Suggested Answers, Problem Set 1
ECON 30331

Bill Evans
Spring 2018

1. a. $$\int_{50}^{100}\frac{dx}{50}=\frac{x}{50}\Big|_{50}^{100}=\frac{100-50}{50}=1$$

b. $$\Pr(x\le 65)+\Pr(X>90)=\int_{50}^{65}\frac{dx}{50}+\int_{90}^{100}\frac{dx}{50}=\frac{x}{50}\Big|_{50}^{65}+\frac{x}{50}\Big|_{90}^{100}=$$
$$\frac{65-50}{50}+\frac{100-90}{50}=0.30+0.20=0.50$$

2. a. $\Pr(x\le a)=\int_0^a \beta e^{-\beta x}dx.$ Let $-\beta x=u$ and $-\beta dx=du$

$$\Pr(x\le a)=\int_0^a -e^u du=-\mathrm{e}^u\Big|_0^a=-\mathrm{e}^{-\beta x}\Big|_0^a=-\mathrm{e}^{-\beta a}--e^0=1-\mathrm{e}^{-\beta a}$$

b. $\Pr(x\le a)=1-\mathrm{e}^{-\beta a}$ if β=0.001 and a=500

$$\Pr(x\le 500)=1-\mathrm{e}^{-0.001(500)}=1-e^{-0.5}=0.393$$

c. $$\Pr(x>1500)=1-\Pr(\mathrm{x}\le 1500)=1-(1-\mathrm{e}^{-0.001(1500)})=e^{-0.001(1500)}=e^{-1.5}=0.223$$

3. a. Prob(Z ≤ 0.35)=0.6368
b. Prob(Z > -1.50)= 1 - Prob(Z ≤ -1.5) = 1 - 0.0669 = 0.9332
a. Prob(-1.22< z ≤ 0.57)= Prob(z ≤ 0.57) – Prob(z ≤ -1.22) = 0.7157 – 0.1112 = 0.6045

4. Let s be snowfall
$$s\sim N(\mu,\sigma^2)=N(64,12^2)$$

a. $$\Pr(r\le 12)=\Pr\left(z=\frac{s-\mu}{\sigma}\le\frac{12-\mu}{\sigma}\right)=\Pr\left(z\le\frac{12-64}{12}\right)=\Pr\left(z\le -4.33\right)=\Phi(-4.33)=0.000$$
It is essentially zero

b. $$\Pr(r>80)=1-\Pr(r\le 80)=1-\Pr\left(z=\frac{r-\mu}{\sigma}\le\frac{80-\mu}{\sigma}\right)=1-\Pr\left(z\le\frac{80-64}{12}\right)$$
$$=1-\Pr\left(z\le 1.33\right)=1-\Phi(1.33)=1-0.9082=0.0918$$

5. If you plot out the points, you will see that the points lie along a positively-sloped straight line, so X perfectly predicts Y and vice versa. This means that the estimated correlation coefficient $\hat{\rho}$ must equal 1.

6. I=a + bGPA + cLSAT =15GPA + LSAT
a=0, b=800, c=1

$\mu_g = 3.2$ and $\mu_{lsat} = 150$ $\sigma_g = 1.2$ and $\sigma_s = \varrho_{gs} = 0.5$ $\sigma_{gs} = \varrho_{gs}\ \sigma_g\ \sigma_s$

