Intermediate Micro

- Workhorse model of intermediate micro
  - Utility maximization problem
  - Consumers Max $U(x,y)$ subject to the budget constraint,
    \[ I = P_x x + P_y y \]
- Problem is made easier by the fact that we assume all variables are known with certainty
  - Consumers know prices and income
  - Know exactly the quality of the product

- Many cases, there is uncertainty about some variables
  - Uncertainty about income?
  - What are prices now? What will prices be in the future?
  - Uncertainty about quality of the product?
- This section, will review utility theory under uncertainty

- Will emphasize the special role of insurance in a generic sense
  - Why insurance is ‘good’?
  - How much insurance should people purchase?
  - Compare that to how insurance is usually structured in health care
Special problems of health care insurance

- Moral hazard
  - Reimbursement structure of health insurance encourages more use of medical care

- Adverse selection
  - Those with the most needs for medical care are attracted to insurance

What these problems do to markets?

What these problems due to welfare?

Definitions

- Probability - likelihood discrete event will occur
  - n possible events, i=1,2,...,n
  - \( P_i \) be the probability event i happens
  - \( 0 \leq P_i \leq 1 \)
  - \( P_1+P_2+P_3+...P_n=1 \)

- Probabilities can be 'subjective' or 'objective', depending on the model
- In our work, probabilities will be know with certainty

- Expected value —
  - Weighted average of possibilities, weight is probability
  - Sum of the possibilities times probabilities

- \( x=\{x_1,x_2,...,x_n\} \)
- \( P=\{P_1,P_2,...,P_n\} \)

- \( E(x) = P_1X_1 + P_2X_2 + P_3X_3 + ... + P_nX_n \)

- Roll of a die, all sides have (1/6) prob. What is expected roll?

- \( E(x) = 1(1/6) + 2(1/6) + ... + 6(1/6) = 3.5 \)

- Suppose you have: 25% chance of an A, 50% B, 20% C, 4% D and 1% F

- \( E[\text{quality points}] = 4(.25) + 3(.5) + 2(.2) + 1(.04) + 0(.01) = 2.94 \)
Expected utility

- Suppose income is random. Two potential values ($Y_1$ or $Y_2$)
- Probabilities are either $P_1$ or $P_2 = 1 - P_1$
- When incomes are realized, consumer will experience a particular level of income and hence utility
- But, looking at the problem beforehand, a person has a particular ‘expected utility’

However, suppose an agent is faced with choice between two different paths
- Choice a: $Y_1$ with probability $P_1$ and $Y_2$ with $P_2$
- Choice b: $Y_3$ with probability $P_3$ and $Y_4$ with $P_4$

Example: You are presented with two option
- a job with steady pay or
- a job with huge upside income potential, but one with a chance you will be looking for another job soon

- How do you choose between these two options?

Assumptions about utility with uncertainty

- Utility is a function of one element (income or wealth), where $U = U(Y)$
- Marginal utility is positive
  - $U' = dU/dY > 0$
- Standard assumption, declining marginal utility $U'' < 0$
  - Implies risk averse but we will relax this later
Von Neumann-Morgenstern Utility

- $N$ states of the world, with incomes defined as $Y_1, Y_2, \ldots, Y_n$
- The probabilities for each of these states is $P_1, P_2, \ldots, P_n$
- A valid utility function is the expected utility of the gamble
- $U(Y) = Y^{0.5}$

$E(U) = P_1U(Y_1) + P_2U(Y_2) + \ldots + P_nU(Y_n)$

$E(U)$ is the sum of the possibilities times probabilities

Example:
- 40% chance of earning $2500/month
- 60% chance of $1600/month
- $U(Y) = Y^{0.5}$

$E(U) = 0.4(2500)^{0.5} + 0.6(1600)^{0.5}$

$= 0.4(50) + 0.6(40) = 44$
• Note that expected utility in this case is very different from expected income
  \[ E(Y) = 0.4 \times 2500 + 0.6 \times 1600 = 1960 \]

• Expected utility allows people to compare gambles

• Given two gambles, we assume people prefer the situation that generates the greatest expected utility
  \[ \text{People maximize expected utility} \]

Example

• Job A: certain income of $50K
  \[ \text{EY}_a = 50000 = 10.82 \]

• Job B: 50% chance of $10K and 50% chance of $90K
  \[ \text{EY}_b = 0.5 \times 10000 + 0.5 \times 90000 = 44 \]

• Expected income is the same ($50K) but in one case, income is much more certain
• Which one is preferred?

