How to Maximize a Likelihood Function
ECON 60303

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Problem setup

$f(y_i \mid x_i, \hat{\beta})$ is a pdf that describes the data generating process
There are n observations in the sample
x_i is (1 x j)
$\hat{\beta}$ is (k x 1)

Log likelihood function $\ell = \sum_{i=1}^{n} \ln f(y_i \mid x_i, \hat{\beta})$

Gradient (k x 1): $g = \frac{\partial \ell}{\partial \hat{\beta}}$ $cc = g'(-H^{-1})g$

Hessian (k x k): $H = \frac{\partial^2 \ell}{\partial \hat{\beta} \partial \hat{\beta}'}$

$Cov(\hat{\beta}) = -H^{-1}$

Convergence criteria: $cc = g'(-H^{-1})g$

Quasi-Newton search method:

Let $\hat{\beta}_t$ be the t'th iteration of search algorithm.

Corresponding values for the other parameters are $g_t, (-H_t)^{-1}$

Initially fix λ=1

$$\hat{\beta}_{t+1} = \hat{\beta}_t + \lambda(-H_t)^{-1} g_t$$

Basic Algorithm:

1. Obtain starting values $\hat{\beta}_o$
2. Calculate $g_o, (-H_0)^{-1}$
3. Update $\hat{\beta}_1 = \hat{\beta}_o + \lambda(-H_o)^{-1} g_o$
4. Is $\ell(\hat{\beta}_1) > \ell(\hat{\beta}_o)$
5. If no, cut λ in half and go 3
6. If yes, calculate $g_o, (-H_0)^{-1}$
7. Has model converged?
8. If no, go to 3 and update beta
9. If yes, printout results

Flow Chart for MLE Search Routine
Quasi-Newton Method