Hypothesis tests for one parameter

Couple of definitions

- Standard normal distribution
  $z_i \sim N(0,1)$
- Normal distribution
  $y_i \sim N(\mu, \sigma^2)$
- Normal can always be turned into a standard normal by subtracting mean and dividing by standard deviation
  $z_i = (y_i - \mu)/\sigma \sim N(0,1)$

Some Prob/Stat Review

- y_i is a normal random variable
- $y_i \sim N(0,\sigma^2)$
- Suppose there are n independent y_i's

$$W_1 = \sum_{i=1}^{n} y_i$$

- Then $W_1 \sim N(0,n\sigma^2)$
- (Last question on Problem set #1)

Likewise

- $y_i \sim N(0,\sigma^2)$
- Suppose there are n independent y_i's

$$W_2 = \sum_{i=1}^{n} b_i y_i$$

- Where b_i is a constant

$$W_2 \sim N\left[0, \left(\sum_{i=1}^{n} b_i^2\right)\sigma^2\right]$$

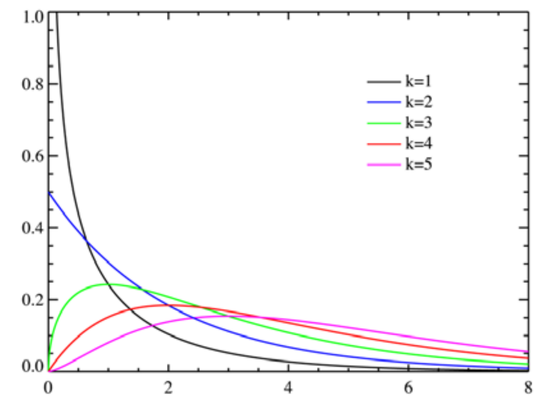
Some Prob/Stat Review

- z_i is a standard normal random variable
- $z_i \sim N(0,1)$
- Suppose there are n independent z_i's

$$W_3 = \sum_{i=1}^{n} z_i^2$$

- Then W_3 is a Chi-squared random variable with n degrees of freedom

5



[In this graph, K is the degrees of freedom] 6

- Suppose
  - W_k is a chi-squared distribution with k degrees of freedom
  - W_n is a chi-squared distribution with n degrees of freedom
- Then $S=(W_k/K)/(W_n/n)$ is an F distribution with (k,n) degrees of freedom
- Defined over all S>0

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- Suppose z is standard normal z~N[0,1]
- Suppose W is a chi-squared with n degrees of freedom

$$t = \frac{z}{\sqrt{\frac{W}{n}}}$$

is distributed as a student t with n DOF

8

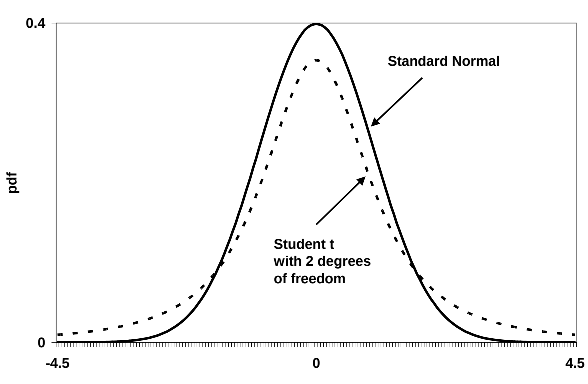
Student t

- William Sealy Gosset
  - (1876-1937)
- Statistician
- Employee of Guinness
- Used statistical models to isolate the highest yielding varieties of barley
- Homer is correct

9

- Symmetric, uni-modal PDF
- Defined over all real numbers
- Shape is a function of the degrees of freedom
- Sigmoid CDF
- E[t]=0
- V[t]>1 but approaches 1 as DOF approach ∞
- PDF shape very similar to standard normal with "fatter tails"

10



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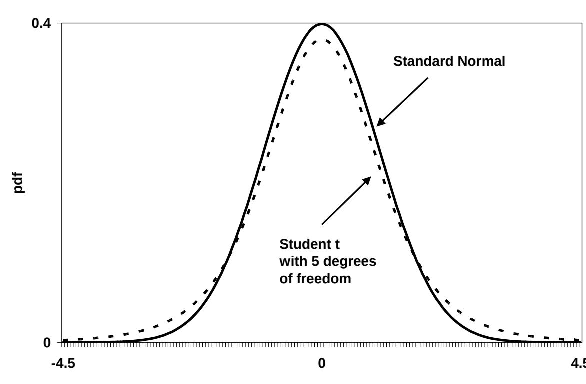

