, and within-country heterogeneity in the income of individuals. The model is also constructed to match facts about trade between countries of differing income levels. Similar to Markusen (1986), in both the model and data, rich countries trade very similar goods with one another, but trade different goods with poor countries. Yet poor countries also trade very different goods with one another. Likewise, for poor countries there are many goods that are only imported or only exported from a given partner, while this is less true for rich countries. These aspects of the data motivate the need for non-homotheticities, as absolute income levels seem to be an important determinant of trade.

We then ask if the non-homothetic nature of this model generates qualitatively new predictions for the gains from international trade compared to existing homothetic models. In order to make this comparison, we make use of the main result in Arkolakis, Costinot and Rodriguez-Clare (2012), referred to hereafter as ACR. Their result is that, under some specific conditions, the predicted gains from trade can be computed by a simple formula that is only a function of readily observable statistics from the data. Hence, any model that matches those statistics predicts gains from trade given by their simple formula. Their main result is that most of the widely used models of international trade satisfy their conditions and, therefore, have the same predicted gains from trade whenever they match those statistics.
However, the class of models considered in ACR does not include models with non-homotheticities. Therefore, the gains predicted by our model do not, in general, coincide with those predicted by homothetic models. Our main question, then, is quantitative: how large is the difference between the two? Our strategy for answering this question is to apply the ACR calculation to the output from our model and compare that to the exact gains that we can get by changing trade costs in the model\(^1\). In this way, the ACR calculation is useful because it stands in for any of a large class of homothetic models, including versions of our model that remove the non-homotheticity.

Our main finding is that the difference between these two measures is large for some countries and that the relationship between them is systematic. Homothetic models overestimate gains for some countries and underestimate them for others. Countries with larger populations and higher productivities tend to be overestimated. The United States and Japan respectively have a 1.9% and 1.6% gain in the non-homothetic model, but are predicted to have gains of 2.2% and 2.0% in the homothetic model. Meanwhile, smaller and less productive countries tend to be underpredicted. Spain has a 4.3% gain in the non-homothetic model but is predicted to have a 3.4% gain by the homothetic model. However, not all countries exhibit large differences. For instance, India and China, which have very large populations and low productivity, have gains of 2.8% and 1.5% in both the non-homothetic model, and in the ACR calculation. Our interpretation is that the effects of their large population and low productivity offset one another.

To build intuition for these results, we provide a simple, two country non-homothetic model of trade that can be solved analytically. In this environment we are able to prove this relationship analytically. When one country is larger or more productive than the other, their real income is relatively higher, which generates a systematic bias in the ACR calculation due to the non-homotheticity. We are able to sign this bias based on the relative sizes and productivities of the countries. Moreover, we show that, when the countries are equal, this bias disappears and the gains from trade in the non-homothetic model and the ACR equation (and, therefore, the homothetic model) exactly coincide. In this model, we show that the bias is exactly equal to the elasticity of relative income with respect to changes in trade costs.

This highlights our main conclusion, which is that non-homotheticities are important when studying countries that vary substantially in size and income. The difference is more quantitatively important the larger is the difference between the countries. Moreover, our results suggest that further study of the nature of non-homotheticities may, in fact, be

\(^{1}\)We model all trade costs as iceberg transportation costs rather than tariffs. Throughout the paper, when we say "gains from trade" we mean comparing observed levels of trade with autarky.
informative for predicting gains from trade.

1.1 Literature Review

This paper draws upon the large literature on models of trade with non-homotheticities. A recent study by Fieler (2011) uses a model in which goods vary in their income elasticity of demand to match the patterns of trade across countries of different level of output and population, and studies its relationship to a standard gravity model. In the model, poor countries concentrate consumption on goods with low income elasticity, while rich countries consume high elasticity goods. This model is then used to assess the effects of productivity shocks to countries of different incomes. Our model shares many of the properties of this model, such as the pattern of consumption by income level, so that we are also able to match these facts about the pattern of international trade. We then ask, in this model, which is consistent with these empirical observations, are the gains from trade different than in a model with homothetic preferences?

Markusen (2011) provides a detailed analysis of results from models of non-homothetic international trade. His emphasis is on the role that per capita income plays in determining international trade flows. These roles are highlighted in papers that use models with non-homothetic preferences to match facts about the price and quality of goods traded between countries of differing income levels. Simonovska (2011) uses non-homotheticities to match the observed relationship between the income levels of different countries, and the prices of their tradable goods. Fajgelbaum, et al (2011) use a model of vertical product differentiation to match facts about the quality differences of products exported from rich and poor countries, and find that the gains from trade liberalization vary across the profile of income levels within each country due to non-homotheticities. Choi, et al (2009) use a model of within-country income differences and non-homotheticities to match patterns of trade between countries with differing income distributions.

Matsuyama (2000) shares some of the structural features of the model we develop, such as a positive relationship between the income of an individual and the number of varieties consumed. This feature again allows us to match some of the patterns of bilateral trade between countries of different income levels.

Our model has some similarities with Markusen (1986) and Markusen and Wigle (1990) regarding trade flows. In their models, world trade is divided between a pair of rich, northern countries that takes the usual New Trade form and trade between these northern countries and a poorer partner in the south, which takes a Ricardian-type of trade. Our enriched
model could easily boil down to a similar structure, if we were to avoid the production of the luxury good in the poorer countries. Instead, we allow these countries to produce some of the more luxurious goods. This makes a great difference between the structure of production in our model and theirs.

The paper is organized as follows: Section 2 describes the data and the facts. Section 3 develops a simple model of trade and non-homotheticities and derives some analytical results. Section 4 develops an enriched model of trade matching facts from Section 2. Section 5 has the quantitative exercise comparing gains in the non-homothetic model and the class of homothetic models considered by ACR. Section 6 concludes.

2 Data

To motivate the usefulness of non-homothetic models of trade, we first demonstrate some facts that homothetic models of trade cannot replicate. We analyze disaggregated trade data from Comtrade, using the 5,227 6 digit Harmonized System (HS6) categories from 2005. Our goal in using HS6 is to have very narrowly defined categories.

2.1 Three Facts and Three Examples

We want to first establish three facts about bilateral trade:

1) Trade among pairs of high income countries (G7 countries) is characterized by a "hump shaped" relationship between their relative income and how similar their imports and exports are.\(^3\)

2) This relationship disappears for low income countries (BRICS). The similarity of their imports from and exporters to their trading partners do not depend on their relative income levels.

3) The number of goods categories that are only imported or exported (but not both) is large for low income countries, and is smaller the higher is the country’s income.

\(^2\)We chose 2005 because it does not overlap with the financial crisis that many countries experienced in the latter part of that decade.
\(^3\)That is, countries with similar income levels trade more similar goods with one another than they do with countries that are either richer or poorer.
It is important here to notice that homothetic models of trade will not be able to match any of these three facts. Homothetic utility and production functions eliminate any role for income differences in consumption and production patterns. That is, by their nature, wealthier consumers (or countries, in this context) consume proportionally more goods than poorer consumers.

These results are similar to those in Markusen (1986): trade between rich countries is characterized by high levels of intra-industry trade, while trade among poor countries is not. To this we add that within the set of rich countries, the degree of similarity of imports and exports depends on the trading partners’ relative incomes. Furthermore, we add an element related to the extensive margin: countries at low income levels have zero trade in many categories that they import, while they do export that category.

Before formally establishing the 3 facts that we are interested in, we depict three examples of bilateral trade between trading partners to graphically illustrate these facts. The three relationships are 1) Germany and France (two high income countries); 2) France and Russia (one high income and one middle income); and 3) Russia and Turkey (two middle income countries). Figures 1, 2 and 3 respectively show each of these bilateral relations. Each figure contains a scatter plot in which each point is an HS6 category. The units are the logarithm of the value plus one, in order to display the categories with zero trade volume. Furthermore, in each figure the correlation coefficient is written and the Grubel-Lloyd index (after Grubel and Lloyd, 1971).^4

[Figures 1, 2 and 3 around here]

a) In Figure 1, we depict trade between France and Germany. We observe the following: most categories are both imported and exported, and being intensively exported is highly correlated with being intensively imported.

b) In Figure 2, we show trade between France and Russia. There are many more trade categories with zero trade volumes, and there are many categories that one country exports but does not import. Furthermore, the relationship between how intensively goods are imported and exported has disappeared.

c) Finally, Figure 3 shows trade between Russia and Turkey. Again, there is very little relationship between the import and export intensity of different product categories. Also, trade is dominated by categories that one country exports, but does not import.

