I. Online Appendix
A. Proof of Proposition 1

(i) Consider first a firm that already paid the fixed cost $f_x$. We can write the dynamic problem of the firm using cash-on-hand as the unique state variable as follows:

$$V^x(a, z) = \max_{d, k, b'} \left[ d + q(1 - \delta)V^x(a', z) \right]$$

subject to

$$d + (1 - \delta)qa' \leq \pi^x(a)$$
$$d \geq 0$$

It can be shown that $V^x$ is differentiable and concave in $a$ and $V^{x'}(a) \geq 1$ with $V^{x'}(a) = 1$ for all $a \geq a^*$ and $V^{x'}(a) > 1$ for $a < a^*$. Letting $\lambda$ and $\eta$ be the multiplier associated with the budget constraint and the non-negativity on dividends respectively, the focs for the problem are:

$$(1) \quad d : 0 = 1 - \lambda + \eta$$
$$(2) \quad a' : 0 = q(1 - \delta)\lambda - q(1 - \delta)V^{x'}(a')$$

and the envelope condition:

$$V'^{x'}(a) = \lambda \pi^{x'}(a)$$
We want to show that if $a' < a^*$ then $\eta > 0$. Suppose for contradiction that $\eta = 0$. Then (1) implies that $\lambda = 1$ and in turn (??) implies that

$$1 - V^{x'}(a') = 0$$

but $V^{x'}(a) > 1$ if $a < a^*$ thus $1 - V^{x'}(a') < 0$ yielding a contradiction. Then it must be that $\eta > 0$ and $d = 0$.

Consider a non-exporter now. If it is never optimal to export, the same logic we used for an exporter goes through (notice that in this case $V^{nx}$ is concave and differentiable). Instead, if it will be optimal to export at some date, $V^{nx}$ is not necessarily concave and differentiable everywhere. Letting $T$ be the period in which a firm with initial cash on hand $a$ will start to export, we can write the problem as follows:

$$V^{nx}(a) = \max_{\{d_t,a_{t+1}\}} \sum_{t=0}^{T} \beta^t(1 - \delta)^t d_t + \beta^{T+1}(1 - \delta)^{T+1} V^{x}(a_{T+1})$$

subject to

$$d_t + q(1 - \delta)a_{t+1} \leq \pi^{nx}(a_t) \quad \text{for} \quad t = 0, \ldots, T - 1$$

$$d_T + q(1 - \delta)a_{T+1} \leq \pi^{nx}(a_T - f_x) - f_x$$

$$d_t \geq 0$$

Letting $\beta^t(1 - \delta)^t \lambda_t$ and $\beta^t(1 - \delta)^t \eta_t$ be the multiplier associated with the budget constraint
and the non-negativity on dividends respectively, the focs for the problem are:

\[
(3) \quad 0 = 1 - \lambda_t + \eta_t \quad \text{for} \ t = 0, \ldots, T
\]

\[
(4) \quad 0 = \lambda_t - \lambda_{t+1}\pi'(a_{t+1}) \quad \text{for} \ t = 0, \ldots, T - 1
\]

\[
(5) \quad 0 = \lambda_T - V'^x(a_{T+1})
\]

Starting at \( T + 1 \), suppose that \( a_{T+1} < a^* \) and for contradiction that \( \eta_T = 0 \). Then it must be that \( \lambda_T = 1 \). This and (5) imply that

\[
1 - V'^x(a') = 0
\]

but \( V'^x(a_{T+1}) > 1 \) thus \( 1 - V'^x(a') < 0 \) yielding a contradiction. Hence \( \eta_T > 0 \) and \( d_T = 0 \).

Now combine (3) at \( t \) and \( t + 1 \) with (4) at \( t \) we obtain:

\[
\eta_t = \lambda_t - 1 = \lambda_{t+1}\pi'(a_{t+1}) - 1 \geq \lambda_{t+1} - 1 = \eta_{t+1}
\]

Thus, if \( \eta_{t+1} > 0 \) then \( \eta_t > 0 \). This is turn implies that as long as any borrowing constraint is binding in the future then there is no dividend distributions as wanted. When no borrowing constraint in the future are binding then the firms optimal dividend policy is indeterminate.

Thus, without loss of generality we can set \( d = 0 \) to characterize the firm’s value and policy functions.