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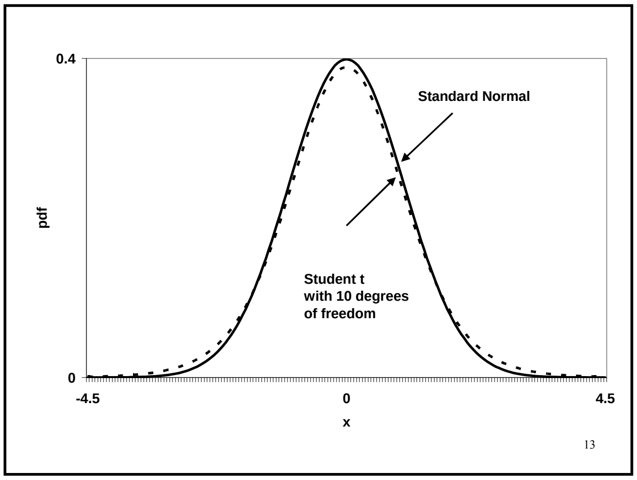
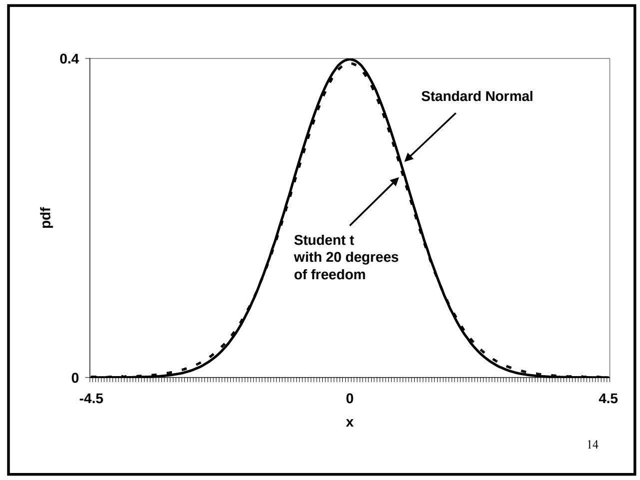
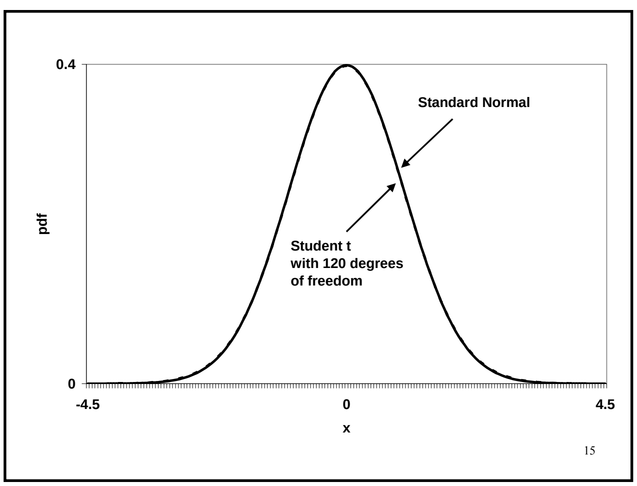
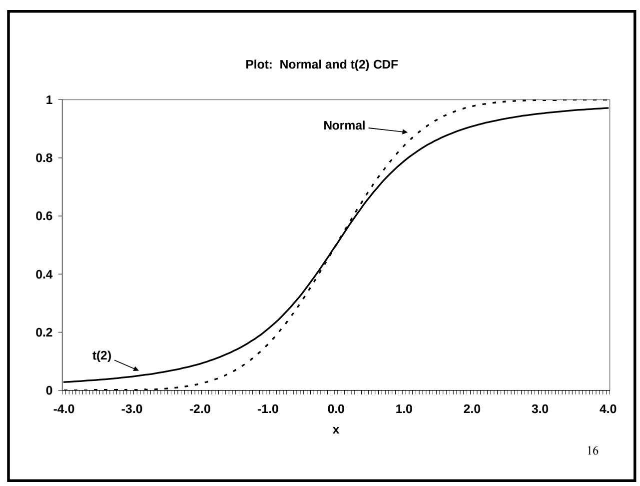
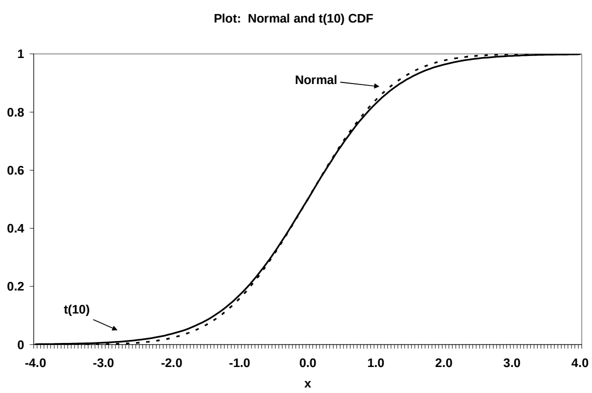

0.4
0
pdf
Standard Normal
Student t
with 10 degrees
of freedom
-4.5
0
4.5
x
13

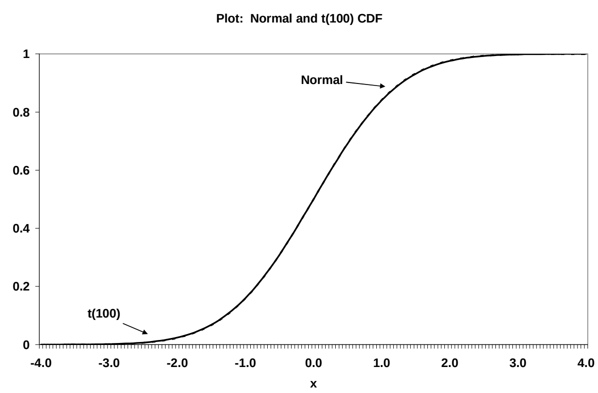

0.4
0
pdf
Standard Normal
Student t
with 20 degrees
of freedom
-4.5
0
4.5
x
14


0.4
0
pdf
Standard Normal
Student t
with 120 degrees
of freedom
-4.5
0
4.5
x
15


Plot: Normal and t(2) CDF
1
0.8
0.6
0.4
0.2
0
Normal
t(2)
-4.0
-3.0
-2.0
-1.0
0.0
1.0
2.0
3.0
4.0
x
16



17



18

Normality of ε

- $y_i = \beta_0 + x_{1i}\beta_1 + x_{2i}\beta_2 + \ldots. x_{ki}\beta_k + \varepsilon_i$
- There are n observations
- k+1 parameters to be estimated
- n-k-1 degrees of freedom
- Assume ε_i is normally distributed.
- What does that assumption buy us?

19

$$y_i = \beta_0 + x_i\beta_1 + \varepsilon_i$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{\sum_{i=1}^{n}(x_i - \overline{x})^2} = \beta_1 + \frac{\sum_{i=1}^{n}(x_i - \overline{x})\varepsilon_i}{\sum_{i=1}^{n}(x_i - \overline{x})^2}$$

$$= \beta_1 + \sum_{i=1}^{n} w_i\varepsilon_i \qquad where \quad w_i \frac{(x_i - \overline{x})}{\sum_{i=1}^{n}(x_i - \overline{x})^2}$$

20

- Note that $\hat{\beta}_1$ is a linear estimator, that is

$$\hat{\beta}_1 = \beta_1 + \sum_{i=1}^{n} w_i \varepsilon_i$$

- Note $\hat{\beta}_1$ is a linear function of the ε_i's
- A linear function of normal variables is also normally distributed
- Since the ε_i's are assumed to be normal….