^4We take correlation to be the most relevant measure of the similarity of imports and exports. A coefficient from a linear regression, for example, would be confounded by the balance of bilateral trade.
2.2 Establishing the Facts

To demonstrate the first fact, we use trade data from the OECD countries excluding Turkey and Mexico. First, for each country pair we compute the correlation of imports and exports in the 5,227 HS6 categories. We then take the set of these correlations and run the following regression:

\[
corr(I_{x-y}, X_{x-y}) = \alpha + \beta_1 \frac{GDP_{cx}}{GDP_{cy}} + \beta_2 \left( \frac{GDP_{cx}}{GDP_{cy}} \right)^2 + \varepsilon_{x,y}
\]

The results of this regression are in the first panel of Table 1. The estimated coefficients imply that \(\beta_1 > 0, \beta_2 < 0\) (which indicates a hump shape) and they are both significant at 1 percent level. Furthermore, the implied maximum, \(\frac{\beta_1}{2\beta_2} = 1.07\). Hence, we interpret this to mean that, within the set of high income countries, the further a country pair’s ratio of incomes is from 1, the less similar are their imports and exports.

For the second fact, we show that low income countries exhibit no such relationship between the correlation of imports and exports and the relative income of the trading partner. In order to analyze this, we run the same regression again. The results of this regression are in the second panel of Table 2. We find that neither of the two coefficients is significant at the 5% level.

[Table 1 around here]

For the third fact, Figure 4 shows, for each bilateral pair of countries, the number of goods either imported or exported (but not both) divided by the total number of traded goods in that bilateral relationship plotted against the income of the per capita GDP of one of the trading partners. The point is colored red if the other partner is in the OECD (except Mexico and Turkey), and is labeled blue otherwise. We see that the trade of poor countries is dominated by categories that only have positive trade flows in one direction. For rich countries, this is true to a lesser extent.

[Figure 4 around here]

We use these three facts to motivate the model developed in later sections. The first fact shows that countries tend to export and import more similar goods the more similar they are. In the next section we develop a very stylized model of bilateral trade with non-homotheticities where we capture this simple idea, and we analytically compare the gains
from trade implied by this model to those that would be implied by standard homothetic models.

In the following sections we expand this model in order to account for the second and third facts, and analyze the gains from trade of a richer, calibrated model.

3 Welfare Gains and Non-Homothetic Preferences

In this section we develop a simple, yet useful model to think about the role that non-homothetic preferences play in the analysis of gains from trade. The section is divided into three parts. We first develop a simple, stylized, two country model consistent with the first fact discussed previously. Importantly, in this model the two countries are asymmetric in their productivity levels. Second, we analytically compare the results with the non-homotheticity to homothetic models (described by the ACR calculation). Third, we prove two theorems about the relationship between the gains from trade predicted by this model and those predicted by the ACR calculation. We find that the two coincide for countries with the same income level, but disagree when the countries have different income levels. In particular, the ACR calculation overestimates gains from trade for the poor country, and underestimates the gains for the rich country.

3.1 A Simple Model with Non-Homothetic Preferences

We develop a static, 2 equal-sized country model with a continuum of goods. The representative household in country \( L \) chooses patterns of consumption according to

\[
\begin{align*}
\max_{j} & \left( \int_0^M (c(j) + j)^{\rho} dj \right)^{\frac{1}{\rho}} \\
\text{st : } & \int_0^M p(j)c(j) dj \leq w_l
\end{align*}
\]

where \( M \) is a very large constant.

Next, there is a competitive producer of each good \( i \) in each country. In country 1, that firm chooses how many inputs from the domestic market to purchase \((x_{1,1})\) and how many foreign ones, \((x_{1,2})\), in order to maximize profits. In order to get one unit of the country 2 good, the country 1 producer needs to purchase \((1 + \tau)\) units.

\[
\begin{align*}
\max_{j} & p_1(i)c_1(i) - q_1(i)x_{1,1}(i) - q_2(i)x_{1,2}(i) \left( 1 + \tau \right) \\
\text{st : } & c_1(i) = (\alpha_{1,1} x_{1,1}(i) \mu + \alpha_{1,2} x_{1,2}(i) \mu)^{\frac{\rho}{\mu}}
\end{align*}
\]
where $\alpha_{1,1}$ and $\alpha_{1,2}$ determine the shares of inputs from each country, and $\mu \in (0, 1)$ governs the elasticity of substitution. The problem for the competitive producer in country $2$ is symmetric.

We will assume throughout that $\alpha_{1,1} = \alpha_{2,1} = (L_1 z_1)^{1-\mu}$ and $\alpha_{1,2} = \alpha_{2,2} = (L_2 z_2)^{1-\mu}$ so that we maintain the following two properties: 1) if a country splits in two, the consumption from the other country is still the sum of the two countries separately and 2) if the other country doubles size but halves productivity, consumption stays the same. It is easily to prove that the stated conditions imply these two properties.

Finally, there is a continuum of competitive intermediate goods producers in country $1$ that chooses to maximize profits according to

$$\max q_1(i) \left( x_{1,1}(i) + x_{2,1}(i) (1 + \tau) \right) - w_1 l_1(i)$$

$$st : x_{1,1}(i) + x_{2,1}(i) (1 + \tau) \leq z_1 l_1(i)$$

We allow the two countries to differ in productivity $z_i$ and population size $L_i$.

This preferences' structure implies a cutoff

$$J = c(0) \left( \frac{p(0)}{p(J)} \right)^{\frac{1}{1-\rho}} = (c(j) + j) \left( \frac{p(j)}{p(J)} \right)^{\frac{1}{1-\rho}} \text{ for all } j < J$$

(3)

This cutoff demonstrates the role of non-homotheticities. Figure 5 shows the pattern of consumption between the two countries. A useful property of the model is that $p_1(i) = p_1(j) \equiv p_1$ for all $i$. This follows from the facts that all goods have the same marginal cost, and all markets are competitive. Figure 6 shows the pattern of trade between the two countries: the rich country enjoys a larger set of goods to be consumed than the poor one, and hence the poor country produces some goods that are not consumed domestically. In Figure 7 we show the correlation that is implied by our model as a function of relative incomes. As it was shown in the data, the maximum is exactly $1$ when the two countries have the very same productivity.

Equilibrium wages are such that the labor market clears

$$\int_0^{J_1} x_{1,1}(i) di + \int_0^{J_2} x_{2,1}(i) di (1 + \tau) = z_1$$

(4)

and trade balances.

\footnote{We always have $M$ being so large that $J < M$.}
\begin{equation}
\int_0^{J_1} q_2(i)x_{1,2}(i)di = \int_0^{J_2} q_1(i)x_{2,1}(i)di
\end{equation}

We now characterize the equilibrium of the model. First, we show how welfare is linked to the non-homotheticity of the demand system, by showing that total welfare has a one-to-one mapping to the set of goods consumed.

**Lemma 1** Welfare for country \( i \) in the model is given by \( \frac{w_i}{p_i} = \frac{J_2^2}{2} \)

**Proof.** See Appendix 7.1

Second, we show that the country that has larger productivity is also the richer country, as measured by total income. We further show that the ratio of productivities is indeed larger than the ratio of incomes.

**Lemma 2** Suppose \( L_1 = L_2 \). If \( z_1 \geq z_2 \) then \( w_1 \geq w_2 \) and \( \frac{z_1}{z_2} \geq \left( \frac{w_1}{w_2} \right)^{\frac{1}{\mu}} \).

Suppose \( z_1 = z_2 \) and \( L_1 = 1 \). Then, \( L_1 \geq L_2 \) implies \( w_1 \geq w_2 \) and \( \frac{L_1}{L_2} \geq \left( \frac{w_1}{w_2} \right)^{\frac{1}{1-\mu}} \)

**Proof.** See Appendix 7.2.

In order to proceed, we assume some parameter restrictions in the model. In particular, we assume that the ratio of productivities is large enough. The exact specification is in Condition 1.

**Condition 1** Parameters satisfy \( z_1^{1+\mu}l_1^{1-\mu} > (1 + \tau)^{2\mu} z_2^{1+\mu}l_2^{1-\mu} \)

The previous condition is useful in order to characterize further results regarding the bias of welfare gains.

Finally, we show that following trade liberalization, countries with higher productivity benefit less than low productivity countries.

**Lemma 3** The derivative of the ratio of wages increases with increases in trade costs as long as Condition 1 is satisfied.