Another Example

• Job 1
  \[ \text{EY}_1 = 0.4 \times 2500 + 0.6 \times 1600 = 1960 \]
  \[ \text{EU}_1 = (0.4)(2500)^{0.5} + (0.6)(1600)^{0.5} = 44 \]

• Job 2
  \[ \text{EY}_2 = 0.25 \times 5000 + 0.75 \times 1000 = 2000 \]
  \[ \text{EU}_2 = 0.25(5000)^{0.5} + 0.75(1000)^{0.5} = 41.4 \]

• Job 1 is preferred to 2, even though 2 has higher expected income
The Importance of Marginal Utility:
The St. Petersburg Paradox

- Bet starts at $2. Flip a coin and if a head appears, the bet doubles. If tails appears, you win the pot and the game ends.

- So, if you get H, H, H T, you win $16

- What would you be willing to pay to ‘play’ this game?

- Probabilities?
  - $Pr(h)=Pr(t)=0.5$

- All events are independent
  - $Pr(h \text{ on 2nd | h on 1st}) = Pr(h \text{ on 2nd})$

- Recall definition of independence
  - If A and B and independent events
    - $Pr(A \cap B) = Pr(A)Pr(B)$

- Note, $Pr(\text{first tail on kth toss}) = Pr(\text{h on 1st})Pr(\text{h on 2nd})...Pr(\text{t on kth}) = (1/2)(1/2)...(1/2) = (1/2)^k$

- What is the expected pot on the kth trial?
  - 2 on 1st or 2^1
  - 4 on 2nd or 2^2
  - 8 on 3rd, or 2^3
  - So the payoff on the kth is 2^k

- What is the expected value of the gamble
  - $E = (1/2)$2^1 + (1/2)$2^2 + (1/2)$2^3 + (1/2)$2^4$
  - $E = \sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^k \left(2^k\right) = \sum_{k=1}^{\infty} (1) = \infty$

- The expected payout is infinite
<table>
<thead>
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<th>Round</th>
<th>Winnings</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>5th</td>
<td>$32</td>
<td>0.03125</td>
</tr>
<tr>
<td>10th</td>
<td>$1,024</td>
<td>0.000977</td>
</tr>
<tr>
<td>15th</td>
<td>$32,768</td>
<td>3.05E-5</td>
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<tr>
<td>20th</td>
<td>$1,048,576</td>
<td>9.54E-7</td>
</tr>
<tr>
<td>25th</td>
<td>$33,554,432</td>
<td>2.98E-8</td>
</tr>
</tbody>
</table>

Suppose Utility is $U = Y^{0.5}$. What is $E[U]$?

$$E = \sum_{k=1}^{\infty} \left( \frac{1}{2} \right)^{k/2} \left( 2^k \right)^{1/2} = \sum_{k=1}^{\infty} \left[ \left( \frac{1}{2} \right)^{1/2} \right] \left( 2^{1/2} \right)^{k} = \sum_{k=1}^{\infty} \left( \frac{1}{\sqrt{2}} \right)^{k}$$

Can show that $\sum_{k=1}^{\infty} \left( \frac{1}{\sqrt{2}} \right)^{k} = \frac{1}{1 - \frac{1}{\sqrt{2}}} - 1 = 2.414$

How to represent graphically

- Probability $P_1$ of having $Y_1$
- $(1-P_1)$ of having $Y_2$
- $U_1$ and $U_2$ are utility that one would receive if they received $Y_1$ and $Y_2$ respectively
- $E(Y) = P_1Y_1 + (1-P_1)Y_2 = Y_3$
- $U_3$ is utility they would receive if they had income $Y_3$ with certainty
• Notice that \( E(U) \) is a weighted average of utilities in the good and bad states of the world

\[ E(U) = P_1 U(Y_1) + (1 - P_1) U(Y_2) \]

• The weights sum to 1 (the probabilities)

• Draw a line from points \((a, b)\)

• Represent all the possible ‘weighted averages’ of \( U(Y_1) \) and \( U(Y_2) \)

• What is the one represented by this gamble?

