Intermediate Micro

- Workhorse model of intermediate micro
  - Utility maximization problem
  - Consumers Max $U(x,y)$ subject to the budget constraint, $I = P_x x + P_y y$
- Problem is made easier by the fact that we assume all parameters are known
  - Consumers know prices and income
  - Know exactly the quality of the product
  - Simple optimization problem

Many cases, there is uncertainty about some variables
  - Uncertainty about income?
  - What are prices now? What will prices be in the future?
  - Uncertainty about quality of the product?

This section will review utility theory under uncertainty

Will emphasize the special role of insurance in a generic sense
  - Why insurance is ‘good’ — consumption smoothing across states of the world
  - How much insurance should people purchase?

Applications: Insurance markets may generate incentives that reduce the welfare gains of consumption smoothing
  - Moral hazard
  - Adverse selection
Definitions

- **Probability** - likelihood discrete event will occur
  - $n$ possible events, $i=1,2,...,n$
  - $P_i$ be the probability event $i$ happens
  - $0 \leq P_i \leq 1$
  - $P_1 + P_2 + P_3 + ... + P_n = 1$
- Probabilities can be 'subjective' or 'objective', depending on the model
- In our work, probabilities will be known with certainty

Expected value —
- Weighted average of possibilities, weight is probability
- Sum of the possibilities times probabilities

- $x = \{x_1, x_2, ..., x_n\}$
- $P = \{P_1, P_2, ..., P_n\}$
- $E(x) = P_1x_1 + P_2x_2 + P_3x_3 + ... + P_nx_n$

- Roll of a die, all sides have (1/6) prob. What is expected roll?

- $E(x) = 1(1/6) + 2(1/6) + ... + 6(1/6) = 3.5$

- Suppose you have: 25% chance of an A, 50% B, 20% C, 4% D and 1% F
- $E[\text{quality points}] = 4(.25) + 3(.5) + 2(.2) + 1(.04) + 0(.01) = 2.94$

Expected utility

- Suppose income is random. Two potential values ($Y_1$ or $Y_2$)
- Probabilities are either $P_1$ or $P_2 = 1 - P_1$
- When incomes are realized, consumer will experience a particular level of income and hence utility
- But, looking at the problem beforehand, a person has a particular 'expected utility'
• However, suppose an agent is faced with choice between two different paths
  – Choice a: \( Y_1 \) with probability \( P_1 \) and \( Y_2 \) with \( P_2 \)
  – Choice b: \( Y_3 \) with probability \( P_3 \) and \( Y_4 \) with \( P_4 \)

• Example: You are presented with two option
  – a job with steady pay or
  – a job with huge upside income potential, but one with a chance you will be looking for another job soon

• How do you choose between these two options?

Assumptions about utility with uncertainty

• Utility is a function of one element (income or wealth), where \( U = U(Y) \)

• Marginal utility is positive
  – \( U' = \frac{dU}{dY} > 0 \)

• Standard assumption, declining marginal utility \( U'' < 0 \)
  – Implies risk averse but we will relax this later
Von Neumann-Morganstern Utility

- N states of the world, with incomes defined as $Y_1, Y_2, ..., Y_n$
- The probabilities for each of these states is $P_1, P_2, ..., P_n$
- A valid utility function is the expected utility of the gamble
  \[ E(U) = P_1U(Y_1) + P_2U(Y_2) + ... + P_nU(Y_n) \]

- $E(U)$ is the sum of the possibilities times probabilities

Example:
- 40% chance of earning $2500/month
- 60% chance of $1600/month
- $U(Y) = Y^{0.5}$
- Expected utility
  \[ E(U) = 0.4(2500)^{0.5} + 0.6(1600)^{0.5} = 0.4(50) + 0.6(40) = 44 \]

Note that expected utility in this case is very different from expected income
- $E(Y) = 0.4(2500) + 0.6(1600) = 1960$

Expected utility allows people to compare gambles

Given two gambles, we assume people prefer the situation that generates the greatest expected utility
- People maximize expected utility
Example

• Job A: certain income of $50K
• Job B: 50% chance of $10K and 50% chance of $90K
• Expected income is the same ($50K) but in one case, income is much more certain
• Which one is preferred?