**Proof.** See Appendix 7.3

This analysis of the model is very useful in order to be able to properly derive results that compare our model to ACR. We do so in the next subsection.
3.2 Welfare Gains from Trade

To understand the role that non-homotheticities play in the welfare gains from trade in the model, it is useful to compare the gains implied by our model to those that would be computed in a homothetic model of trade. The computation in ACR provides a benchmark for a large class of homothetic models. The basic comparison that we will make throughout is the following: what are the exact computed gain from trade implied by the model compared to the ACR computation applied to the output of our model?

The ACR computation requires two statistics. If $X_{ij}$ is the final use by country $j$ of goods produced in country $i$, then the two statistics are: 1) the import penetration ratio, $1 - \lambda_{ij} = X_{ij}/X_{jj}$, and 2) the trade elasticity, $\varepsilon^{ii}_j = \partial \ln (1 - \lambda_{ij}) / \partial \ln (1 + \tau_{ij})$. The ACR computation says that the gains from being open to trade (that is, the welfare difference between the observed level of trade and autarky) is given by:

$$W^{ACR} = 1 - \lambda_{ij}^{-1/\varepsilon^{ii}_j}$$

The main result of ACR is that this computation coincides with those that can be computed in any trade model whose model output match these two statistics if those models meet the following criteria: trade is balanced, profits are a constant fraction of revenue (perfect competition or constant markups, for instance) and the "import demand system is CES". This last assumption is that $\varepsilon^{ii}_j = \varepsilon < 0$ if $i = i'$ and is 0 otherwise. That is, if a country opens to trade with one country, the proportion of goods consumed from all other countries relative to one another is unchanged. This is an implication of models with homothetic preferences. In our environment, the first two assumptions are certainly satisfied. The third assumption certainly is not due to the non-homotheticity.

We first compute the ACR measure applied to this model.

**Proposition 4** The ACR formula applied to our model implies welfare gains given by

$$\log(\hat{W}^{ACR}) = \frac{1 - \mu}{\mu} \frac{1}{1 - \frac{w_2}{w_1} (1 + \tau)} \frac{\partial w_1}{\partial (1 + \tau)} \log \left( \frac{1 + \frac{L_2 z_2}{L_1 z_1}}{1 + \frac{L_2 z_2}{L_1 z_1} \left( \frac{w_1}{w_2} \frac{1}{1 + \tau} + z_2 \right)} \right)$$  \hspace{1cm} (6)

**Proof.** See Appendix 7.4.  

We then compare this measure to the actual welfare gains in the model.
Proposition 5  Welfare gains in our model are given by

\[ \log(\hat{W}) = \frac{1 - \mu}{\mu} \log \left( \frac{1 + \frac{L_2 z_2}{L_1 z_1} \left( \frac{w_1}{w_2} \frac{1}{1 + \tau} \frac{z_2}{z_1} \right)}{1 + \frac{L_2 z_2}{L_1 z_1} \left( \frac{w_1}{w_2} \frac{1}{1 + \tau} \frac{z_2}{z_1} \right)} \right) \]  

(7)

Proof. See Appendix 7.5. 

From equations (6) and (7) we then find the following:

\[ \frac{\log(\hat{W}^{ACR}) - \log(\hat{W})}{\log(\hat{W}^{ACR})} = \frac{\partial \frac{w_1}{w_2}}{\partial (1 + \tau)} \frac{1 + \tau}{w_1/w_2} \]

That is, the bias in the estimated gains in welfare (approximated by \( \log(\hat{W}) \) and \( \log(\hat{W}^{ACR}) \)) is exactly equal to the elasticity of relative wages with respect to changes in trade costs. This allows us to prove the following theorems about the sign of the bias.

Theorem 6  The ACR formula coincides with ours if \( L_1 = L_2 \) and \( z_1 = z_2 \).

Proof. Using Lemma 3

Therefore, even though the model does not satisfy the assumptions of the ACR result, welfare computed within the model nonetheless coincides with that implied by the ACR result if the productivity levels of the two countries are the same. Hence, when countries have the same income level, the ACR calculation is correct even in this model. However, when productivity (and therefore, income) levels and population sizes differ between countries the ACR computation is biased, as described in the following theorem:

Theorem 7  The ACR formula overestimates the welfare of the rich and large country and underestimates the welfare of the poor and small country as long as Condition 1 is satisfied.

Proof. Using Lemma 3

The intuition behind this theorem is simple. A fall in trade costs makes each individual better off by consuming more goods. However, trade must balance. The poor individual gets more marginal utility for the same increase in consumption, and hence his welfare increases relatively more. This translates in a higher relative increase in her wage (see Lemma 3). This increase in wage implies a larger relative increase for the cost of imported goods for the rich individual, and hence, a relatively smaller increase in his imports. This, in turn, translates into a smaller elasticity (see Proposition 4), which biases the ACR prediction for welfare upward.
This simple model is useful in establishing analytical, qualitative predictions. In the next section, we wish to quantify these differences to determine if using non-homotheticities gives meaningfully different predictions for gains from trade. We will expand our model to include many things: goods that are specific in their country of origin, within-country income inequality, and multiple countries. We will show that the expanded model is consistent with all three of the facts described in Section 2, and will demonstrate for what countries and in what situations the model with non-homotheticities disagrees significantly with the ACR calculation.

4 Model of Trade with Non-Homothetic Preferences

4.1 Household

There are $N$ different countries, each with different population sizes $L_j$. We assume there is a continuum of differently endowed households in each country. Each household $k$ in country $m$ has labor endowment $l_m(k)$. We further assume that $l_m$ follows a truncated Pareto distribution.

Households consume two types of goods: country specific goods (denoted with $S$ subscripts) and luxury goods (denoted with $E$ subscripts). The set of country specific goods is partitioned into $N$ sets that are each assigned to a different country. Those goods are only produced in those countries, but are purchased by all other countries.

Household $k \in [0, L_m]$ in country $m$ solves the following problem.

$$\max \left( \int_0^M (c_{L,m}(j,k) + j)^p dj + \sum_{n=1}^N \int_0^{A_n} (c_{S,n,m}(i_n,k))^p di_n \right)^{\frac{1}{p}}$$

$$\text{st} : \int_0^M p_{L,m}(j)c_{L,m}(j,k) dj + \sum_{n=1}^N (1 + \tau_{n,m}) \int_0^{A_n} p_{S,n,m}(i_n)c_{S,n,m}(i_n,k) di_n = w_m l_m(k)$$

where country specific goods $i_n \in (0, A_n)$, are produced in country $n$, and $[0, M]$ is the set of potential varieties$^6$ of luxury goods produced in the world. The iceberg transportation cost between countries $n$ and $m$ is $\tau_{n,m}$.

$^6$As in the previous model, $M$ is a very large, exogenous constant.
4.2 Country Specific Good Firms

Country specific goods are produced by competitive firms in each country. Such a firm in country $m$ producing good $i_n$, solves the problem

$$\max \pi_{S,m}(i_n) = \max \sum_{m=1}^{N} (1 + \tau_{m,n}) p_{S,m,n}(i_n) \int_{0}^{L_n} c_{S,m,n}(i_n,k)dk - w_m l_{S,m}(i_n)$$

$$\text{st} : \sum_{m=1}^{N} (1 + \tau_{m,n}) \int_{0}^{L_n} c_{S,m,n}(i_n,k)dk = z_{S,m}(i_n)l_{S,m}(i_n)$$

where $z_{S,m}(i_n)$ is the country-specific efficiency of the variety. We assume throughout that $\forall i,j \in (0,A_m), z_{S,m}(i) = z_{S,m}(j)$. Hence, although all such firms have the same marginal cost, the number of firms that operate in each country (and in the world) is determined in equilibrium.