• Draw vertical line from \( E(Y) \) to the line segment. This is \( E(U) \)

\[ U_4 = E(U) = P_1 U(Y_1) + (1 - P_1) U(Y_2) \]

• Suppose offered two jobs

– Job A: Has chance of a high \( (Y_1) \) and low \( (Y_2) \) wages

– Job B: Has chance of high \( (Y_3) \) and low \( (Y_4) \) wages

– Expected income from both jobs is the same

\[ P_a Y_1 + (1 - P_a) Y_2 = P_b Y_3 + (1 - P_b) Y_4 = E(Y) \]

Numeric Example

• Job A

– 20% chance of $150,000

– 80% chance of $20,000

\[ E(Y) = 0.2(150K) + 0.8(20K) = 46K \]

• Job B

– 60% chance of $50K

– 40% chance of $40K

\[ E(Y) = 0.6(50K) + 0.4(40K) = 46K \]
• Notice that Job A and B have the same expected income
• Job A is riskier – bigger downside for Job A
• Prefer Job B (Why? Will answer in a moment)

Example
• Suppose have $200,000 home (wealth).
• Small chance that a fire will damage you house. If does, will generate $75,000 in loss (L)
• \( U(W) = \ln(W) \)
• Prob of a loss is 0.02 or 2%
• Wealth in “good” state = W
• Wealth in bad state = W - L

• The prior example about the two jobs is instructive. Two jobs, same expected income, very different expected utility
• People prefer the job with the lower risk, even though they have the same expected income
• People prefer to ‘shed’ risk – to get rid of it.
• How much are they willing to pay to shed risk?
• $E(W) = (1-P)W + P(W-L)$
  • $E(W) = 0.98(200,000) + 0.02(125,000) = $198,500$

• $E(U) = (1-P) \ln(W) + P \ln(W-L)$
  • $E(U) = 0.98 \ln(200K) + 0.02 \ln(200K-75K) = 12.197$

• Suppose you can add a fire detection/prevention system to your house.
  • This would reduce the chance of a bad event to 0 but it would cost you $C to install
  • What is the most you are willing to pay for the security system?
    • $E(U)$ in the current situation is 12.197
    • Utility with the security system is $U(W-C)$
    • Set $U(W-C)$ equal to 12.197 and solve for $C$

• $\ln(W-C) = 12.197$
  • Recall that $e^{\ln(x)} = x$
  • Raise both sides to the e
    • $e^{\ln(W-C)} = W-C = e^{12.197} = 198,128$
    • $198,500 - 198,128 = $372$

• Expected loss is $1500
  • Would be willing to pay $372 to avoid that loss
• Will earn $Y_1$ with probability $p_1$
  – Generates utility $U_1$
• Will earn $Y_2$ with probability $p_2=1-p_1$
  – Generates utility $U_2$
• $E(I) = p_1Y_1 + (1-p_1)Y_2 = Y_3$
• Line $(ab)$ is a weighted average of $U_1$ and $U_2$
• Note that expected utility is also a weighted average
• A line from $E(Y)$ to the line $(ab)$ gives $E(U)$ for given $E(Y)$

• Take the expected income, $E(Y)$. Draw a line to $(ab)$. The height of this line is $E(U)$.
• $E(U)$ at $E(Y)$ is $U_4$
• Suppose income is known with certainty at $I_3$. Notice that utility would be $U_3$, which is greater than $U_4$
• Look at $Y_4$. Note that the $Y_4 < E(Y)$ but these two situations generate the same utility – one is expected, one is known with certainty

