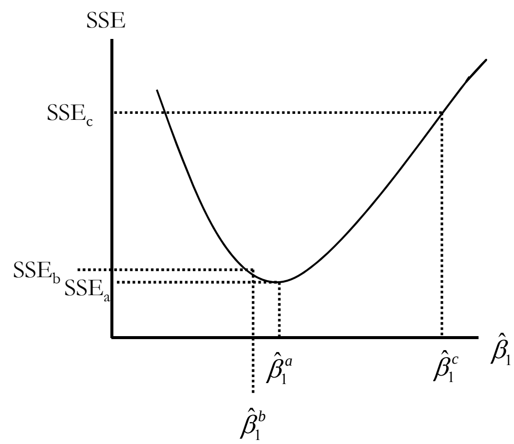
Test of multiple parameters



Null hypothesis

- Model
  - $y_i = \beta_0 + x_{1i}\beta_1 + x_{2i}\beta_2 + x_{3i}\beta_3 + \ldots\ldots x_{ki}\beta_k + \varepsilon_i$
- H_o: $\beta_1 = \beta_2 = \beta_3 = \ldots.\beta_q = 0$
- Subset of the parameters all equal zero
- q restrictions in total

- Unrestricted model
- $y_i = \beta_0 + x_{1i}\beta_1 + x_{2i}\beta_2 + x_{3i}\beta_3 + \ldots\ldots x_{ki}\beta_k + \varepsilon_i$
- Restricted model
- $y_i = \beta_0 + x_{q+1i}\beta_1 + x_{q+2i}\beta_2 + \ldots\ldots x_{ki}\beta_k + \varepsilon_i$

$$\hat{F} = \frac{\frac{SSE_r - SSE_u}{q}}{\frac{SSE_u}{n-k-1}} =$$

$$\frac{\frac{SSE_r - SSE_u}{q}}{\hat{\sigma}_{\varepsilon}^2(u)} \sim F(q, n-k-1)$$

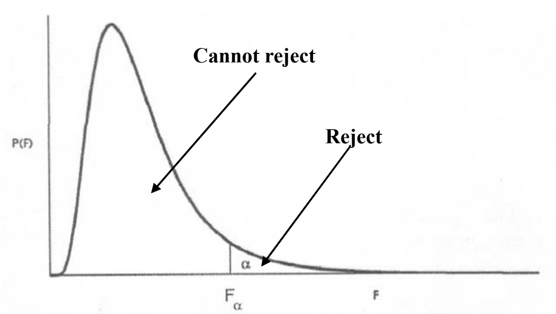
5

- If f is distributed as an F(a,b), then
- E[f] = a/(b-2)
- With a=4 and b=102, E[f]=0.04
- Therefore, if the null is correct, then the constructed $\hat{F}$ should be “small”

6

- Intuition: If the null is true, then imposing the restriction will not change the SSE much, and therefore, the numerator in $\hat{F}$ should be close to zero

7



F_α is critical value of F. There is only α chance that drawn at random, and F Would be > F_α. Therefore, if F-test < F_α, cannot reject

8

5% Critical values of F-Distribution

degrees of freedom in denominator	Degrees of Freedom in numerator: 1	2	3	4	5	6
10	4.96	4.10	3.71	3.48	3.33	3.22
11	4.84	3.98	3.59	3.36	3.20	3.09
12	4.75	3.89	3.49	3.26	3.11	3.00
13	4.67	3.81	3.41	3.18	3.03	2.92
14	4.60	3.74	3.34	3.11	2.96	2.85
15	4.54	3.68	3.29	3.06	2.90	2.79
16	4.49	3.63	3.24	3.01	2.85	2.74
17	4.45	3.59	3.20	2.96	2.81	2.70
18	4.41	3.55	3.16	2.93	2.77	2.66
19	4.38	3.52	3.13	2.90	2.74	2.63
20	4.35	3.49	3.10	2.87	2.71	2.60
21	4.32	3.47	3.07	2.84	2.68	2.57
22	4.30	3.44	3.05	2.82	2.66	2.55
23	4.28	3.42	3.03	2.80	2.64	2.53
24	4.26	3.40	3.01	2.78	2.62	2.51
30	4.17	3.32	2.92	2.69	2.53	2.42
40	4.08	3.23	2.84	2.61	2.45	2.34
60	4.00	3.15	2.76	2.53	2.37	2.25
90	3.95	3.10	2.71	2.47	2.32	2.20
120	3.92	3.07	2.68	2.45	2.29	2.18
infinity	3.84	3.00	2.61	2.37	2.21	2.10