4.3 Luxury Goods’ Production - Final Producer

There is a competitive final firm $j$ for each luxury variety in each country that produces using domestic and foreign goods according to a standard Dixit-Stiglitz aggregator.

$$\pi_{L,m}(j) = \max p_{L,m}(j) \int_{0}^{L_m} c_{L,m}(j,k)dk - \sum_{n=1}^{N} \int_{s \in \Omega_n,j} q_{n,m}(j,s)x_{n,m}(j,s)(1 + \tau_{n,m})d$$

$$\text{st} : \int_{0}^{L_m} c_{L,m}(j,k)dk = \left( \sum_{n=1}^{N} \int_{s \in \Omega_n} x_{n,m}(j,s)^{\rho}ds \right)^{\frac{1}{\rho}}$$

where $\Omega_n$ is the varieties in country $n$ producing the intermediate good used in the production of good $j$, and $\rho$ governs the elasticity of substitution among varieties.
4.4 Luxury Goods’ Production

We assume each variety of intermediate good is operated by a perfectly competitive firm that solves the following problem.

$$\max \pi_{X,m}(j, s) = \max \sum_{n=1}^{N} q_{m,n}(j, s)x_{m,n}(j, s)(1 + \tau_{m,n}) - l_{X,m}(j, s)w_m$$

$$st : \sum_{n=1}^{N} x_{m,n}(j, s)(1 + \tau_{m,n}) = z_{X,m}l_{X,m}(j, s)$$

4.5 Market Clearing Conditions and Trade Balance

Finally, the market for labor clears:

$$\int_{0}^{A_m} l_{S,m}(i)di + \int_{0}^{M} \int_{s \in \Omega_n} l_{X,m}(j, s)dsdj = \int_{0}^{L_m} l_{m}(k)dk$$

and trade balances:

$$\sum_{n=1}^{N} x_{m,n}(j, s)(1 + \tau_{m,n}) + \sum_{n=1}^{N} (1 + \tau_{m,n}) \int_{0}^{L_n} c_{S,m,n}(i_n, k)dk$$

$$= \sum_{m=1}^{N} x_{n,m}(j, s)(1 + \tau_{n,m}) + \sum_{m=1}^{N} (1 + \tau_{n,m}) \int_{0}^{L_n} c_{S,n,m}(i_n, k)dk$$

4.6 Definition of Equilibrium

In this section we define what an equilibrium is in this economy.

**Definition 8** Given the distribution of skills, the set of iceberg costs, $\tau_{n,m}$, the sets of varieties operated in each country, $\Omega_m$ and the mass of country specific goods, $A_m$, an equilibrium is a vector of functions of prices, $(q_{n,m}(j, s), w_m, p_{E,m}(j), p_{S,m,n}(i_n))$ and a vector of functions of allocations $(c_{S,m,n}(i_n, k), c_{L,m}(j, k), x_{m,n}(j, s), l_{X,m}(j, s), l_{S,m}(i_n), W_m(k))$ such that:

a) Households solve problem (8)

b) Country-specific competitive final firms solve problem (9)

c) Final prodcers of luxury goods solve problem (10)
d) Intermediate luxury goods’ producers solve problem (11)
e) No firm makes profits
f) Markets of labor clear (equation 12)
g) Trade balances (equation 13)

The following proposition characterizes the equilibrium:

**Proposition 9** The equilibrium of the model is given by

\[ p_{L,m} = \left( \sum_{n=1}^{N} \Omega_n \left( (1 + \tau_{n,m}) \frac{w_n}{z_{X,n}} \right)^{\frac{\rho}{1-\rho}} \right)^{\frac{1}{\rho-1}} \]

\[ W_m(k) = w_ml_m(k); p_{S,n,m}(i_n) = \frac{w_n}{z_{S,n}(i_n)}; q_n,m = \frac{w_n}{z_{X,n}} \]

\[ c_{L,m}(j,k) = (J_m(k) - j); c_{S,n,m}(i_n, k) = J_m(k) \left( \frac{p_{L,m}}{p_{S,n,m}(i_n) (1 + \tau_{n,m})} \right)^{\frac{1}{\rho}} \]

\[ \int_{s \in \Omega_s} \int_{0}^{+\infty} x_{n,m}(j, s) dj ds = \frac{\Omega_n^{1/2} \int J_m^2(k) dk}{((1 + \tau_{n,m}) q_n,m)^{1/\rho} \left( \sum_{r=1}^{N} \Omega_r \left( (1 + \tau_{r,m}) \frac{w_r}{z_{X,r}} \right)^{\frac{\rho}{1-\rho}} \right)^{1/\rho}} \]

\[ l_{X,m}(j, s) = \frac{\sum_{n=1}^{N} x_{m,n}(j, s) (1 + \tau_{m,n})}{z_{X,m}} \]

\[ l_{S,m}(i_n) = \frac{\sum_{m=1}^{N} (1 + \tau_{m,n}) \int_{0}^{L_m} c_{S,n,m}(i_n, k) dk}{z_{S,m}(i_n)} \]

where \( J_m(k) = \sqrt{\left( \frac{p_{L,m}^{\frac{1}{\rho}} \sum_{n=1}^{N} \int_{0}^{A_n} (p_{S,n,m}(i_n) (1 + \tau_{n,m}))^{\frac{\rho}{1-\rho}} di_n \right)^2 + 2W_m(k)}} \]

and the wage satisfies equation 12

**Proof.** See Appendix 7.6. ■

In the next section, we calibrate this economy to match salient features of bilateral trade data, and perform a quantitative exercise showing the similarities and differences between this type of models and standard ACR predictions.

## 5 Quantitative Exercise

In this section we calibrate the economy of the previous section and perform a quantitative exercise assessing the importance of non-homothetic preferences for welfare gains from trade.
5.1 Calibration

Given a list of countries, our economy is governed by the following parameters: \( \{\alpha_n, b_{0,n}, z_{X,n}, z_{S,n}(i), \Omega_n, A_n, \{\tau_{n,m}\}_m=1^N, L_n\}_n=1^N \) and \( \rho \). Next we explain how we calibrate the parameters. Table 2 summarizes the calibration.

We use a truncated Pareto distribution and calibrate the \( \alpha_n \) to match the Gini index. The data for the Gini index is taken from the World Bank.\(^8\)

We set \( z_{x,n} = z_{s,n} \) for all \( n \) and calibrate it to match GDP per capita in each country. The data for GDP per capita in each country is taken from the Penn World Tables. We calibrate \( L_n \) to match the population of each country, relative to the US. The calibration of \( \Omega_n \) follows the properties of the shares of goods that are bought from each country in the Simple Model. This implies that \( \Omega_n = (L_n z_n \int l^{-\alpha_n} dl)^{1-\rho} \). We set the country-specific shares of each goods, \( A_n \) to be a fraction of \( \Omega_n \). The fraction is constant across countries and matches the fraction of goods that the US exports, but doesn’t import (approximately 0.2%).

We calibrate the \( \tau_{n,m} \) matrix in order to match the entire import matrix among all countries in the model. Since our world is only a fraction of the total world, we make total trade flows to actually match each country’s imports over GDP in the data. This data is taken form the World Bank, although the ratios use data from Comtrade.

We set \( \rho = 0.90 \) in order to match the trade elasticity that one would get from a gravity regression. Anderson and Van Wincoop (2004) show that this number is between \(-5\) and \(-10\). The computed elasticities in our model are in the range of \(-5.03\) to \(-10.63\).

5.2 Model Performance

We now compare our model’s results to the empirical relationships discussed in Section 2. First, in order to have comparable magnitudes, we discretize the space of luxury goods and

\(^7\)In this exercise, we calibrate the model to the following countries: United States, United Kingdom, Germany, France, Canada, Japan, Italy, Spain, Turkey, India, China, Brazil, South Africa, Russia and Mexico. These countries have been chosen since they are the G8+5, plus Spain and Turkey. We added these last two because we wanted more countries that are middle income, but geographically close to Europe.

\(^8\)Unfortunately, we do not have the Gini coefficients for all the countries for 2005. For those that this coefficient is not available, we use the Gini index for the closest year in which it is available.
account how much each country consumes from the inputs of the others. We use the luxury good final producer’s problem and solve for \( j = 0 \):

\[
\int x_{m,m}(0,k)dk = \frac{\int J_m(k)dk}{\left( \sum_{\Omega_n} \left( \frac{q_m}{q_n} \frac{1}{1+\tau_{n,m}} \right)^{\frac{p}{p-1}} \right)^\frac{1}{p}}
\]

Then, using the first order condition of \( x_{m,m}(j) \) we get that:

\[
x_{m,m}(j,k) = \begin{cases} 
  x_{m,m}(0) \left( \frac{J_m(k)-j}{J_m(k)} \right)^{\frac{1}{p}} & \text{if } j \leq J_m(k) \\
  0 & \text{if } j > J_m(k)
\end{cases}
\]

Then, bilateral trade flows are given by

\[
x_{m,n}(j,k) = x_{m,m}(j,k) \left( \frac{q_m}{q_n (1 + \tau_{m,n})} \right)^{\frac{1}{p}}
\]

We then compute the total trade of each category \( j \). We set a grid point of 4500 points that covers up to the highest \( J_m \) in the world economy.

Then, we compute the correlation between any pair of two countries’ trade flows. Following the exercise from Section 2, we regress this correlation measure against relative GDP per capita and relative GDP per capita squared, in which one of the trading partners is always a rich country.