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then $\hat{\beta}_1$ *is normally distributed*

$$\hat{\beta}_1 \sim Normal[\beta_1, V(\beta_1)]$$

$$E[\hat{\beta}_1] = \beta_1$$

$$V(\hat{\beta}_1) = \frac{\sigma_\varepsilon^2}{\sum_{i=1}^{n}(x_i - \overline{x})^2}$$

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General case: $y_i = \beta_0 + x_{1i}\beta_1 + x_{2i}\beta_2 + \ldots.x_{ki}\beta_k + \varepsilon_i$

then $\hat{\beta}_j$ *is normally distributed*

$$\hat{\beta}_j \sim Normal[\beta_j, V(\beta_j)]$$

$$E[\hat{\beta}_j] = \beta_j$$

$$V(\hat{\beta}_j) = \frac{\sigma_\varepsilon^2}{SST_j(1 - R_j^2)}$$

where $SST_j = \sum_{i=1}^{n}(x_{ji} - \overline{x}_j)^2$ *and* $R_j^2 =$ *the* R^2 *from the regression of* x_j *on all the other* x*'s*

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because $\hat{\beta}_j$ *is normally distributed, we could use the std. normal distribution for test of hypotheses IF WE KNEW* σ_ε^2

$$\frac{\hat{\beta}_j - \beta_j}{\sqrt{V(\hat{\beta}_j)}} = \frac{\hat{\beta}_j - \beta_j}{\sqrt{\dfrac{\sigma_\varepsilon^2}{SST_j(1 - R_j^2)}}} \sim N(0,1)$$

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Problem?

- σ^2_ε is unknown and must be estimated
- Unbiased estimate is

$$\hat{\sigma}_\varepsilon^2 = \frac{\sum_{i=1}^{n} \hat{\varepsilon}_i^{\,2}}{n-k-1}$$

25

$$\hat{\varepsilon}_i = y_i - \hat{\beta}_0 - x_{1i}\hat{\beta}_1 - x_{2i}\hat{\beta}_2 - \dots x_{ki}\hat{\beta}_k$$

each $\hat{\beta}_j$ *is normally distributed*

therefore, $\hat{\varepsilon}_i$ *a linear combination of normally distributed variables*

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$$\hat{\sigma}_\varepsilon^2 = \frac{\sum_{i=1}^{n} \hat{\varepsilon}_i^{\,2}}{n-k-1}$$

The numerator in the estimate looks something like a chi-squared. But because $\hat{\varepsilon}_i \sim N(0, \sigma_\varepsilon^2)$ *(it does not have a var. of 1) it is not exactly in the correct form.*

Technically, $(n-k-1)\hat{\sigma}_\varepsilon^2 / \sigma_\varepsilon^2 \sim \chi^2(n-k-1)$ 27

- Because the n observations are already used to get k+1 parameters, there are only n-k-1 unique estimated errors
- Therefore, the degrees of freedom of the chi-squared distribution are n-k-1

28

Recall that

$$\frac{\hat{\beta}_j - \beta_j}{\sqrt{V(\hat{\beta}_j)}} = \frac{\hat{\beta}_j - \beta_j}{\sqrt{\dfrac{\sigma_\varepsilon^2}{SST_j(1-R_j^2)}}} \sim N(0,1)$$

and $$\frac{(n-k-1)\hat{\sigma}_\varepsilon^2}{\sigma_\varepsilon^2} \sim \chi^2(n-k-1)$$

and $$t = \frac{z}{\sqrt{\dfrac{W}{n}}} \sim t(n)$$

29

The theoretical variance for $\hat{\beta}_j$

$$V(\hat{\beta}_j) = \frac{\sigma_\varepsilon^2}{SST_j(1-R_j^2)}$$

The estimated variance is then

$$\hat{V}(\hat{\beta}_j) = \frac{\hat{\sigma}_\varepsilon^2}{SST_j(1-R_j^2)}$$

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Standard normal

$$\frac{\dfrac{\hat{\beta}_j - \beta_j}{\sqrt{V(\hat{\beta}_j)}}}{\sqrt{\dfrac{\dfrac{(n-k-1)\hat{\sigma}_\varepsilon^2}{\sigma_\varepsilon^2}}{n-k-1}}} = \frac{N(0,1)}{\sqrt{\dfrac{\chi^2(n-k-1)}{n-k-1}}} \sim t(n-k-1)$$

Chi-squared

Degrees of freedom

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$$\frac{\dfrac{\hat{\beta}_j - \beta_j}{\sqrt{\dfrac{\sigma_\varepsilon^2}{SST_j(1-R_j^2)}}}}{\sqrt{\dfrac{\left[\dfrac{(n-k-1)\hat{\sigma}_\varepsilon^2}{\sigma_\varepsilon^2}\right]}{n-k-1}}} = \frac{\dfrac{\hat{\beta}_j - \beta_j}{\sqrt{\dfrac{\sigma_\varepsilon^2}{SST_j(1-R_j^2)}}}}{\sqrt{\dfrac{(n-k-1)\hat{\sigma}_\varepsilon^2}{(n-k-1)\sigma_\varepsilon^2}}} = \frac{\hat{\beta}_j - \beta_j}{\sqrt{\dfrac{\hat{\sigma}_\varepsilon^2}{SST_j(1-R_j^2)}}}\left(\frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2}\right)$$

$$= \frac{\hat{\beta}_j - \beta_j}{\sqrt{\dfrac{\hat{\sigma}_\varepsilon^2}{SST_j(1-R_j^2)}}} = \frac{\hat{\beta}_j - \beta_j}{\sqrt{Est.Var(\hat{\beta}_j)}} \sim t(n-k-1)$$