Some numbers
• Person has a job that has uncertain income
  – 50% chance of making $30K, $U(30K) = 18$
  – 50% chance of making $10K, $U(10K) = 10$
• Another job with certain income of $16K
  – Assume $U($16K$)=14$
• $E(I) = (0.5)($30K$) + (0.05)($10K$) = $20K$
• $E(U) = 0.5U(30K) + 0.5U(10K) = 14$

• The line segment $(cd)$ is the “Risk Premium.” It is the amount a person is willing to pay to avoid the risky situation.
• If you offered a person the gamble of $Y_3$ or income $Y_4$, they would be indifferent.
• Therefore, people are willing to sacrifice cash to ‘shed’ risk.
• Expected utility. Weighted average of U(30) and U(10).
  \[ E(U) = 14 \]
• Notice that a gamble that gives expected income of $20K is equal in value to a certain income of only $16K.
• This person dislikes risk.
  – Indifferent between certain income of $16 and uncertain income with expected value of $20
  – Utility of certain $20 is a lot higher than utility of uncertain income with expected value of $20

• Although both jobs provide the same expected income, the person would prefer the guaranteed $20K.
• Why? Because of our assumption about diminishing marginal utility
  – In the ‘good’ state of the world, the gain from $20K to $30K is not as valued as the 1st $10
  – In the ‘bad’ state, because the first $10K is valued more than the last $10K, you lose lots of utility.

• Notice also that the person is indifferent between a job with $16K in certain income and $20,000 in uncertain
• They are willing to sacrifice up to $4000 in income to reduce risk, risk premium
Example

- $U = y^{0.5}$
- Job with certain income
  - $400/week
  - $U = 400^{0.5} = 20$
- Can take another job that
  - 40% chance of $900/week, $U = 30$
  - 60% chance of $100/week, $U = 10$
  - $E(I) = 420, E(U) = 0.4(30) + 0.6(10) = 18$

Notice that utility from certain income stream is higher even though expected income is lower
- What is the risk premium?
- What certain income would leave the person with a utility of 18? $U = y^{0.5}$
  - So if 18 = $y^{0.5}$, $18^2 = y = 324$
- Person is willing to pay 400 - 324 = $76 to avoid moving to the risky job

Risk Loving

- The desire to shed risk is due to the assumption of declining marginal utility of income
- Consider the next situation.
- The graph shows increasing marginal utility of income
  - $U'(Y_1) > U'(Y_2)$ even though $Y_1 > Y_2$
What does this imply about tolerance for risk?
• Notice that at $E(Y) = Y_3$, expected utility is $U_3$.
• Utility from a certain stream of income at $Y_3$ would generate $U_4$. Note that $U_3 > U_4$.
• This person prefers an uncertain stream of $Y_3$ instead of a certain stream of $Y_3$.
• This person is ‘risk loving’. Again, the result is driven by the assumption are $U''$.

Risk Neutral
• If utility function is linear, the marginal utility of income is the same for all values of income
  - $U' > 0$
  - $U'' = 0$
• The uncertain income $E(Y)$ and the certain income $Y_3$ generate the same utility.
• This person is considered risk neutral.
• We usually make the assumption firms are risk neutral.
Example

- 25% chance of $100
- 75% chance of $1000
- $E[Y] = 0.25(100) + 0.75(1000) = $775
- $U = Y$
- Compare to certain stream of $775

Utility

$U = a + bY$

Income

Benefits of insurance

- Assume declining marginal utility
- Person dislikes risk
  - They are willing to receive lower certain income rather than higher expected income
- Firms can capitalize on the dislike for risk by helping people shed risk via insurance

Simple insurance example

- Suppose income is know ($Y_1$) but random -- shocks can reduce income
  - House or car is damaged
  - Can pay $ to repair, return you to the normal state of world
- L is the loss if the bad event happens
- Probability of loss is $P_1$
- Expected utility without insurance is
  - $E(U) = (1-P_1)U(Y_1) + P_1U(Y_1-L)$
• Suppose you can buy insurance that costs you PREM. The insurance pay you to compensate for the loss L.
  – In good state, income is
    • Y-Prem
  – In bad state, paid PREM, lose L but receive PAYMENT, therefore, income is
    • Y-Prem-L+Payment
  – For now, lets assume PAYMENT=L, so
  – Income in the bad state is also
    • Y-Prem