Table 3 shows the comparison between the model and the data:

[Table 3 around here]

Given that nothing in the parameterization of the model is targetted, the model qualitatively replicates the regression coefficients from the data quite well. For the regression with only rich countries, it does match the hump-shape relationship in the data. The coefficient on the linear term is positive, and on the quadratic term is negative, and they are both significant. The implied maximum of the hump is higher than the data at 1.29. This is mostly driven by the fact that the coefficient on the linear term is much higher than in the data. Similarly, the same regression for the poor countries delivers no relationship between relative income and the correlation measure, as in the data.

Finally, the model performs well on the third fact discussed in Section 2. Due to the presence of country-specific goods, there are many good categories that are only imported or only exported. Because of the non-homotheticity, this is particularly true for the poor
countries. In the data, we say that there were a large number of goods categories that only had trade flows in one direction, and that this was particularly true for low income countries. Figure 8 shows this relationship for both groups in the model:

![Figure 8 around here](image)

The model does well in matching the magnitude of the relationship for both groups. Comparing Figure 8 to Figure 4, we see that there is less dispersion in the model than in the data, but the patterns are quite clear: low income countries have a very large number of goods with trade in only one direction, and this relationship is roughly constant across the income levels of their partners. For high income countries, then have a somewhat smaller number of categories with trade in one direction, and the number of such goods is decreasing in the income of their partner. These relationships are maintained in the output from the model. Again, the calibration does not target these facts.

### 5.3 Welfare Gains Compared to ACR

In this section we compute the welfare gains from trade in our non-homothetic model, and compare them to the results from a class of homothetic models represented by the ACR computation applied to our model’s output. Table 4 summarizes the quantitative results of the measured welfare gains and of ACR methodology.

![Table 4 around here](image)

In the first column, we have the welfare gains that our model delivers. The second column is the ACR formula applied to the model-generated data for that country. The third gives the percentage difference. The fourth and fifth column give the productivity and population levels of each country relative to the US.

We see wide variations in the disagreement between the two measures. The main pattern is that identified in the simple model: countries with high productivities and large populations are typically underestimated by the homothetic model, and the opposite is true for small and low productivity countries. The simple model further explains us how to aggregate the components- From Condition 1, we can see that productivity and population are aggregated as $\left(\frac{z_i}{z_j}\right)^{1+p} \left(\frac{t_i}{t_j}\right)^{1-p}$. However, actual productivity levels have two problems. First, they are
model specific, and hence this exercise could not be replicated with actual data. The second problem is that countries do not trade equally with the remaining countries.

In our model, productivity of each country is highly correlated with GDP per capita (0.96). Hence, instead of using productivity, we use GDP per capita. In order to address the second problem, we construct an "ideal GDP per capita of the trading partner". Given that the simple model is silent about how to aggregate the different countries, we weight by imports. In particular, for each country $i$, we use

$$\frac{z_i}{z_{j(i)}} \approx \frac{GDP_{pc_i}}{\sum_{j \neq i} GDP_{pc_j} \frac{l_j(1+\tau_{i,j})}{\sum_{k \neq m} l_k(1+\tau_{i,k})}}$$

For the measure of population, we use a similar strategy. In particular, for each country $i$, we use

$$\frac{l_i}{l_{j(i)}} \approx \frac{L_i}{\sum_{j \neq i} L_j \sum_{k \neq m} l_k(1+\tau_{i,k})}$$

Finally, the simple model also highlights the role of iceberg costs. This pattern is made clear by the following simple regression over each country $j$:

$$\% \text{ Difference}_i = \alpha + \beta_1 \left( \frac{z_i}{z_{j(i)}} \right)^{1+\rho} \left( \frac{l_i}{l_{j(i)}} \right)^{1-\rho} + \varepsilon_j$$

The simple model predicts that the difference between the models is negatively related to the measure of productivity and population. Table 10 shows the coefficient and significance of the regression.

[Table 5 around here]

This shows that the results of the analytical model carry through to the quantitative model. This is why we conclude that the degree of overestimation of homothetic models is increasing in the population and productivity of the country.

5.3.1 Decomposition

In the analytical section, we showed that both population size and productivity can bias the ACR calculation in the model with non-homotheticities. Now we show the magnitudes of each in two different ways. First we remove population differences from the model. We recalibrate\(^9\) the model with population constant across countries and get the results summarized in Table 7:

\(^9\)See Table 6 for the parameter values used here.
We broadly see that the countries whose populations increased to U.S. levels had a reduction in the amount that the homothetic model underestimates their gains, while India and China had an increase. In fact, for both of them the homothetic model underestimates their gains when population differences are included, but overestimates them when population differences are removed.

Next, we consider the opposite case: hold productivity fixed across countries and allow population to vary. Recalibrating\(^{10}\) the model in that case gives us the results in Table 9:

These results are driven, to a large extent, by the effect of India and China both becoming as productive as the U.S. Essentially, this has very large effects on some countries that trade a large amount with those two countries (particularly the U.S. and Canada). First, notice that, like with holding population constant, when giving all countries U.S. productivity levels, those that increased their productivity had a reduction in the amount homothetic models underestimate their gains, and the opposite for those countries whose productivity decreased. Second, since there is less variation across countries the magnitude of the differences in general declined. Again, where this is not true is, for example, in Mexico, which has a large degree of trade with large countries, such as the U.S. Again, this demonstrates the effect of population differences on how homothetic models underestimate gains from trade.

Our last exercise is a decomposition of the effects of all the different parameters that vary across countries. Here we keep all parameters at their levels from Table 2, then, in each case, make one of those parameters constant across countries (without otherwise changing the calibration).

In the case of constant population, there are two notable features. First, the amount that the homothetic model underestimates gains goes down for European countries. This is consistent with the fact that all their populations increase (which is an increase in real income), and they mostly trade with one another. Second, like in Table 7, removing the large populations from India and China has a large effect on the countries they trade with.

\(^{10}\)See Table 8 for the parameter values for this case.
The case of constant productivity is similar to the constant population case. Constant productivity has a large effect on Canada, partly because of its large volume of trade with China and Mexico. Notice that the magnitudes of differences between the baseline case and the constant population case is very similar to that of the constant productivity case.

When all trade costs are removed, countries that were closed become relatively more richer, and those that were very open become relatively poorer. For example, Canada, a relatively open country, has a large increase in the amount that homothetic models underestimate its gains, while the U.S. has a decline. This pattern explains most of the changes in that case. The other two cases do not have a very large effect on the estimation of gains in the non-homothetic model.

6 Conclusion

This paper contributes to the literature on computing the gains from international trade by showing that models with non-homotheticities exhibit markedly different gains from trade than models without. We demonstrated this by developing a model with non-homotheticities, matching patterns of trade between countries, and comparing the gains from trade in that model to a measure that summarizes the gains in a large class of homothetic trade models. Our results demonstrate that homothetic models overstate the gains from trade for high income and large countries, and understate them for low income and small countries. For some countries, though not all, these differences are large.

We interpret our results as demonstrating that homothetic models (and the ACR calculation) are useful when comparing countries of similar income levels, but are not when comparing countries with very different income levels (such as the U.S. and China). Notice that the results for many of the European countries indicate that there is little difference between the predictions of our non-homothetic model and the class of homothetic models. Our theoretical results suggest that this is due to the fact that European countries mostly trade with one another, and they mostly have similar income levels. On the other hand, for the U.S., the ACR calculation and our model give quite different predictions. We interpret this as being due to the U.S.’s high level of trade with countries of lower income levels like Mexico and China.

In future work we hope to explore the role of micro level details about the export activities of firms in non-homothetic models. The ACR result suggests that, in homothetic models, more information about how firms make export decisions, and how their decisions respond to trade costs is irrelevant for computing gains from trade. In future work we wish to explore
the extent to which this is true in non-homothetic models. In particular, what aspects of firm-level decision making is important in the class of non-homothetic models? We consider this an important avenue of future research.

References


7 Appendix

This Appendix covers the proofs for all the Lemmas and Propositions of the paper. In most cases, due to symmetry, we only show the result for country 1, and that for country 2 follows by doing the appropriate changes.