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Instead of working with

$$\frac{\hat{\beta}_j - \beta_j}{\sqrt{\dfrac{\sigma_\varepsilon^2}{SST_j(1-R_j^2)}}} \sim N(0,1)$$

we use

$$\frac{\hat{\beta}_j - \beta_j}{\sqrt{\dfrac{\hat{\sigma}_\varepsilon^2}{SST_j(1-R_j^2)}}} \sim t(n-k-1)$$

33

$se(\hat{\beta}_j)$ standard error of $\hat{\beta}_j$

$$se(\hat{\beta}_j) = \sqrt{\frac{\hat{\sigma}_\varepsilon^2}{SST_j(1-R_j^2)}}$$

$$\frac{\hat{\beta}_j - a}{se(\hat{\beta}_j)} \sim t(n-k-1)$$

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Testing Hypotheses about a Single Parameter: 2 tailed tests

- Basic model
- $y_i = \beta_0 + x_{1i}\beta_1 + x_{2i}\beta_2 + \ldots\, x_{ki}\beta_k + \varepsilon_i$
- Economic theory suggests a particular value of the parameter
- H_0: $\beta_j = a$
- H_a: $\beta_j \neq a$

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Two-tailed test

- These are called two tail tests because falsification of the null hypothesis can be due to either large + or – values (in absolute value)
- Therefore, we use both "tails" of the underlying t-distribution

36

- The distribution for $\hat{\beta}_j$

$$\frac{\hat{\beta}_j - a}{se(\hat{\beta}_j)} \sim t(n-k-1)$$

- Given the hypothesis is true, we can replace "a" for β_j

37

$$\hat{t} = \frac{\hat{\beta}_j - a}{se(\hat{\beta}_j)} \sim t(n-k-1)$$

- If the hypothesis is true, the constructed test statistic should be centered on zero. How "far" from zero does it have to be to reject the null?

38

- Need to set the "confidence level" of the test.
  - Usually 95%
- Let 1-confidence level = α
  - With 95% confidence level, α =0.05
- α is the probability you reject the null when it is in fact true …return to this later

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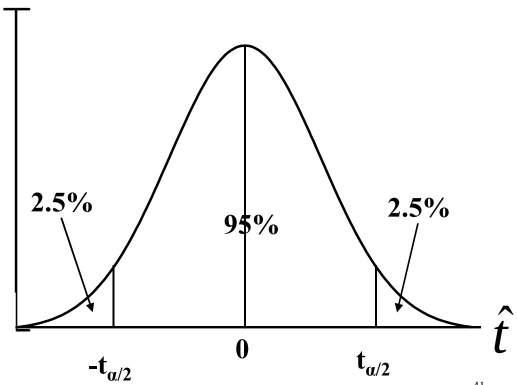
- $t_{\alpha/2}$(dof) be the cut-off from a t-distribution with dof degrees of freedom where only $\alpha/2$ percent of the distribution lies above
- Given symmetry, $\alpha/2$ percent lies below $-t_{\alpha/2}$(dof)

40



41

- If we were to draw $\hat{t}$ at random, 95% of the time it would be between ($-t_{\alpha/2}$, $t_{\alpha/2}$)

- Therefore, if the hypothesized value of β_j is true, there is a 95% chance $\hat{t}$ will be between ($-t_{\alpha/2}$, $t_{\alpha/2}$)

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Decision Rule

$$\hat{t} = \frac{\hat{\beta}_j - a}{se(\hat{\beta}_j)} \sim t(n-k-1)$$

$$if\ |\hat{t}\,| \geq t_{\alpha/2}(n-k-1)\ reject\ null$$
$$if\ |\hat{t}\,| < t_{\alpha/2}(n-k-1)\ cannot\ reject\ null$$

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Most basic test

- $y_i = \beta_0 + x_{1i}\,\beta_1 + x_{2i}\,\beta_2 + \ldots.\ x_{ki}\,\beta_k + \varepsilon_i$

- H_0: $\beta_j = 0$
- H_a: $\beta_j \neq 0$

- Is the parameter estimate statistically distinguishable from zero?

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Baseball example

- Regress attendance on wins – do winning teams attract more fans
- Data on 30 teams, 2 parameters, DOF=n-2=28
- Look at table in the back of the book for critical value of t
  - Vertical axis is DOF
  - Horizontal axis is the value of α

45

Significance Level

one-tailed tests		**0.100**	**0.050**	**0.025**	**0.010**	**0.005**
two-tailed tests		**0.200**	**0.100**	**0.050**	**0.020**	**0.010**
	1	**3.078**	**6.314**	**12.706**	**31.821**	**63.657**
	2	**1.886**	**2.920**	**4.303**	**6.965**	**9.925**
	3	**1.638**	**2.353**	**3.182**	**4.541**	**5.841**
	4	**1.533**	**2.132**	**2.776**	**3.747**	**4.604**
Degrees	**5**	**1.476**	**2.015**	**2.571**	**3.365**	**4.032**
of	**15**	**1.341**	**1.753**	**2.131**	**2.602**	**2.947**
freedom	**16**	**1.337**	**1.746**	**2.120**	**2.583**	**2.921**
	17	**1.333**	**1.740**	**2.110**	**2.567**	**2.898**
	18	**1.330**	**1.734**	**2.101**	**2.552**	**2.878**
	19	**1.328**	**1.729**	**2.093**	**2.539**	**2.861**
	20	**1.325**	**1.725**	**2.086**	**2.528**	**2.845**
	21	**1.323**	**1.721**	**2.080**	**2.518**	**2.831**
	22	**1.321**	**1.717**	**2.074**	**2.508**	**2.819**
	23	**1.319**	**1.714**	**2.069**	**2.500**	**2.807**
	24	**1.318**	**1.711**	**2.064**	**2.492**	**2.797**
	25	**1.316**	**1.708**	**2.060**	**2.485**	**2.787**
	26	**1.315**	**1.706**	**2.056**	**2.479**	**2.779**
	27	**1.314**	**1.703**	**2.052**	**2.473**	**2.771**
	28	**1.313**	**1.701**	**2.048**	**2.467**	**2.763**
	29	**1.311**	**1.699**	**2.045**	**2.462**	**2.756**
	30	**1.310**	**1.697**	**2.042**	**2.457**	**2.750**