9

Descriptive stats from ed_attain.dta

```
. sum droprate lnchprg pupil_staff salary benefit

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
    droprate |       408    5.066422    5.485072          0       61.9
     lnchprg |       408    25.20147    13.61008        1.4       79.5
 pupil_staff |       408    26.94982    26.32721   1.388206   157.9592
      salary |       408    31774.51    5038.304      19764      52812
    benefits |       408    6463.429    1456.338          0      11618
```

10

SSE_u **DOF** $\hat{\sigma}_{\varepsilon}^{2}$ *for unrest.*

```
. * run unrestricted model
. reg droprate lnchprg pupil_staff salary benefit expend

      Source |       SS       df       MS              Number of obs =     408
-------------+------------------------------           F(  5,   402) =    5.13
       Model |  734.13002     5  146.826004            Prob > F      =  0.0001
    Residual |   11510.88   402  28.6340299            R-squared     =  0.0600
-------------+------------------------------           Adj R-squared =  0.0483
       Total | 12245.0101   407  30.0860198            Root MSE      =  5.3511

------------------------------------------------------------------------------
    droprate |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     lnchprg |   .0986281   .0214838     4.59   0.000     .0563935    .1408627
 pupil_staff |   .0057106   .0117324     0.49   0.627     -.017354    .0287752
      salary |  -.0000206   .0000876    -0.23   0.814    -.0001928    .0001516
    benefits |   -.000077   .0002259    -0.34   0.733    -.0005211    .0003671
      expend |   .0004369    .000458     0.95   0.341    -.0004635    .0013372
       _cons |   1.666554   2.261232     0.74   0.462    -2.778764    6.111871
------------------------------------------------------------------------------
```

10% point ↑ in sch lunch ↑dropout rate by 0.9 % points

Zippy going on for any purchased inputs

11

Why might the results for purchased inputs be so weak?

12

```
. corr pupil_staff salary benefits expend
(obs=408)

             | pupil_~f   salary benefits   expend
-------------+------------------------------------
 pupil_staff |   1.0000
      salary |   0.5100   1.0000
    benefits |   0.3113   0.5667   1.0000
      expend |   0.3115   0.6503   0.4714   1.0000
```

13

- Hypothesis:
- H_o: $\beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$
- q=4

14

Restricted Model

SSE$_r$

```
.
. * run the brute force ftest
. * unrestricted model is one without
. * purchased inputs
. reg droprate lnchprg

      Source |       SS       df       MS              Number of obs =     408
-------------+------------------------------           F(  1,   406) =   24.42
       Model |  694.854867     1  694.854867           Prob > F      =  0.0000
    Residual |  11550.1552   406  28.4486581           R-squared     =  0.0567
-------------+------------------------------           Adj R-squared =  0.0544
       Total |  12245.0101   407  30.0860198           Root MSE      =  5.3337

------------------------------------------------------------------------------
    droprate |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     lnchprg |    .096004   .0194255     4.94   0.000     .0578168    .1341912
       _cons |    2.64698    .556227     4.76   0.000     1.553535    3.740424
------------------------------------------------------------------------------
```

What does the fact that deleting the school inputs does not change the coefficient on lnchprg much tell us?