U = ln(y)

• EU_a = ln(50,000) = 10.82
• EU_b = 0.5 ln(10,000) + 0.5 ln(90,000) = 10.31
• Job (a) is preferred

Another Example

• Job 1
  – 40% chance of $2500, 60% of $1600
  – E(Y_1) = 0.4*2500 + 0.6*1600 = $1960
  – E(U_1) = (0.4)(2500)^{0.5} + (0.6)(1600)^{0.5} = 44
• Job 2
  – 25% chance of $5000, 75% of $1000
  – E(Y_2) = 0.25(5000) + 0.75(1000) = $2000
  – E(U_2) = 0.25(5000)^{0.5} + 0.75(1000)^{0.5} = 41.4
• Job 1 is preferred to 2, even though 2 has higher expected income

The Importance of Marginal Utility: The St Petersburg Paradox

• Bet starts at $2. Flip a coin and if a head appears, the bet doubles. If tails appears, you win the pot and the game ends.
• So, if you get H, H, H T, you win $16
• What would you be willing to pay to ‘play’ this game?
• Probabilities
  • Pr(h) = Pr(t) = 0.5
• All events are independent
  • Pr(h on 2nd | h on 1st) = Pr(h on 2nd)
  • Recall definition of independence
  • If A and B and independent events
    • Pr(A ∩ B) = Pr(A)Pr(B)

• Note, Pr(first tail on kth toss) =
  • Pr(h on 1st)Pr(h on 2nd)…Pr(t on kth) =
    • (1/2)(1/2)…(1/2) = (1/2)k
• What is the expected pot on the kth trial?
  • 2 on 1st or 2
  • 4 on 2nd or 2
  • 8 on 3rd or 2
  • So the payoff on the kth is 2k

• What is the expected value of the gamble
  • E = (1/2)$21 + (1/2)$22 + (1/2)$23 + (1/2)$24
  \[ E = \sum_{k=1}^{\infty} \left( \frac{1}{2} \right)^k (2^k) = \sum_{k=1}^{\infty} (1) = \infty \]
• The expected payout is infinite

<table>
<thead>
<tr>
<th>Round</th>
<th>Winnings</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>5th</td>
<td>$32</td>
<td>0.03125</td>
</tr>
<tr>
<td>10th</td>
<td>$1,024</td>
<td>0.000977</td>
</tr>
<tr>
<td>15th</td>
<td>$32,768</td>
<td>3.05E-5</td>
</tr>
<tr>
<td>20th</td>
<td>$1,048,576</td>
<td>9.54E-7</td>
</tr>
<tr>
<td>25th</td>
<td>$33,554,432</td>
<td>2.98E-8</td>
</tr>
</tbody>
</table>
Suppose Utility is $U = Y^{0.5}$. What is $E[U]$?

$$E = \sum_{k=1}^{\infty} \left( \frac{1}{2} \right)^{\frac{k}{2}} \left( 2^{\frac{k}{2}} \right) = \sum_{k=1}^{\infty} \left( \frac{1}{2} \right)^{\frac{k}{2}} \left( 2^{\frac{k}{2}} \right)$$

$$= \sum_{k=1}^{\infty} \left( \frac{\sqrt{2}}{2} \right)^k \left( \sqrt{2} \right)^k = \sum_{k=1}^{\infty} \left( \frac{\sqrt{2}}{2} \right)^k = \sum_{k=1}^{\infty} \left( \frac{1}{\sqrt{2}} \right)^k =$$

Can show that $= \sum_{k=1}^{\infty} \left( \frac{1}{\sqrt{2}} \right)^k = \left( \frac{1}{1 - \frac{1}{\sqrt{2}}} \right) - 1 = 2.414$

How to represent graphically

- Probability $P_1$ of having $Y_1$
- $(1 - P_1)$ of having $Y_2$
- $U_1$ and $U_2$ are utility that one would receive if they received $Y_1$ and $Y_2$ respectively
- $E(Y) = P_1Y_1 + (1-P_1)Y_2 = Y_3$
- $U_3$ is utility they would receive if they had income $Y_3$ with certainty