P-value for two tailed test

46

Significance Level

one-tailed tests		**0.100**	**0.050**	**0.025**	**0.010**	**0.005**
two-tailed tests		**0.200**	**0.100**	**0.050**	**0.020**	**0.010**
	1	**3.078**	**6.314**	**12.706**	**31.821**	**63.657**
	2	**1.886**	**2.920**	**4.303**	**6.965**	**9.925**
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	28	**1.313**	**1.701**	**2.048**	**2.467**	**2.763**
	29	**1.311**	**1.699**	**2.045**	**2.462**	**2.756**
	30	**1.310**	**1.697**	**2.042**	**2.457**	**2.750**

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```
. * run simple regression
. reg attendance wins

      Source |       SS       df       MS              Number of obs =      30
-------------+------------------------------           F(  1,    28) =    8.89
       Model |   606784507     1   606784507           Prob > F      =  0.0059
    Residual |  1.9110e+09    28  68249360.1           R-squared     =  0.2410
-------------+------------------------------           Adj R-squared =  0.2139
       Total |  2.5178e+09    29  86819537.6           Root MSE      =  8261.3

------------------------------------------------------------------------------
  attendance |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        wins |   310.0473   103.9825     2.98   0.006     97.04894    523.0457
       _cons |    3095.14   8539.507     0.36   0.720    -14397.25    20587.53
------------------------------------------------------------------------------
```

$$\hat{t} = \frac{\hat{\beta}_j - a}{se(\hat{\beta}_j)} = \frac{310.05 - 0}{103.98} = 2.98$$

$$|\hat{t}| = |2.98| > t_{\alpha/2}(n-k-1) = 2.048 \quad \therefore \textit{reject null}$$

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Statistical significance

- When we reject the null hypothesis that H_0: $\beta_j=0$, we say that a variable is "statistically significant"
- Which is short hand for saying the variable is statistically distinguishable from 0
- Statistically insignificant variables are those that we cannot reject the null H_0: $\beta_j=0$

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College GPA example

- Data on 141 students
- 2 continuous variables:
  - HS GPA
  - ACT Score
- One intercept
- DOF = n-k-1 = 141-3 = 138
- On the table, there is no 138, just find the closest one

50

Significance Level

one-tailed tests		0.100	0.050	0.025	0.010	0.005
two-tailed tests		0.200	0.100	0.050	0.020	0.010
Degrees of Freedom	1	3.078	6.314	12.706	31.821	63.657
	2	1.886	2.920	4.303	6.965	9.925
	3	1.638	2.353	3.182	4.541	5.841
	4	1.533	2.132	2.776	3.747	4.604
	5	1.476	2.015	2.571	3.365	4.032
	15	1.341	1.753	2.131	2.602	2.947
	35	1.306	1.690	2.030	2.438	2.724
	36	1.306	1.688	2.028	2.434	2.719
	37	1.305	1.687	2.026	2.431	2.715
	38	1.304	1.686	2.024	2.429	2.712
	39	1.304	1.685	2.023	2.426	2.708
	40	1.303	1.684	2.021	2.423	2.704
	60	1.296	1.671	2.000	2.390	2.660
	90	1.291	1.662	1.987	2.368	2.632
	120	1.289	1.658	1.980	2.358	2.617
	infinity	1.282	1.645	1.960	2.326	2.576

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```
. * run multivariate regression
. reg college_gpa act hs_gpa

      Source |       SS       df       MS              Number of obs =     141
-------------+------------------------------           F(  2,   138) =   14.78
       Model |  3.42365506     2  1.71182753           Prob > F      =  0.0000
    Residual |  15.9824444   138  .115814814           R-squared     =  0.1764
-------------+------------------------------           Adj R-squared =  0.1645
       Total |  19.4060994   140  .138614996           Root MSE      =  .34032

------------------------------------------------------------------------------
 college_gpa |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         act |    .009426   .0107772     0.87   0.383    -.0118838    .0307358
      hs_gpa |   .4534559   .0958129     4.73   0.000     .2640047    .6429071
       _cons |   1.286328   .3408221     3.77   0.000      .612419    1.960237
------------------------------------------------------------------------------
```

$$\hat{t}_{act} = 0.87$$

$$|\hat{t}_{act}| < 1.98 \quad \therefore \textit{cannot reject null}$$

$$\hat{t}_{hsgpa} = 4.73$$

$$|\hat{t}_{hsgpa}| \geq 1.98 \quad \therefore \textit{ reject null}$$

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Confidence intervals

- The CI represent the 95% most likely values of the parameter β_j
- If the hypothesized value "a" (H_0: β_j=a) is not part of the confidence interval, it is not a likely value and we reject the null
- If interval contains "a" we cannot reject null
- The t-test and CI should provide the same decision – if not, you did something wrong