15

5% Critical values of F-Distribution

degrees of freedom in denominator	Degrees of Freedom in numerator 1	2	3	4	5	6
10	4.96	4.10	3.71	3.48	3.33	3.22
11	4.84	3.98	3.59	3.36	3.20	3.09
12	4.75	3.89	3.49	3.26	3.11	3.00
13	4.67	3.81	3.41	3.18	3.03	2.92
14	4.60	3.74	3.34	3.11	2.96	2.85
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16	4.49	3.63	3.24	3.01	2.85	2.74
17	4.45	3.59	3.20	2.96	2.81	2.70
18	4.41	3.55	3.16	2.93	2.77	2.66
19	4.38	3.52	3.13	2.90	2.74	2.63
20	4.35	3.49	3.10	2.87	2.71	2.60
21	4.32	3.47	3.07	2.84	2.68	2.57
22	4.30	3.44	3.05	2.82	2.66	2.55
23	4.28	3.42	3.03	2.80	2.64	2.53
24	4.26	3.40	3.01	2.78	2.62	2.51
30	4.17	3.32	2.92	2.69	2.53	2.42
40	4.08	3.23	2.84	2.61	2.45	2.34
60	4.00	3.15	2.76	2.53	2.37	2.25
90	3.95	3.10	2.71	2.47	2.32	2.20
120	3.92	3.07	2.68	2.45	2.29	2.18
infinity	3.84	3.00	2.61	2.37	2.21	2.10

16

$$F = \frac{\dfrac{SSE_r - SSE_u}{q}}{\dfrac{SSE_u}{n-k-1}} = \frac{\dfrac{SSE_r - SSE_u}{q}}{\hat{\sigma}_{\varepsilon}^{2}(u)}$$

$$= \frac{\dfrac{11550.16 - 11510.88}{4}}{28.64} = 0.34$$

Cannot reject null that all purchased inputs have zero impact

- . * test that purchased inputs are all zero
- . test pupil_staff salary benefits expend
- (1) pupil_staff = 0
- (2) salary = 0
- (3) benefits = 0
- (4) expend = 0
- F(4, 402) = 0.34
- Prob > F = 0.8489

18

```
. * run unrestricted model
. reg droprate lnchprg pupil_staff salary benefit expend

      Source |       SS       df       MS              Number of obs =     408
-------------+------------------------------           F(  5,   402) =    5.13
       Model |   734.13002     5  146.826004           Prob > F      =  0.0001
    Residual |    11510.88   402  28.6340299           R-squared     =  0.0600
-------------+------------------------------           Adj R-squared =  0.0483
       Total |  12245.0101   407  30.0860198           Root MSE      =  5.3511

------------------------------------------------------------------------------
    droprate |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     lnchprg |   .0986281   .0214838     4.59   0.000     .0563935    .1408627
 pupil_staff |   .0057106   .0117324     0.49   0.627     -.017354    .0287752
      salary |  -.0000206   .0000876    -0.23   0.814    -.0001928    .0001516
    benefits |   -.000077   .0002259    -0.34   0.733    -.0005211    .0003671
      expend |   .0004369    .000458     0.95   0.341    -.0004635    .0013372
       _cons |   1.666554   2.261232     0.74   0.462    -2.778764    6.111871
------------------------------------------------------------------------------
```

This F-test is for the null hypothesis that all the non-constants are zero

$H_0: \beta_1=\beta_2= \ldots\ldots\beta_k=0$

19

5% Critical values of F-Distribution

degrees of freedom in denominator	Degrees of Freedom in numerator: 1	2	3	4	5	6
10	4.96	4.10	3.71	3.48	3.33	3.22
11	4.84	3.98	3.59	3.36	3.20	3.09
12	4.75	3.89	3.49	3.26	3.11	3.00
13	4.67	3.81	3.41	3.18	3.03	2.92
14	4.60	3.74	3.34	3.11	2.96	2.85
15	4.54	3.68	3.29	3.06	2.90	2.79
16	4.49	3.63	3.24	3.01	2.85	2.74
17	4.45	3.59	3.20	2.96	2.81	2.70
18	4.41	3.55	3.16	2.93	2.77	2.66
19	4.38	3.52	3.13	2.90	2.74	2.63
20	4.35	3.49	3.10	2.87	2.71	2.60
21	4.32	3.47	3.07	2.84	2.68	2.57
22	4.30	3.44	3.05	2.82	2.66	2.55
23	4.28	3.42	3.03	2.80	2.64	2.53
24	4.26	3.40	3.01	2.78	2.62	2.51
30	4.17	3.32	2.92	2.69	2.53	2.42
40	4.08	3.23	2.84	2.61	2.45	2.34
60	4.00	3.15	2.76	2.53	2.37	2.25
90	3.95	3.10	2.71	2.47	2.32	2.20
120	3.92	3.07	2.68	2.45	2.29	2.18
infinity	3.84	3.00	2.61	2.37	2.21	2.10