- Notice that $E(U)$ is a weighted average of utilities in the good and bad states of the world
- $E(U) = P_1U(Y_1) + (1-P_1)U(Y_2)$
- The weights sum to 1 (the probabilities)
- Draw a line from points $(a,b)$
- Represent all the possible ‘weighted averages’ of $U(Y_1)$ and $U(Y_2)$
- What is the one represented by this gamble?
• Draw vertical line from $E(Y)$ to the line segment. This is $E(U)$
• $U_4$ is Expected utility
• $U_4 = E(U) = P_1U(Y_1) + (1-P_1)U(Y_2)$

• Suppose offered two jobs
  – Job A: Has chance of a high $(Y_1)$ and low $(Y_2)$ wages
  – Job B: Has chance of high $(Y_3)$ and low $(Y_4)$ wages
  – Expected income from both jobs is the same
  – $P_a$ and $P_b$ are the probabilities of getting the high wage situation

  $$P_aY_1 + (1-P_a)Y_2 = P_bY_3 + (1-P_b)Y_4 = E(Y)$$

Numeric Example

• Job A
  – 20% chance of $150,000
  – 80% chance of $20,000
  – $E(Y) = 0.2(150K) + 0.8(20K) = 46K$

• Job B
  – 60% chance of $50K$
  – 40% chance of $40K$
  – $E(Y) = 0.6(50K) + 0.4(40K) = 46K$
• Notice that Job A and B have the same expected income
• Job A is riskier – bigger downside for Job A
• Prefer Job B (Why? Will answer in a moment)

• The prior example about the two jobs is instructive. Two jobs, same expected income, very different expected utility
• People prefer the job with the lower risk, even though they have the same expected income
• People prefer to ‘shed’ risk – to get rid of it.
• How much are they willing to pay to shed risk?

Example
• Suppose have $200,000 home (wealth).
• Small chance that a fire will damage your house. If does, will generate $75,000 in loss (L)
• U(W) = ln(W)
• Prob of a loss is 0.02 or 2%
• Wealth in “good” state = W
• Wealth in bad state = W-L.

• E(W) = (1-P)W + P(W-L)
  E(W) = 0.98(200,000) + 0.02(125,000) = $198,500

• E(U) = (1-P) ln(W) + P ln(W-L)
  E(U) = 0.98 ln(200K) + 0.02 ln(200K-75K) = 12.197
• Suppose you can add a fire detection/prevention system to your house.
• This would reduce the chance of a bad event to 0 but it would cost you $C to install
• What is the most you are willing to pay for the security system?
• $U(W) = 12.197$
• Utility with the security system is $U(W-C)$
• Set $U(W-C) = 12.197$ and solve for $C$

\[
\ln(W-C) = 12.197
\]

Recall that $e^{\ln(x)} = x$

Raise both sides to the $e$

\[
e^{\ln(W-C)} = W-C = e^{12.197} = 198,128
\]

$198,500 - 198,128 = $372

• Expected loss is $1500$
• Would be willing to pay $372 to avoid that loss

Utility

<table>
<thead>
<tr>
<th>Wealth</th>
<th>Y1</th>
<th>Y2 = E(W)</th>
<th>Y3 = E(W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

• Will earn $Y_1$ with probability $p_1$
  - Generates utility $U_1$
• Will earn $Y_2$ with probability $p_2 = 1 - p_1$
  - Generates utility $U_2$
• $E(I) = p_1Y_1 + (1-p_1)Y_2 = Y_3$
• Line (ab) is a weighted average of $U_1$ and $U_2$
• Note that expected utility is also a weighted average
• A line from $E(Y)$ to the line (ab) gives $E(U)$ for given $E(Y)$
• Take the expected income, $E(Y)$. Draw a line to (ab). The height of this line is $E(U)$.
• $E(U)$ at $E(Y)$ is $U_4$.
• Suppose income is known with certainty at $I_3$. Notice that utility would be $U_3$, which is greater than $U_4$.
• Look at $Y_4$. Note that the $Y_4 < E(Y)$ but these two situations generate the same utility – one is expected, one is known with certainty.