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Confidence intervals

if the null is true, then

$$-t_{\alpha/2}(n-k-1) \le \frac{\hat{\beta}_j - a}{se(\hat{\beta}_j)} \le t_{\alpha/2}(n-k-1)$$

which means that

$$\hat{\beta}_j - se(\hat{\beta}_j)t_{\alpha/2}(n-k-1) \ \le \ a$$

$$\le \ \hat{\beta}_j + se(\hat{\beta}_j)t_{\alpha/2}(n-k-1)$$

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Confidence interval

$$\hat{\beta}_j \pm se(\hat{\beta}_j)t_{\alpha/2}(n-k-1)$$

55

```
. * run simple regression
. reg attendance wins

      Source |       SS       df       MS              Number of obs =      30
-------------+------------------------------           F(  1,    28) =    8.89
       Model |   606784507     1   606784507           Prob > F      =  0.0059
    Residual |  1.9110e+09    28  68249360.1           R-squared     =  0.2410
-------------+------------------------------           Adj R-squared =  0.2139
       Total |  2.5178e+09    29  86819537.6           Root MSE      =  8261.3

------------------------------------------------------------------------------
  attendance |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        wins |   310.0473   103.9825     2.98   0.006     97.04894    523.0457
       _cons |    3095.14   8539.507     0.36   0.720    -14397.25    20587.53
------------------------------------------------------------------------------
```

confidence int.

$$\hat{\beta}_j \pm t_{\alpha/2}(n-k-1)se(\hat{\beta}_j)$$

$$310.05 \pm 2.0484(103.98)$$

$$[97.04, 523.05] \therefore \textit{reject null } \beta_j = 0$$

56

Significance Level

		0.100	0.050	0.025	0.010	0.005
one-tailed tests		0.100	0.050	0.025	0.010	0.005
two-tailed tests		0.200	0.100	0.050	0.020	0.010
Degrees of freedom	1	3.078	6.314	12.706	31.821	63.657
	2	1.886	2.920	4.303	6.965	9.925
	3	1.638	2.353	3.182	4.541	5.841
	4	1.533	2.132	2.776	3.747	4.604
	5	1.476	2.015	2.571	3.365	4.032
	15	1.341	1.753	2.131	2.602	2.947
	16	1.337	1.746	2.120	2.583	2.921
	17	1.333	1.740	2.110	2.567	2.898
	18	1.330	1.734	2.101	2.552	2.878
	19	1.328	1.729	2.093	2.539	2.861
	20	1.325	1.725	2.086	2.528	2.845
	21	1.323	1.721	2.080	2.518	2.831
	22	1.321	1.717	2.074	2.508	2.819
	23	1.319	1.714	2.069	2.500	2.807
	24	1.318	1.711	2.064	2.492	2.797
	25	1.316	1.708	2.060	2.485	2.787
	26	1.315	1.706	2.056	2.479	2.779
	27	1.314	1.703	2.052	2.473	2.771
	28	1.313	1.701	2.048	2.467	2.763
	29	1.311	1.699	2.045	2.462	2.756
	30	1.310	1.697	2.042	2.457	2.750

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```
. * run multivariate regression
. reg college_gpa act hs_gpa

      Source |       SS       df       MS              Number of obs =     141
-------------+------------------------------           F(  2,   138) =   14.78
       Model |  3.42365506     2  1.71182753           Prob > F      =  0.0000
    Residual |  15.9824444   138  .115814814           R-squared     =  0.1764
-------------+------------------------------           Adj R-squared =  0.1645
       Total |  19.4060994   140  .138614996           Root MSE      =  .34032

------------------------------------------------------------------------------
 college_gpa |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         act |    .009426   .0107772     0.87   0.383    -.0118838    .0307358
      hs_gpa |   .4534559   .0958129     4.73   0.000     .2640047    .6429071
       _cons |   1.286328   .3408221     3.77   0.000      .612419    1.960237
------------------------------------------------------------------------------
```

confidence int.

$$\hat{\beta}_j \pm t_{\alpha/2}(n-k-1)se(\hat{\beta}_j)$$

$$0.0094 \pm 1.98(0.01078)$$

$$[-0.0119, 0.0307] \quad \therefore \textit{cannot reject null } \beta_j = 0$$

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Problem Set 3

- Regress ln(q) on ln(p)
- Test whether the cigarette demand elasticity is an "elastic" response, that is ζ_d=-1
- 20 years worth of data, 51 states = 1020
- DOF = n-2 = 1018

59

Significance Level

one-tailed tests		0.100	0.050	0.025	0.010	0.005
two-tailed tests		0.200	0.100	0.050	0.020	0.010
Degrees of freedom	1	3.078	6.314	12.706	31.821	63.657
	2	1.886	2.920	4.303	6.965	9.925
	3	1.638	2.353	3.182	4.541	5.841
	4	1.533	2.132	2.776	3.747	4.604
	5	1.476	2.015	2.571	3.365	4.032
	25	1.316	1.708	2.060	2.485	2.787
	50	1.299	1.676	2.009	2.403	2.678
	100	1.290	1.660	1.984	2.364	2.626
	250	1.285	1.651	1.969	2.341	2.596
	500	1.283	1.648	1.965	2.334	2.586
	750	1.283	1.647	1.963	2.331	2.582
	1000	1.282	1.646	1.962	2.330	2.581
	1018	1.282	1.646	1.962	2.330	2.581
	infinity	1.282	1.645	1.960	2.326	2.576

Just looking at confidence interval, we can reject the null

```
      Source |       SS       df       MS              Number of obs =    1020
-------------+------------------------------           F(  1,  1018) =  873.39
       Model |  36.0468802     1  36.0468802           Prob > F      =  0.0000
    Residual |  42.0153163  1018  .041272413           R-squared     =  0.4618
-------------+------------------------------           Adj R-squared =  0.4612
       Total |  78.0621965  1019   .07660667           Root MSE      =  .20316