20

Cigarette tax example

- Two sources of variation in taxes: state and federal
- Should be the case that the impact on demand is the same regardless of whether it is a state or federal tax hike
- Test whether this is the case

21

Unrestricted :

$$y_i = \beta_0 + PCIR_i\beta_1 + TIME_i\beta_2 + S\_RTAX_i\beta_3 + F\_RTAX_i\beta_4 + \varepsilon_i$$

$$H_o : \beta_3 = \beta_4$$

Re*stricted* :

$$y_i = \beta_0 + PCIR_i\beta_1 + TIME_i\beta_2 + S\_F_i\beta_6 + \varepsilon_i$$

22

Generate the variables necessary for the restricted and Unrestricted model

In the unrestricted model, will include state and federal taxes by themselves

In the restricted model, will combine the taxes

```
. * generate real taxes
. gen s_rtax=state_tax/cpi

. gen f_rtax=federal_tax/cpi

. gen s_f_rtax=s_rtax+f_rtax

.
. label var s_rtax "state real tax on cigs, cents/pack"

. label var f_rtax "federal real tax on cigs, cents/pack"

. label var s_f_rtax "state+federal real tax on cigs, cents/pack"
```

23

Adding two new variables to the model

a time trend

real per capita income

```
* construct new variables
.
. * time trend
. gen trend=year-1981

.
. * real per capita income
. gen pcir=pci/cpi

.
. label var trend "=1 in 1st year, 2 in second, etc"

. label var pcir "real per capita income"
```

24

Unrestricted model

SSE_u

DOF unrest model

```
. * run regression with tax, real income, trend and two taxes
. * unrestricted model where both taxes enter
. reg packs_pc trend pcir s_rtax f_rtax

      Source |       SS       df       MS              Number of obs =    1020
-------------+------------------------------           F(  4,  1015) =  214.43
       Model |  373614.47     4  93403.6174           Prob > F      =  0.0000
    Residual | 442132.923  1015  435.598939           R-squared     =  0.4580
-------------+------------------------------           Adj R-squared =  0.4559
       Total | 815747.392  1019  800.537186           Root MSE      =  20.871

------------------------------------------------------------------------------
    packs_pc |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       trend |  -1.819112    .154649   -11.76   0.000     -2.12258   -1.515643
        pcir |   .0004883   .0001864     2.62   0.009     .0001226    .0008541
      s_rtax |  -.7979746   .0435066   -18.34   0.000    -.8833477   -.7126015
      f_rtax |  -.2996423   .1930018    -1.55   0.121    -.6783705    .0790858
       _cons |   144.4113   5.477806    26.36   0.000     133.6622    155.1604
------------------------------------------------------------------------------
```

New parameters

What evidence is there we will Reject the null?

$\hat{\sigma}_{\varepsilon}^{2}$ *for unrest.*

25

Restricted model

SSE_r

```
. * do the f-test by brute force
. * run the restricted model
. reg packs_pc trend pcir s_f_rtax

      Source |       SS       df       MS              Number of obs =    1020
-------------+------------------------------           F(  3,  1016) =  282.26
       Model | 370824.591     3  123608.197           Prob > F      =  0.0000
    Residual | 444922.802  1016  437.916143           R-squared     =  0.4546
-------------+------------------------------           Adj R-squared =  0.4530
       Total | 815747.392  1019  800.537186           Root MSE      =  20.926

------------------------------------------------------------------------------
    packs_pc |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       trend |  -1.619607   .1334016   -12.14   0.000    -1.881381   -1.357833
        pcir |   .0004559   .0001864     2.45   0.015     .0000901    .0008218
    s_f_rtax |  -.7760163   .0427458   -18.15   0.000    -.8598965   -.6921361
       _cons |   154.4029   3.807324    40.55   0.000     146.9318     161.874
------------------------------------------------------------------------------
```