• The line segment (cd) is the “Risk Premium.” It is the amount a person is willing to pay to avoid the risky situation.
• If you offered a person the gamble of $Y_3$ or income $Y_4$, they would be indifferent.
• Therefore, people are willing to sacrifice cash to ‘shed’ risk.

Some numbers

• Person has a job that has uncertain income
  – 50% chance of making $30K, $U(30K) = 18$
  – 50% chance of making $10K, $U(10K) = 10$
• Another job with certain income of $16K
  – Assume $U(16K) = 14$

• $E(I) = (0.5)(30K) + (0.5)(10K) = 20K$
• $E(U) = 0.5U(30K) + 0.5U(10K) = 14$

• Expected utility. Weighted average of $U(30)$ and $U(10)$.
  $E(U) = 14$
• Notice that a gamble that gives expected income of $20K is equal in value to a certain income of only $16K
• This person dislikes risk.
  – Indifferent between certain income of $16 and uncertain income with expected value of $20$
  – Utility of certain $20 is a lot higher than utility of uncertain income with expected value of $20$
Although both jobs provide the same expected income, the person would prefer the guaranteed $20K. Why? Because of our assumption about diminishing marginal utility:
- In the 'good' state of the world, the gain from $20K to $30K is not as valued as the 1st $10
- In the 'bad' state, because the first $10K is valued more than the last $10K, you lose lots of utils.

Notice also that the person is indifferent between a job with $16K in certain income and $20,000 in uncertain.
They are willing to sacrifice up to $4000 in income to reduce risk, risk premium.

Example

- $U = y^{0.5}$
- Job with certain income
  - $400$ week
  - $U = 400^{0.5} = 20$
- Can take another job that
  - 40% chance of $900/week, $U = 30$
  - 60% chance of $100/week, $U = 10$
  - $E(I) = 420, E(U) = 0.4(30) + 0.6(10) = 18$
Utility Income

\[ U = f(I) \]

- Notice that utility from certain income stream is higher even though expected income is lower
- What is the risk premium??
- What certain income would leave the person with a utility of 18? \( U = Y^{0.5} \)
- So if \( 18 = Y^{0.5} \), \( 18^2 = Y = 324 \)
- Person is willing to pay 400-324 = $76 to avoid moving to the risky job

Risk Loving

- The desire to shed risk is due to the assumption of declining marginal utility of income
- Consider the next situation.
- The graph shows increasing marginal utility of income
  - \( U'(Y_1) > U'(Y_2) \) even though \( Y_1 > Y_2 \)
What does this imply about tolerance for risk?

- Notice that at $E(Y) = Y_3$, expected utility is $U_3$.
- Utility from a certain stream of income at $Y_3$ would generate $U_4$. Note that $U_3 > U_4$.
- This person prefers an uncertain stream of $Y_3$ instead of a certain stream of $Y_3$.
- This person is 'risk loving'. Again, the result is driven by the assumption are $U$.

Risk Neutral

- If utility function is linear, the marginal utility of income is the same for all values of income:
  - $U' > 0$
  - $U'' = 0$
- The uncertain income $E(Y)$ and the certain income $Y_3$ generate the same utility.
- This person is considered risk neutral.
- We usually make the assumption firms are risk neutral.

Example

- 25% chance of $100
- 75% chance of $1000
- $E[Y] = 0.25(100) + 0.75(1000) = $775$
- $U = Y$
- Compare to certain stream of $775$
Utility

\[ U = a + bY \]

Income

\[ Y_2 \quad Y_1 = E(y) \quad Y_3 \]

Benefits of insurance

- Assume declining marginal utility
- Person dislikes risk
  - They are willing to receive lower certain income rather than higher expected income
- Firms can capitalize on the dislike for risk by helping people shed risk via insurance