------------------------------------------------------------------------------
        ln_q |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        ln_p |  -.8076941   .0273302   -29.55   0.000    -.8613241   -.7540641
       _cons |   8.834473   .1423221    62.07   0.000     8.555195    9.113751
------------------------------------------------------------------------------
```

$\ln(q_i) = \beta_0 + \ln(p_i)\beta_1 + \varepsilon_i$

H_o: $\beta_1 = -1$

$$\hat{t} = \frac{\hat{\beta}_1 - a}{se(\hat{\beta}_1)} = \frac{-0.808 - -1}{0.0273} = \frac{0.192}{0.0273} = 7.03$$

$$t_{\alpha/2}(dof) = t_{0.025}(1018) = 1.96$$

$$|\hat{t}| > 1.96 \quad \therefore \textit{reject null}$$

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P-value

- Alternative way of characterizing the data contained in the t-test
- Given that the null is true, the **p-value** is probability of obtaining a result at least as extreme as the one that was actually observed
- Calculate $\hat{t}$
- In the two tailed test, the p-value is then

$$p - value = \Pr(t \le -|\hat{t}|) + \Pr(t > |\hat{t}|)$$

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```
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       Model |   606784507     1   606784507           Prob > F      =  0.0059
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-------------+------------------------------           Adj R-squared =  0.2139
       Total |  2.5178e+09    29  86819537.6           Root MSE      =  8261.3

------------------------------------------------------------------------------
  attendance |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
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       _cons |    3095.14   8539.507     0.36   0.720    -14397.25    20587.53
------------------------------------------------------------------------------
```
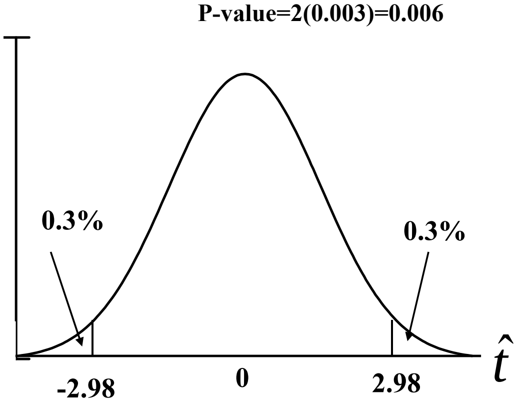

63

P-value=2(0.003)=0.006



64

```
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. reg college_gpa act hs_gpa

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------------------------------------------------------------------------------
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      hs_gpa |   .4534559   .0958129     4.73   0.000     .2640047    .6429071
       _cons |   1.286328   .3408221     3.77   0.000      .612419    1.960237
------------------------------------------------------------------------------
```
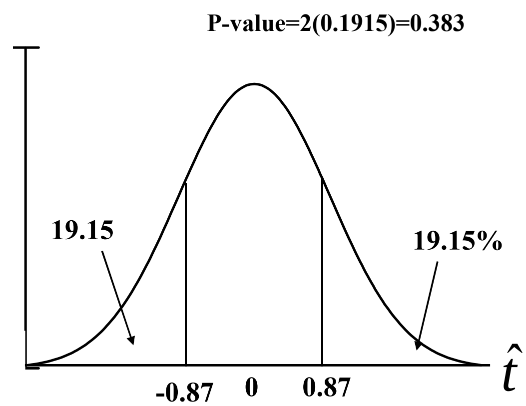

65



66

- A small p-value gives you confidence that you can reject the null hypothesis – you would not get a value this large (in absolute value) at random, therefore, the H_o must be false

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Note

- Using p-value, t-test, confidence interval are three ways to get the same results
- The decision rule (reject or not reject) should not vary across test methods
- Good check on your work --

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Errors in Prediction

- Statistical tests can be used as tests of theoretical hypothesis
  - Do demand curves slope down?
  - Do wages increase w/more education?
- These are only statistical tests – they ask, in a probabilistic sense, what is the likely state of the world
- Consider H_o: β_j=0
- Suppose the t-test is small, so you *cannot reject* the null hypothesis. There is always a chance that you are wrong.

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Example 1: New Drug

- Reduces deaths from stroke by 10%. However a clinical trial cannot reject the null hypothesis that there is no effect
- t-statistic on the active ingredient is 1.12
- Cannot reject null that β_j=0

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Two possible situations

- Drug does not work and your test is correct
- Drug does work, but the statistical model did not have enough power to detect a statistically significant impact

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Type I and II Errors

		Decision	
		Cannot rejectH_o	Reject H_o
True State	H_o true	Correct decision	Type I error Reject true hypothesis
	H_o false	Type II error Accept false hypothesis	Correct decision

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- Type I – false positive
- Type II – false negative

- In regression, H_0: $\beta_j=0$
  - Type I – you reject that $\beta_j=0$ when it equals 0
  - Type II – you cannot reject $\beta_j=0$ when $\beta_j\neq0$

73

What is the probability you will make a "wrong" decision

- Type I error – reject null when it is in fact true
  - H_o: $\beta_j=0$
  - Get large t-statistic so reject null
- There is a chance that, by accident, you will get a large t-stat
- What is that chance? 1 - confidence level = α so α is the probabilty

74

- Type II errors: Do not reject null when it is in fact false
  - H_o: $\beta_j=0$
  - Get small t-statistic so do not reject null
- What is the probability this will happen?
  - The type II error rate (false negative) labeled β
  - 1- β called the "power of the test"
  - Factors that increase power
    - Increase sample size
    - Increase variation in X's

75

- Depending on the problem, need to balance the probabilities of Type I and II errors

- If concerned about Type I errors, so you increase the size of the confidence interval – Increase the chance of Type II error

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Example: Criminal Court

- Consider criminal court:
  - H_o: not guilty
  - Job of jury – decide guilty or not guilty
  - Type I error – reject true hypothesis
    - convict an innocent man
  - Type II error – accept a false hypothesis
    - let guilty man go free
- Decision rule: guilt beyond a reasonable doubt
- Requires low p-value, high confidence level (99.99% confidence interval) to convict – mininimze Type I

77

Example: Mammography

- Low level radiation exam to detect breast exam
- H_o: no breast cancer
  - Type I error – False positive – find a cancer growth but it does not exist
  - Type II error – False negative – fails to detect a growth
- What do you minimize?

78

- Consider the doctor's liability –
  - Suppose a Type II error happens – failed to find tumor -- patient dies – gets sued for malpractice
  - Suppose a Type I error – detect tumor, perform surgery when none was needed –
- For the doctor, what type of error has more "downside" risk?