To prove the remaining parts of Proposition 1 we will consider the forward and back-
Forward-Looking Constraint In this case it is convenient to write the problem in (??) and (??) using their dual formulation. This can be thought of as an optimal contracting problem between the entrepreneur and competitive, risk-neutral financial intermediaries. Financial intermediaries offer the entrepreneur long-term contracts that specify production plans and the value of the dividends paid to the entrepreneur. It is then straightforward to write this problem recursively using the discounted sum of promised dividend payments $v$ as well as the export status of the firm as state variables. Denote the value functions of the financial intermediaries as $W^{nx}(v, z, \phi)$ and $W^x(v, z, \phi)$. Letting $r \equiv [1 - q(1 - \delta)(1 - \delta_k)]/[q(1 - \delta)]$, the problem of the financial intermediary can be written as:

\[
W^x(v, z, \phi) = \max \quad rk + p_d(y_d)y_d + p_x(y_x)y_x - w l - d + q(1 - \delta)w^x(v', z, \phi)
\]

subject to

\[
y_d + y_x \leq z k^\alpha l^{1-\alpha} \\
d + q(1 - \delta) = v \\
v \geq \frac{\theta}{q(1 - \delta)} k + \frac{\xi}{q(1 - \delta)} v_0
\]
and for a firm that has not paid the fixed cost already:

\[ W^{nx}(v, z, \phi) = -rk + p_d(y_d)y_d - wl - d - xf_x + q(1 - \delta) [xW^x(v', z, \phi) + (1 - x)W^{nx}(v', z, \phi)] \]

subject to

\[ y_d + y_x \leq zk^{\alpha}l^{1-\alpha} \]

\[ d + q(1 - \delta) = v \]

\[ v \geq \frac{\theta}{q(1 - \delta)} k + \frac{\xi}{q(1 - \delta)} v_0 \]

For notational convenience define

\[ \Pi^x(v, z) = \max_{y_d, y_x, l, k} [p_d(y_d)y_d + p_x(y_x)y_x - wl] - rk \]

subject to

\[ y_d + y_x \leq zF(k, l) \]

\[ v \geq \frac{\theta}{q(1 - \delta)} k + \frac{\xi}{q(1 - \delta)} v_0 \]

and

\[ \Pi^{nx}(v, z) = \max_{y_d, l, k} [p_d(y_d)y_d + p_x(y_x)y_x - wl] - rk \]
subject to

\[ y_d \leq zF(k, l) \]

\[ v \geq \frac{\theta}{q(1 - \delta)} k + \frac{\xi}{q(1 - \delta)} v_0 \]

By part (i) we can set \( d_t = 0 \) without loss for all \( t \) and rewrite the intermediary’s problem as follows:

\[
W^{nx}(v, z, \phi) = \max\{\Pi^{nx}(v, z) + q(1 - \delta)W^d\left(\frac{v}{q(1 - \delta)}, z, \phi\right); \\
\Pi^x(v, z) - f_x + q(1 - \delta)W^x\left(\frac{v}{q(1 - \delta)}, z, \phi\right)\}
\]

\[
W^x(v, z, \phi) = \Pi^x(v, z) + q(1 - \delta)W^x\left(\frac{v}{q(1 - \delta)}, z\right)
\]

Finally, the minimum equity value for the firm to operate at its efficient scale is given by:

\[
v^*(z, \phi) \equiv \min\{\arg \max_v \{\max\{\Pi^{nx}(v, z), \phi\Pi^x(v, z)\}\}\}\}
\]

A firm will eventually reach \( v^* \), because \( v_t = \frac{v_0}{((1 - \delta)q)^t} \). Then, for \( v' \geq v^* \) a domestic firm with inside equity value \( v' \) will start exporting iff

\[
\frac{\Pi^{nx'}(z)}{1 - (1 - \delta)q} - \frac{\Pi^{nx^*}(z)}{1 - (1 - \delta)q} \geq f_x
\]
as in a standard Melitz model. Since the LHS is strictly increasing in $z$, there exists a cut-off $z_x$ s.t. the above condition holds for all $z \geq z_x$.

We now prove part (iii) and (iv). To this end consider

$$W^x(v, z) - W^{nx}(v, z) = \Pi^x(v, z) + q(1 - \delta) W^x \left( \frac{v}{q(1 - \delta)}, z \right) - \max \left\{ \Pi^{nx}(v, z) + q(1 - \delta) W^{nx} \left( \frac{v}{q(1 - \delta)}, z \right); W^x(v, z) - f_x \right\}$$

$$= \min \{ \Pi^x(v, z) - \Pi^x(v, z) + + q(1 - \delta) \left( W^x \left( \frac{v}{q(1 - \delta)}, z \right) - W^{nx} \left( \frac{v}{q(1 - \delta)}, z \right) \right) ; f_x \}$$

$$= \min \left\{ \Delta \Pi(v, z) + q(1 - \delta) \left( W^x \left( \frac{v}{q(1 - \delta)}, z \right) - W^{nx} \left( \frac{v}{q(1 - \delta)}, z \right) \right); f_x \right\}$$

The following lemma shows that the value of becoming an exporter weakly increases with $v$.