• Notice that insurance has made income certain. You will always have income of Y-PREM
• What is the most this person will pay for insurance?
• The expected loss is p1L.
• Expected income is E(Y)
• The expected utility is U2
• People would always be willing to pay a premium that equaled the expected loss

• But they are also willing to pay a premium to shed risk (line cd)
• The maximum amount they are willing to pay is expected loss + risk premium
Suppose income is $50K, and there is a 5% chance of having a car accident that will generate $15,000 in loss.

- Expected loss is $0.05(15K) = $750
- $U = \ln(y)$

Some properties of logs:
- $Y = \ln(x)$ then $e^y = \exp(y) = x$
- $Y = \ln(x^a) = a \ln(x)$
- $Y = \ln(xz) = \ln(x) + \ln(z)$

$E(U) = P \ln(Y - L) + (1-P)\ln(Y)$

- $E(U) = 0.05 \ln(35,000) + 0.95 \ln(50,000)$
- $E(U) = 10.8$

What is the most someone will pay for insurance?

- People would purchase insurance so long as utility with certainty is at least 10.8 (expected utility without insurance).
- $U_a = U(Y - Prem) \geq 10.8$
- $\ln(Y - Prem) \geq 10.8$
- $Y - Prem = \exp(10.8)$
- $Prem = 50,000 - 49,021 = 979$

Recall that the expected loss is $750 but this person is willing to pay more than the expected loss to avoid the risk.

- Pay $750 (expected loss), plus the risk premium ($979 - $750) = 229$
Supply of Insurance

- Suppose there are a lot of people with the same situation as in the previous slide
- Each of these people have a probability of loss $P$ and when a loss occurs, they have $L$ expenses
- A firm could collect money from as many people as possible in advance. If bad event happens, they pay back a specified amount.

- Firms are risk neutral, so they are interested in expected profits
- Expected profits = revenues – costs
  - Revenues are known
  - Some of the costs are random (e.g., exactly how many claims you will pay)

- Think of the profits made on sales to one person
- A person buys a policy that will pay them $q$ dollars ($q \leq L$) back if the event occurs
- To buy this insurance, person will pay “$a$” dollars per dollar of coverage
- Cost per policy is fixed $t$
• Revenues = \(aq\)
  – \(a\) is the price per dollar of coverage
• Costs = \(pq + t\)
  – For every dollar of coverage (\(q\)) expect to pay this \(p\) percent of time
• \(E(\pi) = aq - pq - t\)
• Let assume a perfectly competitive market, so in the long run \(\pi = 0\)
• What should the firm charge per dollar of coverage?
• \(E(\pi) = aq - pq - t = 0\)

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• \(a = p + (t/q)\)
• The cost per dollar of coverage is proportion to risk
• \(t/q\) is the loading factor. Portion of price to cover administrative costs
• Make it simple, suppose \(t=0\).
  – \(a = p\)
  – If the probability of loss is 0.05, will change 5 cents per $1.00 of coverage

• In this situation, if a person buys a policy to insure \(L\) dollars, the ‘actuarially fair’ premium will be \(LP\)
• An actuarially fair premium is one where the premium equals the expected loss
• In the real world, no premiums are ‘actuarially fair’ because prices include administrative costs called ‘loading factors’

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How much insurance will people purchase when prices are actuarially fair?
• With insurance
  – Pay a premium that is subtracted from income
  – If bad state happens, lose \(L\) but get back the amount of insurance \(q\)
  – They pay \(p + (t/q)\) per dollar of coverage. Have \(q\) dollars of coverage – so they to pay a premium of \(pq + t\) in total
• Utility in good state
  – \(U = U[Y - pq - t]\)
• Utility in bad state
  - $U'[Y - L + q - pq - t]$  
• $E(u) = (1-p)U'[Y - pq - t] + pU'[Y-L+q-pq-t]$  
• Simplify, let $t=0$ (no loading costs)  
• $E(u) = (1-p)U'[Y - pq] + pU'[Y-L+q-pq]$  
• Maximize utility by picking optimal $q$  
• $dE(u)/dq = 0$