$E[I] = 15\ \mu_g + \mu_{last} = 15(3.2) + 150 = 198$

$Var(I) = b^2\ \sigma_g{}^2 + c^2\ \sigma_{last}{}^2 + 2bc\sigma_{gl}$

Because I did not give you σ_{gl} but instead, I provided ϱ_{gl}, you must use the fact that $\sigma_{gs} = \varrho_{gs}\ \sigma_g\ \sigma_s$

$$Var(I) = 15^2\ (1.2)^2 + (1^2)(5)^2 + 2(15)(1)(1.2)(5)(0.5) = 439$$

7. a. Pr(Died) = 0.556
   b. Pr(Was wearing a helmet) = 0.496
   c. Pr(Died | Was wearing a helmet) = Pr (D |W) = Pr(D∩W)/Pr(W)
      = 0.213/0.496 = 0.429
   d. Pr(Died | Was not wearing a helmet) = Pr(D|~W) = Pr(D∩~W)/Pr(~W)
      = 0.343/0.504 = 0.681
   e. The probability of death in a severe crash falls from 0.681 to 0.429 when wearing a helmet. This is a (0.429-0.681)/0.681 = -0.37 or a 37% reduction in the risk

If you look at the NHTSA document I linked to in the problem set, it notes that the chance of mortality falls by 37% for someone wearing a helmet.

8. In this case, $\bar{x} = 18$ and $s = 8$ and n=25. The 95% confidence interval for μ is:

$$\mu = \bar{x} \pm t(n-1)_{0.025}\ s/n^{0.5}$$

$$t(n-1)_{0.025} = t(15)_{0.025} = 2.064$$

$$\mu = \bar{x} \pm t(n-1)_{0.025}\ s/n^{0.5} = 18 \pm 2.064(8/25^{0.5}) = 18 \pm 3.30 = [14.7, 21.03]$$

$$H_o: \mu = 21$$

Using the confidence interval, we see that 21 is within the confidence interval so we cannot reject the null that the company's statement is correct.

9. a. The 95% confidence interval is

$$d = \Delta \pm t(n_1+n_2-2)s_p[(1/n_1) + (1/n_2)]^{0.5}$$

$$t(20+20-2)_{0.025} = t(38)_{0.025} = 2.024$$

$$s^2_p = [(n_1-1)s^2_w + (n_2-1)s^2_l]/(n_1+n_2-1) = [19*12^2 + 19*8^2]/38 = 104$$

$$s_p = 10.2$$

$$((1/n_1) + (1/n_2))^{0.5} = ((1/20) + (1/20))^{0.5} = 0.316$$

$$x_1 - x_2 = -16 - -7 = -9$$

$d = \Delta \pm t(n_1+n_2-2)s_p[(1/n_1) + (1/n_2)]^{0.5} = -9 \pm 2.024(10.2)(0.316) = -9 \pm 6.3 = [-15.53,-2.47]$

b. Since 0 is not in the confidence interval, we can reject the null that the weight loss is the same in the two progrms.

c. The t-ratio for this problem is $t = | (\Delta - d)/(se(\Delta)|$. Recall that the $se(\Delta) = s_p ((1/n_1) + (1/n_2))^{0.5} = 10.2*.316 = 3.22$, so $t = | (\Delta - d)/(se(\Delta)| = | (-16 - -7)/3.22| = 2.795$ which falls above the 99% critical value of 2.712. So in this case, we can still reject the null.

10. Given a linear combination of random variables, $Z = a + bY_1 + cY_2$, the handout demonstrates that $Var(Z) = b^2Var(Y_1) + c^2Var(Y_2) + 2bcCov(Y_1,Y_2)$. In this case $Z = Y_1 + Y_2$ so a=0, b=1 and c=1. We are also given the fact that Y_1 and Y_2 have the same variance, σ^2_y, plus Y_1 and Y_2 are independent so we also know that $Cov(Y_1,Y_2) = \sigma_{12} = 0$. Plugging all these values into the equation above, $Var(Z) = 2\,\sigma^2_y$

11. Given the results above, if $Z = Y_1 + Y_2 + Y_3 + \ldots\ldots Y_n$ and each value Y_i is independent of Y_j, then $Var(Z) = Var(Y_1 + Y_2 + Y_3 + \ldots\ldots Y_n) = Var(Y_1) + Var(Y_2) + \ldots\ldots Var(Y_n) = n\sigma^2_y$