7.1 Proof of Lemma 1

It follow from (1) and the cutoff rule (3), since \( \int_0^{+\infty} p(j)c(j) dj = p \int_0^J (J - j) dj = p \frac{J^2}{2} \leq W \) and \( U = \int_0^{+\infty} \log(c(j) + j) dj = J \log(J) \). Walras law implies the last holds with strict equality and total wealth of the household is given by the wage, \( W = w \).
7.2 Proof of Lemma 2

We start from
\[
c_1(i) = (z_1^{1-\mu} x_{1,1}(i)^\mu + z_2^{1-\mu} x_{1,2}(i)^\mu)^{1\over \mu}
\]
and the first order condition for \(x_{1,i}\),
\[
x_{1,1}(i) = x_{1,2}(i) \frac{L_1 z_1}{L_2 z_2} \left( \frac{w_2 z_1}{w_1 z_2} (1 + \tau) \right)^{1\over 1-\mu}
\]
and integrating over all the set of goods, we get that
\[
\int_0^{J_1} x_{1,2}(i) \, di = \frac{w_1}{p_1} \frac{1}{L_2 z_2} \frac{1}{1-\mu} \frac{1}{(\frac{L_1 z_1}{L_2 z_2})^{1-\mu} \left( \frac{w_2 z_1}{w_1 z_2} (1 + \tau) \right)^{1\over 1-\mu} + 1}^{1\over \mu}
\]
using the expression for \(p_1\) that arises from substituting the free entry condition and the first order conditions into (2)
\[
p_1 = \frac{w_2 (1 + \tau)}{L_2^{1-\mu} z_2^{1\over \mu}} \left( \frac{L_1 z_1}{L_2 z_2} \left( \frac{w_2 z_1}{w_1 z_2} (1 + \tau) \right)^{1\over 1-\mu} + 1 \right)^{1\over \mu}
\]
which, given symmetry, the expressions for \(\int_0^{J_2} x_{2,1}(i) \, di\) and \(p_2\) are very similar.

Finally, we make use of the labor market clearing equation. Using the expressions for \(\int_0^{J_1} x_{1,1}(i) \, di\) and \(\int_0^{J_2} x_{2,1}(i) \, di\) (similar to equation (15)) and the market clearing condition for labor (4) we find that
\[
\int_0^{J_1} x_{1,1}(i) \, di = \frac{z_1 \left( \frac{L_2 z_2}{L_1 z_1} \left( \frac{w_1 z_2}{w_2 z_1} \left( \frac{w_2 z_1}{w_1 z_2} (1 + \tau) \right)^{1\over 1-\mu} + 1 \right) \right)^{1\over 1-\mu} + 1}{1 + \tau} \frac{w_2 z_1}{w_1 z_2} \frac{1}{w_1} \frac{L_2 z_2}{L_1 z_1} \left( \frac{w_1 z_2}{w_2 z_1} (1 + \tau) \right)^{1\over 1-\mu} + 1}
\]
\[
L_1 = \frac{1}{\left( \frac{L_2 z_2}{L_1 z_1} \left( \frac{w_1 z_2}{w_2 z_1} \left( \frac{w_2 z_1}{w_1 z_2} (1 + \tau) \right)^{1\over 1-\mu} + 1 \right) \right)^{1\over 1-\mu} + 1} + \frac{w_2}{w_1} \frac{1}{\left( \frac{L_2 z_2}{L_1 z_1} \left( \frac{w_1 z_2}{w_2 z_1} (1 + \tau) \right)^{1\over 1-\mu} + 1 \right)}
\]
We write \( L = \frac{L_1}{L_2}, W = \frac{w_1}{w_2} \) and \( Z = \frac{z_1}{z_2} \).
\[
L_1 = \frac{1}{LZ} \left( \frac{W}{Z} \left( \frac{1}{1+\tau} \right) \right)^{\frac{\mu}{1-\mu}} + \frac{1}{W} \frac{1}{LZ} \left( \frac{W}{Z} (1+\tau) \right)^{\frac{\mu}{1-\mu}} + 1
\]  
(17)

We want to prove that if \( L = L_1 = 1 \), then if \( Z > 1 \to W > 1 \) as well. We prove it by contradiction, Suppose \( W < 1 \).
\[
\frac{1}{LZ} \left( \frac{W}{Z} \left( \frac{1}{1+\tau} \right) \right)^{\frac{\mu}{1-\mu}} + \frac{1}{W} \left( \frac{1}{LZ} \left( \frac{W}{Z} (1+\tau) \right)^{\frac{\mu}{1-\mu}} + 1 \right) = \frac{1}{W}
\]

Then, the term in the left hand side is smaller than 1, and the term in the right hand side is larger than 1, which cannot be. Hence, the result is proven.

The second result is that \( Z > 1 \to W^\mu < Z \). Suppose that \( \frac{Z}{W^\mu} < 1 \). Then, the left hand side of the previous equation is larger than one, and this would imply that \( W < 1 \). Hence, \( Z > W^\mu \).

For the second part of the Lemma, it is useful to rewrite equation (17).
\[
\frac{1}{L} \left( W \left( \frac{1}{1+\tau} \right) \right)^{\frac{\mu}{1-\mu}} + \left( \frac{1}{L} W^{\frac{\mu}{1-\mu}} \right)^2 = \frac{1}{W}
\]

Notice that in the previous equation, is equivalent to the one used for the first part of the Lemma, up to the reescalting of \( L \) for \( Z^{\frac{1}{1-\mu}} \).

And this concludes the proof."

7.3 Proof of Lemma 3

Using equation (17), by the implicit function theorem we get that the derivative is positive. In order to proceed, for simplicity, we keep using the same notation with \( Z \) and \( W \).
\[
\frac{\partial W}{\partial (1+\tau)} = \frac{1}{1+\tau} \left( \frac{(W - 1) S (2 + S (T - \frac{1}{T})) + (1 - S^2) \left( \frac{W}{T} - T \right)}{(S + T) \left( \frac{S}{T} + 1 \right) (ST + 1) (S + \frac{1}{T}) W} \right)
\]

\[
\times \frac{1}{TW^{\frac{\mu}{1-\mu}} \left( \frac{S}{T} + 1 \right)^2 + \frac{T}{W^{\frac{\mu}{1-\mu}} (ST + 1)^2} + (ST + 1)WS^{\frac{\mu}{1-\mu}}}
\]

where
\[ T = (1 + \tau)^{\frac{\mu}{1-\mu}} > 1 \]
and \[ S = \frac{1}{LZ} \left( \frac{W}{Z} \right)^{\frac{\mu}{1-\mu}} < 1 \]

Notice that among all the parts in the equation, the only one that determines whether or not the derivative is positive is

\[
A = (W - 1) S \left( 2 + S \left( T - \frac{1}{T} \right) \right) + (1 - S^2) \left( \frac{W}{T} - T \right)
\]
since all the other terms are positive.

We make use of equation (17), which implies that

\[
(W - 1) S = T \left( 1 - \frac{1}{LZ} \frac{W}{Z^{\frac{\mu}{1-\mu}}} \right)
\]

In order to find that

\[
A = T \left( 1 - \frac{1}{LZ} \frac{W}{Z^{\frac{\mu}{1-\mu}}} \right) \left( 2 + S \left( T - \frac{1}{T} \right) \right) + (1 - S^2) \left( \frac{W}{T} - T \right)
\]

Suppose that \( W > T^2 \). Then, \( A \) would trivially be positive. So, suppose it is not. In particular, assume that \( W = KT^2 \), where \( K < 1 \) is the exact value of their ratio.

Then, we can rewrite it as

\[
A = T \left( \left( 1 - K \frac{1}{LZ} \frac{T^2}{Z^{\frac{\mu}{1-\mu}}} \right) \left( 2 + S \left( T - \frac{1}{T} \right) \right) - (1 - S^2) (1 - K) \right)
\]

Notice that \( 2 + S \left( T - \frac{1}{T} \right) > 1 - S^2 \), which implies that as long as \( 1 - K \frac{1}{LZ} \frac{T^2}{Z^{\frac{\mu}{1-\mu}}} > 1 - K \), the term \( A \) is positive. In turn, this is holds when \( T < Z^{\frac{\mu}{1-\mu}} (LZ)^{\frac{1}{2}} \). Getting back to the original parameters, this implies that \( z_1^{1+\mu} l_1^{1-\mu} > (1 + \tau)^{2\mu} z_2^{1+\mu} l_2^{1-\mu} \).

Hence, we have proven that under the condition that \( z_1^{1+\mu} l_1^{1-\mu} > (1 + \tau)^{2\mu} z_2^{1+\mu} l_2^{1-\mu} \), the derivative of the ratio of wages to an increase in \( \tau \), is positive.