26

$$F = \frac{\frac{SSE_r - SSE_u}{q}}{\frac{SSE_u}{n-k-1}} = \frac{\frac{SSE_r - SSE_u}{q}}{\hat{\sigma}_{\varepsilon}^{2}(u)}$$

$$= \frac{\frac{444922.8 - 442132.9}{1}}{435.598} = 6.40$$

27

- Degrees of freedom
  - 1 in numerator
  - 1015 in denominator

28

5% Critical values of F-Distribution

degrees of freedom in denominator	Degrees of Freedom in numerator 1	2	3	4	5	6
10	4.96	4.10	3.71	3.48	3.33	3.22
11	4.84	3.98	3.59	3.36	3.20	3.09
12	4.75	3.89	3.49	3.26	3.11	3.00
13	4.67	3.81	3.41	3.18	3.03	2.92
14	4.60	3.74	3.34	3.11	2.96	2.85
15	4.54	3.68	3.29	3.06	2.90	2.79
16	4.49	3.63	3.24	3.01	2.85	2.74
17	4.45	3.59	3.20	2.96	2.81	2.70
18	4.41	3.55	3.16	2.93	2.77	2.66
19	4.38	3.52	3.13	2.90	2.74	2.63
20	4.35	3.49	3.10	2.87	2.71	2.60
21	4.32	3.47	3.07	2.84	2.68	2.57
22	4.30	3.44	3.05	2.82	2.66	2.55
23	4.28	3.42	3.03	2.80	2.64	2.53
24	4.26	3.40	3.01	2.78	2.62	2.51
30	4.17	3.32	2.92	2.69	2.53	2.42
40	4.08	3.23	2.84	2.61	2.45	2.34
60	4.00	3.15	2.76	2.53	2.37	2.25
90	3.95	3.10	2.71	2.47	2.32	2.20
120	3.92	3.07	2.68	2.45	2.29	2.18
infinity	3.84	3.00	2.61	2.37	2.21	2.10

29

Constructing the test in STATA

- `* run regression with tax, real income, trend and two`
- `* unrestricted model where both taxes enter`
- `reg packs_pc trend pcir s_rtax f_rtax`

- `*test the hypothesis that the coef is the same`
- `test s_rtax=f_rtax`

Right after you estimate the unrestricted model, you can perform all the F-tests necessary

30

Results

```
( 1)  s_rtax - f_rtax = 0

      F(  1,  1015) =    6.40

           Prob > F =    0.0115
```

Can easily reject null in this instance

31

Cobb-Douglas Example

- Unrestricted
  - $\ln(q_t) = \beta_0 + \beta_l \ln(l_t) + \beta_k \ln(k_t) + \varepsilon_t$
- H_o: $\beta_l + \beta_k = 1$
  - $\beta_k = 1 - \beta_l$

- Restricted model
  - $\ln(q_t) = \beta_0 + \beta_l \ln(l_t) + (1-\beta_1)\ln(kt) + \varepsilon_t$
  - $\ln(q_t) - \ln(k_t) = \beta_0 + \beta_l[\ln(lt) - \ln(k_t)] + \varepsilon_t$

32

Data

- Annual data on Q, K and L from the cigarette manufacturing industry
- 1970-2000, 31 observations
- DOF = n-k-1 = 31-2-1 =28
- q=1


```
Contains data from cig_industry.dta
  obs:            31
 vars:             4                          28 Sep 2008 11:53
 size:           682 (99.9% of memory free)
-------------------------------------------------------------------------------
              storage  display     value
variable name   type   format      label      variable label
-------------------------------------------------------------------------------
year            int    %8.0g
q               float  %9.0g                  output, billions of cigarettes
l               float  %9.0g                  thousands of workers
                                                (production+nonprod)
k               float  %9.0g                  capital assets index, 1987=100
-------------------------------------------------------------------------------
```



```
* generate logs of all variables

gen kl=ln(k)
gen ll=ln(l)
gen ql=ln(q)

* run unrestricted model
reg ql ll kl

* test for constant returns to scale
test kl+ll=1
```


