How to estimate an RD model

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Problem specifics:

y_i	outcome of interest
x_i	Treatment dummy variable, =1 if treated, =1 otherwise
z_i	assignment or running variable
$D_i=I(z_i \geq z_0)$	Indicator function that equals 1 if $z_i \geq z_0$ and zero otherwise
w_i	(1 x k) vector of covariates

Fuzzy regression discontinuity design

Structural equation of interest:

(1) $$y_i = \beta_0 + x_i\beta_1 + w_i\beta_2 + h^1(z_i) + \varepsilon_i$$

First-stage (how does treatment change at the discontinuity?)

(2) $$x_i = \pi_0 + D_i\pi_1 + w_i\pi_2 + h^2(z_i) + v_i$$

Reduced form (how do outcomes change at the discontinuity?)

(3) $$y_i = \theta_0 + D_i\theta_1 + w_i\theta_2 + h^3(z_i) + u_i$$

Note that the estimate of $\hat{\beta}_1 = \dfrac{\hat{\theta}_1}{\hat{\pi}_1}$ (2SLS via indirect least squares). Note this only works if the structure of $h^m(z_i)$ is the same for all three equations. The key to the exercise is the definition of $h^m(z_i)$ for m=1, 2 or 3. It must be the case that $h^m(z_i = z_0)$ =0.

Note that $\lim\limits_{z_i \to z_o +} \hat{y}_i = \hat{\theta}_0 + \hat{\theta}_1 + w_i\hat{\theta}_2$ and

$\lim\limits_{z_i \to z_o -} \hat{y}_i = \hat{\theta}_0 + w_i\hat{\theta}_2$ and therefore

$\lim\limits_{z_i \to z_o +} \hat{y}_i - \lim\limits_{z_i \to z_o -} \hat{y}_i = \hat{\theta}_1$

What value of $h^m(z_i)$ produces $h^m(z_i = z_0)$ =0? This one is the industry standard:

$$h^m(z_i) = \sum_{j=1}^{\rho}[D_i\delta_j^{m+}(z_i - z_0)^j + (1 - D_i)\delta_j^{m-}(z_i - z_0)^j]$$

Sharp regression discontinuity design

$x_i = D_i = I(z_i \geq z_0)$ and as a result, $\hat{\pi}_1 = 1$

Therefore $\hat{\beta}_1 = \frac{\hat{\theta}_1}{\hat{\pi}_1} = \hat{\theta}_1$

And equation (1) functionallu equals equation (3).

The effect of treatment is estimated directly by equation (1) – substituting D in for x

$$y_i = \beta_0 + D_i\beta_1 + w_i\beta_2 + h^1(z_i) + \varepsilon_i$$