Simple insurance example

- Suppose income is known \( Y_1 \) but random -- shocks can reduce income
  - House or car is damaged
  - Can pay \( $ \) to repair, return you to the normal state of world
- \( L \) is the loss if the bad event happens
- Probability of loss is \( P_1 \)
- Expected utility without insurance is
  \[ E(U) = (1-P_1)U(Y_1) + P_1U(Y_1-L) \]
- Suppose you can buy insurance that costs you \( PREM \). The insurance pays you to compensate for the loss \( L \).
  - In good state, income is
    - \( Y - \) Prem
  - In bad state, paid \( PREM \), lose \( L \) but receive \( PAYMENT \), therefore, income is
    - \( Y - \) Prem - \( L \) + \( PAYMENT \)
  - For now, lets assume \( PAYMENT = L \), so
  - Income in the bad state is also
    - \( Y - \) Prem
• Notice that insurance has made income certain. You will always have income of \( Y + \text{PREM} \).
• What is the most this person will pay for insurance?
• The expected loss is \( p_1L \).
• Expected income is \( E(Y) \).
• The expected utility is \( U_2 \).
• People would always be willing to pay a premium that equaled the expected loss.

But they are also willing to pay a premium to shed risk (line cd).
• The maximum amount they are willing to pay is expected loss + risk premium.

Suppose income is $50K, and there is a 5% chance of having a car accident that will generate $15,000 in loss.
• Expected loss is \( 0.05(15K) = 750 \).
• \( U = \ln(y) \).
• Some properties of logs
  \( Y = \ln(x) \) then \( e^y = \exp(y) \equiv x \)
  \( Y = \ln(x^a) = a \ln(x) \)
  \( Y = \ln(xz) = \ln(x) + \ln(z) \)

<table>
<thead>
<tr>
<th>Utility</th>
<th>Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U_2 )</td>
<td>( Y )</td>
</tr>
<tr>
<td>( Y - L )</td>
<td>( Y_L )</td>
</tr>
<tr>
<td>( Y_2 )</td>
<td>( E(Y) )</td>
</tr>
</tbody>
</table>

Willingness to pay for insurance.
• $E(U) = P \ln(Y-L) + (1-P)\ln(Y)$
• $E(U) = 0.05 \ln(35,000) + 0.95 \ln(50,000)$
• $E(U) = 10.8$

• What is the most someone will pay for insurance?
• People would purchase insurance so long as utility with certainty is at least 10.8 (expected utility without insurance)
• $U_a = U(Y - Prem) \geq 10.8$
• $\ln(Y-PREM) \geq 10.8$
• $Y-PREM = \exp(10.8)$
• $PREM = Y - \exp(10.8) = 50,000 - 49,021 = 979$

• Recall that the expected loss is $750 but this person is willing to pay more than the expected loss to avoid the risk
• Pay $750 (expected loss), plus the risk premium ($979 - $750) = 229
Supply of Insurance

• Suppose there are a lot of people with the same situation as in the previous slide.
• Each of these people have a probability of loss $P$ and when a loss occurs, they have $L$ expenses.
• A firm could collect money from as many people as possible in advance. If bad event happens, they pay back a specified amount.

Firms are risk neutral, so they are interested in expected profits.

• Expected profits = revenues – costs
  – Revenues are known
  – Some of the costs are random (e.g., exactly how many claims you will pay)

Think of the profits made on sales to one person.

• A person buys a policy that will pay them $q$ dollars ($q \leq L$) back if the event occurs.

• To buy this insurance, person will pay “$a$” dollars per dollar of coverage.

• Cost per policy is fixed $t$.

Revenues = $aq$
  – $a$ is the price per dollar of coverage

Costs = $pq + t$
  – For every dollar of coverage ($q$) expect to pay this $p$ percent of time.

$E(\pi) = aq - pq - t$

Let assume a perfectly competitive market, so in the long run $\pi = 0$.

What should the firm charge per dollar of coverage?