**Lemma 1:** (a) $\forall z \ W^x(v, z) - W^{nx}(v, z)$ is weakly increasing in $v$, and (b) $\forall v \ W^x(v, z) - W^{nx}(v, z)$ is weakly increasing in $z$.

**Proof:**

Define $T : C(\mathbb{R}_+ \times \mathbb{R}_+) \rightarrow C(\mathbb{R}_+ \times \mathbb{R}_+)$ as

$$Tf(v, z) = \min \left\{ \Delta \Pi(v, z) + q(1 - \delta)f \left( \frac{v}{q(1 - \delta)}, z \right); f_x \right\}$$

where $C(\mathbb{R}_+ \times \mathbb{R}_+)$ is the space of continuous and bounded functions. $T$ satisfies the Blackwell’s sufficient conditions for a contraction mapping. Then $T$ is a contraction, and $W^x - W^{nx}$ is its unique fixed point.
To prove (a), let $C'(\mathbb{R}_+ \times \mathbb{R}_+) = \{ f \in C'(\mathbb{R}_+ \times \mathbb{R}_+) \text{ a closed set, hence by Corollary 3.1 in Stokey, Lucas and Prescott (1989) it suffices to show that } \forall f \in C'(\mathbb{R}_+ \times \mathbb{R}_+) \text{ to prove that } W^x - W^{nx} \text{ is increasing in its first argument. Fix } z, \text{ let } f \in C'(\mathbb{R}_+ \times \mathbb{R}_+) \text{ and } v' > v:\n
\begin{align*}
T f(v', z) &= \min \left\{ \Delta \Pi(v', z) + q(1 - \delta)f \left( \frac{v'}{q(1 - \delta)}, z \right); f_x \right\} \\
&\geq \min \left\{ \Delta \Pi(v, z) + q(1 - \delta)f \left( \frac{v}{q(1 - \delta)}, z \right); f_x \right\} = T f(v, z)
\end{align*}

as wanted, because $\Delta \Pi(v, z)$ is increasing in $v$, and $f$ is weakly increasing by assumption. Then we established (a). The exact same argument can be used to prove (b) noticing that $\Delta \Pi(v, z)$ is increasing in $z$ also.

Thus, if $z \leq z_x$ a firm will never export since for all $v$ $W^x(v, z) - W^{nx}(v, z) \leq W^x(v^*, z) - W^{nx}(v^*, z) < f_x$. Vice versa, if $z \geq z_x$, then the firm will eventually export, proving (ii).

To prove (iii), notice that if $z \geq z_x$ the firm will eventually export, and the fact that $W^x(v, z) - W^{nx}(v, z)$ is increasing in $v$ implies that there exists a unique threshold $\tilde{v}(z)$ such that a firm will export iff $v \geq \tilde{v}(z)$.

Lastly, we prove (iv) by showing that if $z' > z$ then $\tilde{v}(z')/v_0(z') \leq \tilde{v}(z)/v_0(z)$, implying $\tilde{T}(z') \leq \tilde{T}(z)$. Let $z' > z \geq z_x$. The fact that $W^{nx}(v, z)$ is strictly increasing in $z$ for all $v$ implies that $v_0(z') > v_0(z)$, since $v_0$ is such that $W^{nx}(v_0(z), z) = 0$. To prove the proposition
it is sufficient to show that \( \tilde{v}(z') < \tilde{v}(z) \). By the previous lemma \( \forall v W^x(v, z) - W^{nx}(v, z) \) is weakly increasing in \( z \). Thus, if \( W^x(\tilde{v}(z), z) - W^{nx}(\tilde{v}(z), z) = f_x \) then \( W^x(\tilde{v}(z), z') - W^{nx}(\tilde{v}(z), z') \geq f_x \) since \( z' > z \), therefore \( \tilde{v}(z') \leq \tilde{v}(z) \) as wanted.

To relate this to the "cash on hand" formulation, notice that \( v = xV^x(a(v), z) + (1 - x)V^{nx}(a(v), z) \), which is a monotone relation in \( v \). Hence, all statements about \( v \) are also true for \( a \).

**Backward-Looking Constraint**  Proof of part (iii):

LEMMA 2: Consider a restricted problem in which firms can only choose to either pay the fixed cost in the first or second Let \( x(a, z) = 0 \) be the decision to not export in the first period, and \( x(a, z) = 1 \) be the decision to export in the first period. Then \( \exists \tilde{a} : \forall a < \tilde{a}, x(a, z) = 0 \), and \( \forall a \geq \tilde{a}, x(a, z) = 1 \).