This concludes the proof. \( \blacksquare \)
7.4 Proof of Proposition 4

Following ACR (footnote 1),\(^{11}\) we proceed to compute the import penetration ratio.

\[
\lambda = 1 - \frac{w_2}{w_1} x_{1,2}(1 + \tau) = \frac{w_2 x_{1,1}}{w_1 x_{1,1}} = \frac{1}{1 + \frac{L_2 z_2}{L_1 z_1} \left( \frac{w_1}{w_2} \frac{z_2}{z_1} \right)^{1-\mu}}
\]

And then we compute the trade elasticity, \(\epsilon = \frac{\partial \ln \left( \frac{x_{ij}}{x_{ji}} \right)}{\partial \ln (1+\tau)}\). In our model, this ratio is given from equation (14):

\[
\frac{\partial \log \left( \frac{x_{1,2}(\tau)}{x_{1,1}(\tau)} \right)}{\partial \log (1+\tau)} = -\frac{1}{1-\mu} \left( 1 + \frac{\partial \log \left( \frac{w_2}{w_1} \right)}{\partial \log (1+\tau)} \right)
\]

\[
= -\frac{1}{1-\mu} \left( 1 - (1 + \tau) \frac{w_2}{w_1} \frac{\partial \left( \frac{w_1}{w_2} \right)}{\partial (1+\tau)} \right)
\]

\[
\frac{1}{\epsilon} = -\frac{1-\mu}{1 - (1 + \tau) \left( \frac{w_2}{w_1} \right) \frac{\partial \left( \frac{w_1}{w_2} \right)}{\partial (1+\tau)}}
\]

We need to make a correction by intermediate goods (see ACR section 5.2). Hence, a change in welfare, according to ACR, is given by

\[
\hat{W}^{ACR} = \left( 1 + \frac{L_2 z_2}{L_1 z_1} \left( \frac{w_1}{w_2} \frac{1}{1+\tau} \frac{z_2}{z_1} \right) \right)^{1-\mu} \right) \frac{1}{1 - \frac{w_1}{w_2} \frac{1}{1+\tau} \frac{\partial \left( \frac{w_1}{w_2} \right)}{\partial (1+\tau)}}
\]

and this concludes the proof. ■

7.5 Proof of Proposition 5

In order to proceed, recall that the manner in which ACR computes their formula is by making use of real income, \(w/p\). We proceed from equation (16)

\[
\frac{w_1}{p_1} = K \left( \frac{L_2 z_2}{L_1 z_1} \left( \frac{w_1}{w_2} \frac{1}{1+\tau} \frac{z_2}{z_1} \right) \right)^{\mu} + 1 \right) \frac{1-\mu}{\mu}
\]

\(^{11}\)ACR, footnote 1, page 95: "Import penetration [...] can be interpreted as a share of (gross) total expenditures allocated to imports (see Norihiko and Ahmad (2006))."
Hence a change in real income due to a change in \( \tau \) to \( \tau' \) implies

\[
\hat{W} = \left( \frac{1 + L_{z_2} \left( \frac{w_1'}{w_2'} \frac{1}{1 + \tau'} \right)}{1 + L_{z_1} \left( \frac{w_2}{w_1} \frac{1}{1 + \tau} \right)} \right) \frac{1 - \mu}{\rho}
\]

And this concludes the proof. \( \blacksquare \)

### 7.6 Proof of Proposition 9

From the first order conditions that arise from (10) and (11), we get that

\[
\frac{1}{p_{L,m}(j)} = \left( \sum_{n=1}^{N} \Omega_n \left( 1 + \tau_{n,m} \right)^{1 - \rho} \left( \frac{w_n}{z_{X,n}} \right)^{1 - \rho} \right)
\]

which is the usual Dixit-Stiglitz price aggregator that does not depend on the variety, \( j \). Hence, \( p_{E,m}(j) = p_{E,m}(k) = p_{E,m} \).

Using the first order conditions from problem (8) and the result that prices are not dependent on \( j \), we get that

\[
W_m(k) = \left( \frac{1 - \rho}{p_{L,m}^{1 - \rho}} \sum_{n=1}^{N} \right) (p_{S,n,m}(i_n)(1 + \tau_{n,m}))^{1 - \rho} \, d_i \, J_M(k) + \frac{1}{2} J_M^2(k)
\]

Using again the FOC from (10) and (11), and combining it with the integral over \( k \) for the previous equation, we get that

\[
\int_{s \in \Omega_n} \int_0^{+\infty} x_{n,m}(j,s) \, dj \, ds = \frac{\Omega_n^{1 - \rho}}{\left( (1 - 1_{n,m}) q_{n,m} \right)^{1 - \rho} \left( \sum_{r=1}^{N} \Omega_r \left( (1 + \tau_{r,m}) \frac{w_r}{z_{X,r}} \right)^{1 - \rho} \right)}
\]

Combining the first order conditions from the household’s problem with those from the country specific good, we get that

\[
c_{S,n,m}(i_n, k) = J_m(k) \left( \frac{p_{L,m}(j)}{w_m} \frac{z_{S,m}(i_n)}{w_m} \frac{1}{1 + \tau_{n,m}} \right)^{1 - \rho}
\]

\[
c_{L,m}(j, k) = (J_m(k) - j)
\]

All the other equations arise by definition the equation for \( W_m(k) \), for labors or by markets being perfectly competitive, \( q_{n,m} \) and \( p_{S,n,m}(i_n) \). This concludes the proof. \( \blacksquare \)
8 Tables and Figures

<table>
<thead>
<tr>
<th>Rich Countries</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>β₀</strong></td>
<td><strong>β₁</strong></td>
<td><strong>β₂</strong></td>
<td></td>
</tr>
<tr>
<td>coefficient</td>
<td>0.06</td>
<td>0.46</td>
<td>-0.21</td>
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<tr>
<td>lower bound</td>
<td>-0.01</td>
<td>0.27</td>
<td>-0.33</td>
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<tr>
<td>upper bound</td>
<td>0.13</td>
<td>0.65</td>
<td>-0.10</td>
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<tr>
<td>fitted maximum</td>
<td>1.07</td>
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<td><strong>β₁</strong></td>
<td><strong>β₂</strong></td>
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<tr>
<td>coefficient</td>
<td>0.12</td>
<td>-0.01</td>
<td>0.00</td>
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<tr>
<td>lower bound</td>
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<tr>
<td>upper bound</td>
<td>0.17</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>fitted maximum</td>
<td>does not apply</td>
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<th>Target</th>
<th>Data</th>
<th>Av.</th>
<th>Dev.</th>
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<td>τₘ,ₙ</td>
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<td>10⁻¹¹ − 0.3</td>
<td>&lt; 10⁻⁷</td>
<td></td>
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<tr>
<td>ρ</td>
<td>0.9</td>
<td>Trade Elasticity</td>
<td>(−5, −10)</td>
<td>(−5.03, −10.63)</td>
<td></td>
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<tr>
<td>zₙₓ,m</td>
<td>0.01 − 1.20</td>
<td>(\frac{GDP_{pcUS}}{GDP_{pcm}})</td>
<td>0.058 − 1</td>
<td>&lt; 10⁻⁵</td>
<td></td>
</tr>
<tr>
<td>αₘ</td>
<td>1.05 − 2.51</td>
<td>Gini</td>
<td>24.9 − 67.4</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Lₘ</td>
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<td>(\frac{Population_{m}}{Population_{USA}})</td>
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<td>0</td>
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</tr>
<tr>
<td>Ωₙ</td>
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<td>((L_n z_n ∫ z^{-α_n} dz)^{1−ρ})</td>
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<td>Aₙ</td>
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*Anderson and Van Wincoop (2003)*
### Table 3: Regression Comparison (data vs. model)

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<tr>
<th>Regression Rich Countries</th>
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<th>Model</th>
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<tr>
<td><strong>Coefficient</strong></td>
<td>$\beta_0$</td>
<td>$\beta_1$</td>
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<tr>
<td></td>
<td>0.06</td>
<td>0.46</td>
</tr>
<tr>
<td><strong>95% Confidence Interval</strong></td>
<td>-0.01</td>
<td>0.27</td>
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<tr>
<td></td>
<td>0.13</td>
<td>0.65</td>
</tr>
<tr>
<td><strong>Implied Maximum</strong></td>
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<table>
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<th>Model</th>
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</thead>
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<td>$\beta_1$</td>
</tr>
<tr>
<td></td>
<td>0.12</td>
<td>-0.01</td>
</tr>
<tr>
<td><strong>95% Confidence Interval</strong></td>
<td>0.08</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>0.17</td>
<td>0.00</td>
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<tr>
<td><strong>Implied Maximum</strong></td>
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### Table 4: Welfare gains