$E(\pi) = aq - pq - t = 0$.
• \( a = p + \frac{t}{q} \)
• The cost per dollar of coverage is proportion to risk
• \( \frac{t}{q} \) is the loading factor. Portion of price to cover administrative costs
• Make it simple, suppose \( t=0 \).
  – \( a = p \)
  – If the probability of loss is 0.05, will change 5 cents per $1.00 of coverage

In this situation, if a person buys a policy to insure \( L \) dollars, the ‘actuarially fair’ premium will be \( LP \)
• An actuarially fair premium is one where the premium equals the expected loss
• In the real world, no premiums are ‘actuarially fair’ because prices include administrative costs called ‘loading factors’

How much insurance will people purchase when prices are actuarially fair?

• With insurance
  – Pay a premium that is subtracted from income
  – If bad state happens, lose \( L \) but get back the amount of insurance \( q \)
  – They pay \( p + \frac{t}{q} \) per dollar of coverage. Have \( q \) dollars of coverage – so they pay a premium of \( pq + t \) in total
• Utility in good state
  – \( U = U[Y - pq - t] \)

• Utility in bad state
  – \( U[Y - L + q - pq - t] \)
• \( E(u) = (1-p)U[Y - pq - t] + pU[Y-L+q-pq-t] \)
• Simplify, let \( t=0 \) (no loading costs)
• \( E(u) = (1-p)U[Y - pq] + pU[Y-L+q-pq] \)
• Maximize utility by picking optimal \( q \)
• \( \frac{dE(u)}{dq} = 0 \)
• \( E(u) = (1-p)U[Y - pq] + pU[Y - L + q - pq] \)

• \( \frac{dE(u)}{dq} = (1-p) U'(y-pq)(-p) + pU'(Y-L+q-pq)(1-p) = 0 \)

• \( p(1-p)U'(Y-L+q-pq) = (1-p)pU'(Y-pq) \)

• \( (1-p)p \) cancel on each side

• \( U'(Y-L+q-pq) = U'(Y-pq) \)

• Optimal insurance is one that sets marginal utilities in the bad and good states equal

• \( Y-L+q-pq = Y-pq \)

• \( Y's \) cancel, \( pq's \) cancel,

• \( q=L \)

• If people can buy insurance that is ‘fair’ they will fully insure loses.

---

Insurance w/ loading costs

• Insurance is not actuarially fair and insurance does have loading costs

• Can show (but more difficult) that with loading costs, people will now under-insure, that is, will insure for less than the loss \( L \).

• Intuition? For every dollar of expected loss you cover, will cost more than a $1

• Only get back $1 in coverage if the bad state of the world happens

• Recall:
  – \( q \) is the amount of insurance purchased
  – Without loading costs, cost per dollar of coverage is \( p \)
  – Now, for simplicity, assume that price per dollar of coverage is \( pK \) where \( K>1 \) (loading costs)

• Buy \( q \$ \) worth of coverage

• Pay \( qpK \) in premiums
• \( E(u) = (1-p)U[Y - pqk] + pU[Y-L+q-pqk] \)
• \( \frac{dE(u)}{dq} = (1-p) U'(y-pqk)(-pk) + pU'(Y-L+q-pqk)(1-pk) = 0 \)
• \( p(1-pk)U'(Y-L+q-pqk) = (1-p)pkU'(Y-pqk) \)
• \( p \) cancel on each side

\[ (1-pk)U'(Y-L+q-pqk) = (1-p)kU'(Y-pkq) \]
\[ (a)(b) = (c)(d) \]
• Since \( k > 1 \), can show that \( (1-pk) < (1-p)k \)
• Since \( (a) < (c) \), must be the case that \( (b) > (d) \)
• \( U'(Y-L+q-pqk) > U'(Y-pkq) \)
• Since \( U'(y1) > U'(y2) \), must be that \( y1 < y2 \)

• \( Y-L+q-pqk \) < \( Y-pqk \)
• \( Y \) and \( -pqk \) cancel
• \( -L + q < 0 \)
• Which means that \( q < L \)
• When price is not ‘fair’ you will not fully insure

