Comparing Analytic and Numeric Derivatives
ECON 6303

Bill Evans
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calcloglike.m

```
%% this subroutine calculates the value
%% of the poisson log likelikood
%% inputs are y,x,beta and the function
%% returns a scaler
%% the log likelihood requires the calculation of factorials
%% which is done with the gamma function.
%% gamma(x+1)=x!
function [llike]=calcloglike(y,x,beta)
ylfact=log(gamma(y+1));
lambda=exp(x*beta);
llikei=-lambda+(x*beta).*y-ylfact;
llike=sum(llikei);
end
```

calcgrad.m

```
%% this subroutine calculates the gradient
%% of the poisson log likelikood
%% inputs are y,x,beta and the function
%% returns a (kx1) vector of 1st derivatives
function [grad]=calcgrad(y,x,beta)
lambda=exp(x*beta);
yml=y-lambda;
gradt=yml'*x;
grad=gradt';
end
```

calchess.m

```
%% this subroutine calculates the hessian
%% of the poisson log likelikood
%% inputs are y,x,beta,n,k and the function
%% returns a (kxk) matrix of 2nd derivatives
function [hess]=calchess(y,x,beta,n,k)
lambda=exp(x*beta);
hess=zeros(k,k);
for i=1:n;
xi=x(i,:);
li=lambda(i,:);
hess=hess-li*xi'*xi;
end;
end
```

poisson_check.m

```
%% load in the data from drvisits.xlsx
%% column 1 is the dependent variable while
%% columns 2-(k+1) are independent variables
%% column 2 contains the constant
[w,varlist]=xlsread('drvisits.xlsx');

dv=varlist{1,1};
%% get dimension of w
nk1=size(w);

% number of observations
n=nk1(1);

% number of independent variables
% k+1 is the no of columns of w.  there are k
% covariates (including the constant)
k=nk1(2)-1;

%% extract y which is in the 1st column of w
y=w(:,1);

%% take the log of the max of 1 or y to generate
%% generate starting values

yl=log(max(1,y));

%% extract x which is in columns 2 through k
x=w(:,2:(k+1));
xpxi=inv(x'*x);
beta_start=xpxi*x'*yl

%% compare analytic and numeric derivatives
%% grada is the analytic derivative
%% gradn is the numeric derivative

%% get analytic derivative
grada=calcgrad(y,x,beta_start);

%% get numeric derivatives

%% establish size of epsilon, 0.001*abs(beta) is reasonably small
epsilon=0.001*abs(beta_start);

%% set a vector of zeros of lenth k
gradn=zeros(k,1);

%% betap is the positive step in beta;
%% betan is the negative step in beta;


for i=1:k;
  betap=beta_start;
  betan=beta_start;
  epsilonk=epsilon(i,:);
  betap(i,:)=beta_start(i,:)+epsilonk;
  betan(i,:)=beta_start(i,:)-epsilonk;
```

```
  gradn(i,:)=gradn(i,:)+(calcloglike(y,x,betap)-calcloglike(y,x,betan))/(2*epsilonk);
end;

%% print out results

file1=fopen('poisson_check.txt','w');
c1='Covariate'; c2='beta_start'; c3='epsilon'; c4='grada'; c5='gradn';
fprintf(file1,'--------------------------------------------------------------\n');
fprintf(file1,'%12s %12s %12s %12s %12s \n', c1,c2,c3,c4,c5);
fprintf(file1,'--------------------------------------------------------------\n');
for i=1:k;
    rowname=varlist{1,i+1};
    fprintf(file1,'%12s %12.6f %12.6f %12.6f %12.6f \n', rowname,beta_start(i,:),
epsilon(i,:), grada(i,1),gradn(i,1));
    end;
fprintf(file1,'--------------------------------------------------------------\n');


%% compare analytic second derivatives along main diagonal with
%% numeric estimates of same derivatives

%% hessda is diagonal of analytic hessian
%% hessdn is the diagonal of the numeric hessian
hessda=diag(calchess(y,x,beta_start,n,k));


%% set a vector of zeros of lenth k
hessdn=zeros(k,1);

%% betap is the positive step in beta;
%% betan is the negative step in beta;

%% get baseline loglike -- needed for second derivative;
ll=calcloglike(y,x,beta_start);

for i=1:k;
  betap=beta_start;
  betan=beta_start;
  epsilonk=epsilon(i,:);
  betap(i,:)=beta_start(i,:)+epsilonk;
  betan(i,:)=beta_start(i,:)-epsilonk;
  hessdn(i,:)=hessdn(i,:)+(calcloglike(y,x,betap)+calcloglike(y,x,betan)-
2*ll)/(epsilonk*epsilonk);
end;

%% print out results
c1='Covariate'; c2='beta_start'; c3='epsilon'; c4='hessda'; c5='hessdn';
fprintf(file1,'--------------------------------------------------------------\n');
fprintf(file1,'%12s %12s %12s %12s %12s \n', c1,c2,c3,c4,c5);
fprintf(file1,'--------------------------------------------------------------\n');
for i=1:k;
    rowname=varlist{1,i+1};
    fprintf(file1,'%12s %12.6f %12.6f %12.6f %12.6f \n', rowname,beta_start(i,:),
epsilon(i,:),hessda(i,:),hessdn(i,:));
    end;
fprintf(file1,'--------------------------------------------------------------\n');
fclose(file1);
```

poisson_check.txt

Covariate	beta_start	epsilon	grada	gradn
constant	0.751755	0.000752	9858.716847	9858.714997
age65	0.124065	0.000124	3284.136281	3284.136265
age70	0.228512	0.000229	2916.138576	2916.138526
age75	0.237209	0.000237	2071.141492	2071.141452
age80	0.258576	0.000259	1023.628178	1023.628151
chronic	0.497184	0.000497	7219.478712	7219.478114
excel	-0.576503	0.000577	325.855311	325.855257
good	-0.376005	0.000376	3095.369128	3095.368974
fair	-0.160205	0.000160	3509.207936	3509.207905
hs_drop	-0.210513	0.000211	4729.232387	4729.232318
hs_grad	-0.088321	0.000088	2892.262674	2892.262666
black	-0.286739	0.000287	1044.590157	1044.590130
hispanic	-0.131932	0.000132	231.434593	231.434592
female	0.123723	0.000124	6041.547279	6041.547247
mcaid	0.141200	0.000141	1181.025476	1181.025469
incomel	0.034159	0.000034	95919.727988	95919.724349

Covariate	beta_start	epsilon	hessda	hessdn
constant	0.751755	0.000752	-19641.283153	-19641.283979
age65	0.124065	0.000124	-6020.863719	-6020.858178
age70	0.228512	0.000229	-5763.861424	-5763.862256
age75	0.237209	0.000237	-4199.858508	-4199.858150
age80	0.258576	0.000259	-2427.371822	-2427.371979
chronic	0.497184	0.000497	-14497.521288	-14497.521243
excel	-0.576503	0.000577	-975.144689	-975.144657
good	-0.376005	0.000376	-6540.630872	-6540.631212
fair	-0.160205	0.000160	-7371.792064	-7371.790478
hs_drop	-0.210513	0.000211	-9303.767613	-9303.765714
hs_grad	-0.088321	0.000088	-5907.737326	-5907.745382
black	-0.286739	0.000287	-1964.409843	-1964.409066
hispanic	-0.131932	0.000132	-557.565407	-557.566272
female	0.123723	0.000124	-12368.452721	-12368.450877
mcaid	0.141200	0.000141	-2214.974524	-2214.974632
incomel	0.034159	0.000034	-1886788.666866	-1886788.598144