79

Changing confidence level

- 95% CI is "industry standard"
  - Only 5% error rate
- But, maybe want to decrease Type I error rate
  - Decrease false positives
  - Increase confidence level to 99%
  - Maybe you really need to be sure something causes cancer before you ban the substance

80

- In contrast, you might want to decrease chance of Type II error
  - Reduce the size of the confidence interval
  - Maybe do not require as definitive evidence before you let on the market a new drug to treat in uncurable disease

81

In STATA

- `reg y x1 x2 x3, level(#)`

- The # is a number from 10 to 99.99
  - the top number has a low Type I (.01%)
  - very high Type II error rare

82

Test score data from CA

- 420 schools
- 6 graders given math/reading exams
- Outcome is average score on both exams
- Four covariates
  - Student/teacher ratio
  - Average family income (in thousands of $)
  - % ESL
  - % on free and reduced lunchs

83

```
. desc average_score student_teacher avg_income esl_pct meal_pct

              storage  display     value
variable name   type   format      label      variable label
-------------------------------------------------------------------------------
average_score   float  %9.0g                  average score (math+read) std
                                                test
student_teacher float  %9.0g                  student/teacher ratio
avg_income      float  %9.0g                  average family income
esl_pct         float  %9.0g                  pct student with  english
                                                second language
meal_pct        float  %9.0g                  % kids on free/reduced prices
                                                meals


. sum average_score student_teacher avg_income esl_pct meal_pct

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
average_sc~e |       420    654.1565    19.05335     605.55     706.75
student_te~r |       420    19.64043    1.891812         14       25.8
  avg_income |       420    15.31659     7.22589      5.335     55.328
     esl_pct |       420    15.76816    18.28593          0   85.53972
    meal_pct |       420    44.70524    27.12338          0        100
```

84

```
. * run mv regression wth 4 x's
. reg average_score student_teacher avg_income esl_pct meal_pct

      Source |       SS       df       MS              Number of obs =     420
-------------+------------------------------           F(  4,   415) =  429.12
       Model |  122493.527     4  30623.3818           Prob > F      =  0.0000
    Residual |  29616.0666   415  71.3640159           R-squared     =  0.8053
-------------+------------------------------           Adj R-squared =  0.8034
       Total |  152109.594   419  363.030056           Root MSE      =  8.4477

------------------------------------------------------------------------------
average_sc~e |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
student_te~r |  -.5603892   .2286116    -2.45   0.015     -1.00977   -.1110081
  avg_income |    .674984    .083331     8.10   0.000     .5111805    .8387875
     esl_pct |  -.1943282   .0313796    -6.19   0.000     -.256011   -.1326454
    meal_pct |  -.3963661   .0274084   -14.46   0.000    -.4502427   -.3424895
       _cons |   675.6082   5.308856   127.26   0.000     665.1726    686.0438
------------------------------------------------------------------------------
```

Notice the t-statistic on Student/teacher ratio is 2.45
At 95% level, can reject null β = 0. Notice the
P-value is 0.015, so a 99% CI would not reject null

85

Significance Level

		0.100	0.050	0.025	0.010	0.005
one-tailed tests		**0.100**	**0.050**	**0.025**	**0.010**	**0.005**
two-tailed tests		**0.200**	**0.100**	**0.050**	**0.020**	**0.010**
Degrees of freedom	**1**	**3.078**	**6.314**	**12.706**	**31.821**	**63.657**
	2	**1.886**	**2.920**	**4.303**	**6.965**	**9.925**
	3	**1.638**	**2.353**	**3.182**	**4.541**	**5.841**
	4	**1.533**	**2.132**	**2.776**	**3.747**	**4.604**
	5	**1.476**	**2.015**	**2.571**	**3.365**	**4.032**
	25	**1.316**	**1.708**	**2.060**	**2.485**	**2.787**
	50	**1.299**	**1.676**	**2.009**	**2.403**	**2.678**
	100	**1.290**	**1.660**	**1.984**	**2.364**	**2.626**
	120	**1.289**	**1.658**	**1.980**	**2.358**	**2.617**
	200	**1.286**	**1.653**	**1.972**	**2.345**	**2.601**
	300	**1.284**	**1.650**	**1.968**	**2.339**	**2.592**
	400	**1.284**	**1.649**	**1.966**	**2.336**	**2.588**
	415	**1.284**	**1.649**	**1.966**	**2.335**	**2.588**
	infinity	**1.282**	**1.645**	**1.960**	**2.326**	**2.576**

To change CI, use this option

```
. * run the same regression but ask for a 99% confidence level
. reg average_score student_teacher avg_income esl_pct meal_pct, level(99)

      Source |       SS       df       MS              Number of obs =     420
-------------+------------------------------           F(  4,   415) =  429.12
       Model |  122493.527     4  30623.3818           Prob > F      =  0.0000
    Residual |  29616.0666   415  71.3640159           R-squared     =  0.8053
-------------+------------------------------           Adj R-squared =  0.8034
       Total |  152109.594   419  363.030056           Root MSE      =  8.4477

------------------------------------------------------------------------------
average_sc~e |      Coef.   Std. Err.      t    P>|t|     [99% Conf. Interval]
-------------+----------------------------------------------------------------
student_te~r |  -.5603892   .2286116    -2.45   0.015    -1.151974    .0311954
  avg_income |    .674984    .083331     8.10   0.000     .4593461    .8906219
     esl_pct |  -.1943282   .0313796    -6.19   0.000    -.2755301   -.1131263
    meal_pct |  -.3963661   .0274084   -14.46   0.000    -.4672916   -.3254406
       _cons |   675.6082   5.308856   127.26   0.000     661.8703    689.3461
------------------------------------------------------------------------------
```

confidence int.

$$\hat{\beta}_j \pm t_{\alpha/2}(n-k-1)se(\hat{\beta}_j)$$

$$0.5604 \pm 2.588(0.2286) = [-1.152,\ 0.031]$$

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