Demand for Insurance
• Both people have income of \( Y \)
• Each person has a potential health shock
  – The shock will leave person 1 w/ expenses of \( E1 \) and will leave income at \( Y1=Y-E1 \)
  – The shock will leave person 2 w/ expenses of \( E2 \) and will leave income at \( Y2=Y-E2 \)
• Suppose that
  – \( E1>E2; Y1<Y2 \)
• Probabilities the health shock will occur are P1 and P2
• Expected Income of person 1
  - \( E(Y)_1 = (1-P1)Y + P1*(Y-E1) \)
  - \( E(Y)_2 = (1-P2)Y + P2*(Y-E2) \)
  - Suppose that \( E(Y)_1 = E(Y)_2 = Y_3 \)

• In this case
  - Shock 1 is a low probability/high cost shock
  - Shock 2 is a high probability/low cost shock
• Example
  - \( Y=60,000 \)
  - Shock 1 is 1% probability of $50,000 expense
  - Shock 2 is a 50% chance of $1000 expense
  - \( E(Y) = 59500 \)

• Expected utility locus
  - Line ab for person 1
  - Line ac for person 2
• Expected utility is
  - \( U_a \) in case 1
  - \( U_b \) in case 2
• Certainty premium –
  - Line (de) for person 1, Difference \( Y_3 - Y_a \)
  - Line (fg) for person 2, Difference \( Y_3 - Y_b \)
Implications

• Do not insure small risks/high probability events
  – If you know with certainty that a costs will happen, or, costs are low when a bad event occurs, then do not insure
  – Example: teeth cleanings. You know they happen twice a year, why pay the loading cost on an event that will happen?

• Insure catastrophic events
  – Large but rare risks
  – As we will see, many of the insurance contracts we see do not fit these characteristics – they pay for small predictable expenses and leave exposed catastrophic events

Some adjustments to this model

• The model assumes that poor health has a monetary cost and that is all.
  – When experience a bad health shock, it costs you $L to recover and you are returned to new
• Many situations where
  – health shocks generate large expenses
  – And the expenses may not return you to normal
  – AIDS, stroke, diabetes, etc.

• In these cases, the health shock has fundamentally changed life.
  • We can deal with this situation in the expected utility model with adjustment in the utility function
  • “State dependent” utility
    – $U(y)$ utility in healthy state
    – $V(y)$ utility in unhealthy state
• Typical assumption
  – \( U(Y) > V(Y) \)
    • For any given income level, get higher utility in the healthy state
  – \( U'(Y) > V'(Y) \)
    • For any given income level, marginal utility of the next dollar is higher in the healthy state

Note that:
• At \( Y_1 \),
  – \( U(Y_1) > V(Y_1) \)
  – \( U'(Y_1) > V'(Y_1) \)
  – Slope of line aa > slope of line bb
• Notice that slope line aa = slope of line cc
  – \( U'(Y_1) = V'(Y_1) \)

What does this do to optimal insurance
• \( E(u) = (1-p)U[Y – pq – t] + pV[Y-L+q-pq-t] \)
• Again, let's set \( t=0 \) to make things easy
• \( E(u) = (1-p)U[Y – pq] + pV[Y-L+q-pq] \)
• \( \frac{dE(u)}{dq} = (1-p)(-p)U'[Y-pq] +p(1-p)V'[Y-L+q-pq] = 0 \)
• \( U'[Y-pq] = V'[Y-L+q-pq] \)
• Just like in previous case, we equalize marginal utility across the good and bad states of the world
• Recall that
  \[-U'(y) > V'(y)\]
  \[-U'(y_1) = V'(y_2) \text{ if } y_1 > y_2\]
• Since \[U'[Y-pq] = V'[Y-l+q-pq]\]
• In order to equalize marginal utilities of income, must be the case that
  \[Y-pq > Y-l+q-pq\]

• Income in healthy state > income in unhealthy state
• Do not fully insure losses. Why?
  – With insurance, you take $ from the good state of the world (where MU of income is high) and transfer $ to the bad state of the world (where MU is low)
  – Do not want good money to chance bad

Allais Paradox

• Which gamble would you prefer
  – 1A: $1 million w/ certainty
  – 1B: (.89, $1 million), (.01, $0), (.1, $5 million)

• Which gamble would you prefer
  – 2A: (.89, $0), (.11, $1 million)
  – 2B: (.09, $0), (.10, $5 million)