<table>
<thead>
<tr>
<th>Country</th>
<th>Gains relative to autarky</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Model</td>
</tr>
<tr>
<td>US</td>
<td>1.88%</td>
</tr>
<tr>
<td>Japan</td>
<td>1.58%</td>
</tr>
<tr>
<td>South Africa</td>
<td>3.72%</td>
</tr>
<tr>
<td>Brazil</td>
<td>1.45%</td>
</tr>
<tr>
<td>Russia</td>
<td>2.81%</td>
</tr>
<tr>
<td>Spain</td>
<td>4.25%</td>
</tr>
<tr>
<td>France</td>
<td>5.02%</td>
</tr>
<tr>
<td>Germany</td>
<td>5.03%</td>
</tr>
<tr>
<td>Canada</td>
<td>14.84%</td>
</tr>
<tr>
<td>China</td>
<td>1.45%</td>
</tr>
<tr>
<td>India</td>
<td>2.76%</td>
</tr>
<tr>
<td>Mexico</td>
<td>5.02%</td>
</tr>
<tr>
<td>Turkey</td>
<td>5.35%</td>
</tr>
<tr>
<td>UK</td>
<td>4.19%</td>
</tr>
<tr>
<td>Italy</td>
<td>3.76%</td>
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Table 5: Determinants of Bias

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Confidence Interval (95%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>4.00** (-5.72, 13.72)</td>
</tr>
<tr>
<td>( \left( \frac{z_i}{z_{(i)}} \right)^{1+\rho} \left( \frac{l_i}{l_{(i)}} \right)^{1-\rho} )</td>
<td>-7.67** (-15.22, -0.13)</td>
</tr>
</tbody>
</table>

\( R^2 = 0.27 \) (** is significant at 95%)

Table 6: Calibrated Parameters and Targets. Constant Population

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Target</th>
<th>Data</th>
<th>Av.</th>
<th>Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_{m,n} )</td>
<td>0 - 4.76</td>
<td>( \frac{M_{m,n}}{GDP_m} )</td>
<td>10^{-11} - 0.3</td>
<td>&lt; 10^{-7}</td>
<td></td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.9</td>
<td>( Trade \ Elasticity )</td>
<td>(-5, -10)*</td>
<td>(-6, -9.4)</td>
<td></td>
</tr>
<tr>
<td>( z_{X,n} )</td>
<td>0.02 - 2.83</td>
<td>( \frac{GDP_{pcUS}}{GDP_{pcm}} )</td>
<td>0.058 - 1</td>
<td>&lt; 10^{-5}</td>
<td></td>
</tr>
<tr>
<td>( \alpha_m )</td>
<td>1.05 - 2.51</td>
<td>Gini</td>
<td>24.9 - 67.4</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>( L_m )</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Omega_n )</td>
<td>0.81 - 1.19</td>
<td>( (L_n z_n \int z^{-\alpha_n} dz)^{1-\rho} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( A_n )</td>
<td>0.0016 - 0.0024</td>
<td>( \frac{TradeUS Omega}{TradeUS An} )</td>
<td>0.002</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

*Anderson and Van Wincoop (2003)*

Table 7: Welfare gains

<table>
<thead>
<tr>
<th>Constant population</th>
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<tbody>
<tr>
<td>Gains relative to autarky</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Country</th>
<th>Model</th>
<th>ACR</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>2.07%</td>
<td>2.12%</td>
<td>-2.3%</td>
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<tr>
<td>Japan</td>
<td>1.37%</td>
<td>1.38%</td>
<td>-1.2%</td>
</tr>
<tr>
<td>South Africa</td>
<td>3.11%</td>
<td>2.88%</td>
<td>8.0%</td>
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<td>1.44%</td>
<td>1.36%</td>
<td>5.7%</td>
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<tr>
<td>Russia</td>
<td>2.23%</td>
<td>2.07%</td>
<td>7.3%</td>
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<tr>
<td>Spain</td>
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<td>3.59%</td>
<td>-0.9%</td>
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<tr>
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<td>4.83%</td>
<td>-16.3%</td>
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<tr>
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<td>3.54%</td>
<td>5.1%</td>
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<td>1.52%</td>
<td>-14.9%</td>
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<td>3.95%</td>
<td>9.0%</td>
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<td>India</td>
<td>2.85%</td>
<td>2.68%</td>
<td>6.7%</td>
</tr>
<tr>
<td>Mexico</td>
<td>4.02%</td>
<td>4.13%</td>
<td>-2.6%</td>
</tr>
<tr>
<td>Turkey</td>
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<td>2.39%</td>
<td>-1.8%</td>
</tr>
<tr>
<td>UK</td>
<td>2.40%</td>
<td>2.23%</td>
<td>7.8%</td>
</tr>
<tr>
<td>Italy</td>
<td>2.42%</td>
<td>2.27%</td>
<td>6.3%</td>
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</table>
### Table 8: Calibrated Parameters and Targets. Constant Productivity

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Target</th>
<th>Data</th>
<th>Av.</th>
<th>Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_{m,n}$</td>
<td>0 – 4.16</td>
<td>$\frac{M_{m,n}}{GDP_m}$</td>
<td>$10^{-11} - 0.3$</td>
<td>$&lt; 10^{-5}$</td>
<td></td>
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<tr>
<td>$\rho$</td>
<td>0.9</td>
<td>Trade Elasticity</td>
<td>$(-5, -10)^*$</td>
<td>$(-7.6, -9.2)$</td>
<td></td>
</tr>
<tr>
<td>$z_{X,n}$</td>
<td>1</td>
<td>1</td>
<td>—</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$\alpha_m$</td>
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<td>Gini</td>
<td>24.9 – 57.4</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$L_m$</td>
<td>0.11 – 4.63</td>
<td>$\frac{Population_n}{Population_{USA}}$</td>
<td>0.11 – 4.63</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$\Omega_n$</td>
<td>0.69 – 1.50</td>
<td>$\left(L_n \int z^{-\alpha_m} , dz\right)^{1-\rho}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_n$</td>
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<td>0.02</td>
<td></td>
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* *Anderson and Van Wincoop (2003)*

### Table 9: Welfare Gains

#### Constant productivity

<table>
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<th>Model</th>
<th>ACR</th>
<th>Difference</th>
<th>$z$</th>
<th>$L$</th>
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<td>Japan</td>
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<td>1.48%</td>
<td>8.8%</td>
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<td>South Africa</td>
<td>1.84%</td>
<td>1.82%</td>
<td>1.1%</td>
<td>1</td>
<td>0.15</td>
</tr>
<tr>
<td>Brazil</td>
<td>0.34%</td>
<td>0.35%</td>
<td>-1.4%</td>
<td>1</td>
<td>0.61</td>
</tr>
<tr>
<td>Russia</td>
<td>2.72%</td>
<td>2.60%</td>
<td>4.7%</td>
<td>1</td>
<td>0.51</td>
</tr>
<tr>
<td>Spain</td>
<td>4.28%</td>
<td>4.20%</td>
<td>2.0%</td>
<td>1</td>
<td>0.15</td>
</tr>
<tr>
<td>France</td>
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<td>17.38%</td>
<td>-1.0%</td>
<td>1</td>
<td>0.21</td>
</tr>
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<td>5.11%</td>
<td>4.94%</td>
<td>3.5%</td>
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<td>0.28</td>
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<td>Canada</td>
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<td>36.85%</td>
<td>5.3%</td>
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<td>0.85%</td>
<td>-3.4%</td>
<td>1</td>
<td>4.63</td>
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<td>-2.6%</td>
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<td>0.22</td>
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<td>0.20</td>
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Table 10: Decomposition of Welfare gains.

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<th>Baseline</th>
<th>Constant Population</th>
<th>Constant Productivity</th>
<th>$\tau = 0$</th>
<th>$A_n = 0$</th>
<th>Constant Inequality</th>
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<td>-8%</td>
<td>-5%</td>
<td>-19%</td>
<td>-7%</td>
<td>-3%</td>
</tr>
<tr>
<td>Japan</td>
<td>-21.7%</td>
<td>9%</td>
<td>-1%</td>
<td>-2%</td>
<td>-16%</td>
<td>-23%</td>
</tr>
<tr>
<td>South Africa</td>
<td>2.7%</td>
<td>-1%</td>
<td>1%</td>
<td>4%</td>
<td>4%</td>
<td>3%</td>
</tr>
<tr>
<td>Brazil</td>
<td>0.9%</td>
<td>-1%</td>
<td>-2%</td>
<td>3%</td>
<td>3%</td>
<td>3%</td>
</tr>
<tr>
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Figure 1: France - Germany bilateral trade
Figure 2: France - Russia bilateral trade

Figure 3: Russia - Turkey bilateral trade
Figure 4: Uniquely traded goods and GDP per capita of the trading partner

Figure 5: Consumption pattern for a country (upper triangle)
Figure 6: Trade Patterns

Figure 7: Relationship between correlation and productivity - Simple model