What variation is used to estimate parameters in a two-way fixed-effect model

Bill Evans
Fall 2008

Two way fixed effect model

(1) $y_{it} = \alpha + x_{it}\beta + u_i + \lambda_t + \varepsilon_{it}$

Balanced panels (this is a necessary assumption for this example)
$i=1,2,\ldots n$
$t=1,2,\ldots T$

Define means

$\bar{y}_i = (1/T)\Sigma_t\, y_{it}$	within-group mean
$\bar{y}_t = (1/n)\Sigma_i\, y_{it}$	within-period mean
$\bar{y} = (1/nT)\Sigma_i\, \Sigma_t\, y_{it}$	within-sample mean

The means for x and ε are defined the same way

Take the means of equation (1) within a group. Note that:

$(1/T)\Sigma_t\, \alpha = \alpha$
$(1/T)\Sigma_t\, u_i = u_i$
$(1/T)\Sigma_t\, \lambda_t = \bar{\lambda}$

And therefore $\bar{y}_i = \alpha + \bar{x}_i\beta + u_i + \bar{\lambda} + \bar{\varepsilon}_i$

Take the means of equation (1) within period. Note that:

$(1/n)\Sigma_i\, \alpha = \alpha$
$(1/n)\Sigma_i\, u_i = \bar{u}$
$(1/n)\Sigma_i\, \lambda_t = \lambda_t$

And therefore $\bar{y}_t = \alpha + \bar{x}_t\beta + \bar{u} + \lambda_t + \bar{\varepsilon}_t$

Take the means of equation (1) within the sample. Note that:

$(1/nT)\ \Sigma_i\ \Sigma_t\ \alpha = \alpha$
$(1/nT)\ \Sigma_i\ \Sigma_t\ u_i = (1/n)\ \Sigma_i\ [(1/T)\Sigma_t\ u_i] = (1/n)\ \Sigma_i\ u_i = \bar{u}$
$(1/nT)\ \Sigma_i\ \Sigma_t\ \lambda_t = (1/T)\ \Sigma_t\ [(1/n)\ \Sigma_i\ \lambda_t] = (1/T)\ \Sigma_i\ \lambda_t = \bar{\lambda}$

And therefore $\bar{y} = \alpha + \bar{x}\beta + \bar{u} + \bar{\lambda} + \bar{\varepsilon}$

Now, subtract within-group and within-year means from equation (1)

$$y_{it} = \alpha + x_{it}\beta + u_i + \lambda_t + \varepsilon_{it}$$

$$- \bar{y}_i = \alpha + \bar{x}_i\beta + u_i + \bar{\lambda} + \bar{\varepsilon}_i$$

$$- \bar{y}_t = \alpha + \bar{x}_i\beta + \bar{u} + \lambda_t + \bar{\varepsilon}_t$$

$$y_{it} - \bar{y}_i - \bar{y}_t = - \alpha + (x_{it} - \bar{x}_i - \bar{x}_t)\beta - \bar{u} - \bar{\lambda} + \varepsilon_{it} - \bar{\varepsilon}_i - \bar{\varepsilon}_t$$

Notice the extra terms floating around, namely negative values for α, $\bar{u}$ and $\bar{\lambda}$. To eliminate these terms subtract the within-sample means of equation (1).

$$y_{it} - \bar{y}_i - \bar{y}_t = - \alpha + (x_{it} - \bar{x}_i - \bar{x}_t)\beta - \bar{u} - \bar{\lambda} + \varepsilon_{it}$$

$$\underline{+ \bar{y} = \alpha + \bar{x}\beta + \bar{u} + \bar{\lambda} + \bar{\varepsilon}}$$

$$y_{it} - \bar{y}_i - \bar{y}_t + \bar{y} = (x_{it} - \bar{x}_i - \bar{x}_t + \bar{x})\beta + \varepsilon_{it} - \bar{\varepsilon}_i - \bar{\varepsilon}_t + \bar{\varepsilon}$$

The two-way fixed-effect model is equivalent to estimate a regression of deviations in means of y on deviations in means on x.

Let $y\sim_{it} = y_{it} - \bar{y}_i - \bar{y}_t + \bar{y}$

$x\sim_{it} = x_{it} - \bar{x}_i - \bar{x}_t + \bar{x}$

$\varepsilon\sim_{it} = \varepsilon_{it} - \bar{\varepsilon}_i - \bar{\varepsilon}_t + \bar{\varepsilon}$

The two-way fixed effect model is equal to simple a regression of the form

$$y\sim_{it} = x\sim_{it}\beta + \varepsilon\sim_{it}$$