SSE_u DOF $\hat{\sigma}_e^2(u)$

```
. * run unrestricted model
. reg ql ll kl

      Source |       SS       df       MS              Number of obs =      31
-------------+------------------------------           F(  2,    28) =   10.22
       Model |  .063440086     2  .031720043           Prob > F      =  0.0005
    Residual |  .08691176     28  .003103991           R-squared     =  0.4219
-------------+------------------------------           Adj R-squared =  0.3807
       Total |  .150351846    30  .005011728           Root MSE      =  .05571

------------------------------------------------------------------------------
          ql |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          ll |   .1009336   .0539433     1.87   0.072    -.0095644    .2114315
          kl |   .2581351   .0574695     4.49   0.000     .1404142     .375856
       _cons |   4.999207   .3941311    12.68   0.000     4.191866    5.806548
------------------------------------------------------------------------------
```

- * generate variables for brute force model
- gen ql_kl=ql-kl
- gen ll_kl=ll-kl

- * run restricted model
- reg ql_kl ll_kl


```
. * run restricted model
. reg ql_kl ll_kl

      Source |       SS       df       MS              Number of obs =      31
-------------+------------------------------           F(  1,    29) =   89.60
       Model |  .687494683     1  .687494683           Prob > F      =  0.0000
    Residual |  .222525857    29  .007673305           R-squared     =  0.7555
-------------+------------------------------           Adj R-squared =  0.7470
       Total |   .91002054    30  .030334018           Root MSE      =   .0876

------------------------------------------------------------------------------
       ql_kl |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       ll_kl |   .4080094   .0431049     9.47   0.000     .3198499    .4961688
       _cons |   2.398959   .0380002    63.13   0.000      2.32124    2.476678
------------------------------------------------------------------------------
```


$$F = \frac{\frac{SSE_r - SSE_u}{q}}{\frac{SSE_u}{n-k-1}} = \frac{\frac{SSE_r - SSE_u}{q}}{\hat{\sigma}_{\varepsilon}^{2}(u)}$$

$$= \frac{\frac{0.222525857 - 0.08691176}{1}}{0.0031} = 43.69$$

5% Critical values of F-Distribution

degrees of freedom in denominator	Degrees of Freedom in numerator: 1	2	3	4	5	6
10	4.96	4.10	3.71	3.48	3.33	3.22
11	4.84	3.98	3.59	3.36	3.20	3.09
12	4.75	3.89	3.49	3.26	3.11	3.00
13	4.67	3.81	3.41	3.18	3.03	2.92
14	4.60	3.74	3.34	3.11	2.96	2.85
15	4.54	3.68	3.29	3.06	2.90	2.79
16	4.49	3.63	3.24	3.01	2.85	2.74
17	4.45	3.59	3.20	2.96	2.81	2.70
18	4.41	3.55	3.16	2.93	2.77	2.66
19	4.38	3.52	3.13	2.90	2.74	2.63
25	4.24	3.39	2.99	2.76	2.60	2.49
26	4.23	3.37	2.98	2.74	2.59	2.47
27	4.21	3.35	2.96	2.73	2.57	2.46
28	4.20	3.34	2.95	2.71	2.56	2.45
29	4.18	3.33	2.93	2.70	2.55	2.43
30	4.17	3.32	2.92	2.69	2.53	2.42
40	4.08	3.23	2.84	2.61	2.45	2.34
60	4.00	3.15	2.76	2.53	2.37	2.25
90	3.95	3.10	2.71	2.47	2.32	2.20
120	3.92	3.07	2.68	2.45	2.29	2.18
infinity	3.84	3.00	2.61	2.37	2.21	2.10

```
.* test for constant returns to scale
. test kl+ll=1
( 1)  ll + kl = 1
F(  1,    28) =   43.69
    Prob > F =    0.0000
```

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