PROOF:

In this restricted problem, the fact that all firms must be exporters after the second period (and the fact that \( z \) does not change) implies that the objective of the firm is equivalent to maximizing third period assets. Then the decision to export today or tomorrow yields the following payouts:

If the firm exports today (here assuming all constraints are binding to simplify notation):

\[
x(a, z) = 1 \implies q(1 - \delta)a_x = \pi^x \left( \frac{\pi^x(a - f_x, z)}{1 - (1 - \delta)q}, z \right)
\]
and if they export the next period:

\[ x(a, z) = 0 \implies q(1 - \delta)a_{nx} = \pi^x \left( \frac{\pi^{nx}(a, z)}{1 - (1 - \delta)q} - f_x, z \right) \]

The firm then chooses whichever is greater. Define \( \Delta(a, z) \equiv q(1 - \delta)[a_x - a_{nx}] \). Let \( F(a, z) \equiv \pi^x(a - f_x, z) - \pi^{nx}(a, z) + (1 - q(1 - \delta))f_x \). Note that \( \text{sign}(F(a, z)) = \text{sign}(\Delta(a, z)) \).

Then any zero of the function \( F \) is also a zero of the function \( \Delta \). We can show that \( F \) is a strictly increasing function of \( a \):

\[ F_1(a, z) \equiv \pi^x_1(a - f_x, z) - \pi^{nx}_1(a, z) > 0 \]

which is true because \( \pi^x \) and \( \pi^{nx} \) are concave, and \( \forall a, z, \pi^x_1(a, z) > \pi^{nx}_1(a, z) \).

Then notice that \( F(f_x, z) < 0 \) and (assuming that \( z \geq z_x \)) \( F(a^*, z) > 0 \). Therefore, \( \exists \hat{a} \in [f_x, a^*] \) that has the cutoff properties described in the statement of the lemma.

To complete the proof, we demonstrate that the cutoff found in the restricted problem corresponds to the cutoff in the general problem.

First, consider firms with asset values \( a < \hat{a}(z) \). Our claim is that the firm does not export with that level of assets. For contradiction, suppose that they did. Then, by the definition of \( \hat{a} \) given in the lemma, we know that the firm could generate strictly greater profits by, instead, delaying their decision to export by one period. Hence, exporting this period is not optimal.
Second, consider firms with asset values \( a \geq \hat{a}(z) \). The next lemma shows that for these firms the restriction on the periods when they can export is not binding.

**LEMMA 3:** Suppose a firm prefers to export this period instead of one period in the future. Then the firm prefers to export this period rather than any period in the future.

**PROOF:**

We prove this by induction. The base step is true by hypothesis. Let \( a^k(t) \) be the asset level of a firm \( k \) periods in the future who chooses to enter the export market in period \( t \).

Using the fact that the firm’s objective is equivalent to maximizing their assets whenever they are constrained, to complete the proof we need only show that \( a^k+1(k) < a^k+1(1) \implies a^k+2(k+1) < a^k+2(1) \). Notice that the fact that \( a'(a, z) \) is increasing in \( a \) means that \( a^k+1(k) < a^k+1(1) \implies a^k+2(k) < a^k+2(1) \), so it is sufficient to show that \( a^k+2(k+1) < a^k+2(k) \). But this follows immediately from the previous lemma, the fact that \( a \geq \hat{a}(z) \), and the fact that \( a'(a, z) \) is increasing in \( a \). This completes the proof.

Therefore, \( \forall a \geq \hat{a}(z) \), the fact that they prefer to export this period rather than the following period implies that they prefer to export this period rather than wait until any other period. Therefore, \( \hat{a}(z) \) is the threshold level of assets that determines export status.

**Proof of part (iv):**

Here we use the fact that \( a'(a, z) \) is increasing in \( z \) and that \( \hat{a}(z) \) is decreasing in \( z \). The fact that \( a'(a, z) \) is increasing in \( z \) follows immediately from the fact that \( \pi^{nx}(a, z) \) is
increasing in $z$. To prove that $\hat{a}(z)$ is decreasing in $z$ we make use of the characterization in the proof to part (iii).

Recall that $\hat{a}(z)$ solves $F(\hat{a}(z), z) = 0$. Then the implicit function theorem implies:

$$\frac{d\hat{a}}{dz} = \frac{\left[ \pi_{2}^{nx}(a, z) - \pi_{2}^{x}(a - f_{x}, z) \right]}{\left[ \pi_{1}^{nx}(a, z) - \pi_{1}^{x}(a - f_{x}, z) \right]} < 0$$

The sign follows from the fact that for $j \in \{nx, x\}$, $\pi^{j}$ is concave in the first argument, $\pi_{21}^{j} > 0, \forall a, z, \pi_{1}^{x}(a, z) > \pi_{1}^{nx}(a, z)$ and $\pi_{2}^{x}(a, z) > \pi_{2}^{nx}(a, z)$.

Therefore, starting from assets $a_{0}$, firms with higher productivity both have faster asset growth and a lower asset threshold to enter the export market. Hence, $T(z)$ is decreasing in $z$. 

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