• $E(u) = (1-p)U'[Y - pq] + pU'[Y-L+q-pq]$  
• $dE(u)/dq = (1-p)U'(y-pq)(-p)$  
• $+ pU'(Y-L+q-pq)(1-p) = 0$  
• $(1-p)p$ cancel on each side

• $U'[Y-L+q-pq] = U'(Y-pq)$  
• Optimal insurance is one that sets marginal utilities in the bad and good states equal  
• $Y-L+q-pq = Y-pq$  
• $Y$'s cancel, $pq$'s cancel,  
• $q=L$  
• If people can buy insurance that is ‘fair’ they will fully insure loses.

Insurance w/ loading costs

• Insurance is not actuarially fair and insurance does have loading costs  
• Can show (but more difficult) that with loading costs, people will now under-insure, that is, will insure for less than the loss $L$.  
• Intuition? For every dollar of expected loss you cover, will cost more than a $1  
• Only get back $1 in coverage if the bad state of the world happens
• Recall:
  – $q$ is the amount of insurance purchased
  – Without loading costs, cost per dollar of coverage is $p$
  – Now, for simplicity, assume that price per dollar of coverage is $pk$ where $k > 1$ (loading costs)
• Buy $q$ $\$ worth of coverage
• Pay $qpk$ in premiums

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\[
\]

\[
\frac{dE(u)}{dq} = (1-p) U'(Y-pqk)(-pk) + pU'(Y-L+q-pqk)(1-pk) = 0
\]

\[
p(1-pk)U'(Y-L+q-pqk) = (1-p)pkU'(Y-pqk)
\]

• $p$ cancel on each side

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\[
(1-pk)U'(Y-L+q-pkq) = (1-p)kU'(Y-pkq)
\]

\[
(a)(b) = (c)(d)
\]

• Since $k > 1$, can show that
• (1-pk) < (1-p)k
• Since (a) < (c), must be the case that
• (b) > (d)
• $U'(Y-L+q-pkq) > U'(Y-pkq)$
• Since $U'(y1) > U'(y2)$, must be that $y1 < y2$

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• $(Y-L+q-pqk) < (Y-pqk)$
• $Y$ and $-pqk$ cancel
• $-L + q < 0$
• Which means that $q < L$
• When price is not ‘fair’ you will not fully insure
Demand for Insurance

- Both people have income of Y.
- Each person has a potential health shock.
  - The shock will leave person 1 w/ expenses of E1 and will leave income at Y1 = Y - E1.
  - The shock will leave person 2 w/ expenses of E2 and will leave income at Y2 = Y - E2.
- Suppose that:
  - E1 > E2, Y1 < Y2.

Probabilities the health shock will occur are P1 and P2.

- Expected Income of person 1:
  - E(Y1) = (1 - P1)Y + P1 * (Y - E1).
  - E(Y2) = (1 - P2)Y + P2 * (Y - E2).
- Suppose that E(Y1) = E(Y2) = Y3.

In this case:
- Shock 1 is a low probability/high cost shock.
- Shock 2 is a high probability/low cost shock.

Example:
- Y = $60,000.
- Shock 1 is 1% probability of $50,000 expense.
- Shock 2 is a 50% chance of $1000 expense.
- E(Y) = $59500.
• Expected utility locus
  – Line ab for person 1
  – Line ac for person 2
• Expected utility is
  – Ua in case 1
  – Ub in case 2
• Certainty premium –
  – Line (de) for person 1, Difference Y3 – Ya
  – Line (fg) for person 2, Difference Y3 - Yb

Implications
• Do not insure small risks/high probability events
  – If you know with certainty that a costs will happen, or, costs are low when a bad event occurs, then do not insure
  – Example: teeth cleanings. You know they happen twice a year, why pay the loading cost on an event that will happen?

Some adjustments to this model
• The model assumes that poor health has a monetary cost and that is all.
  – When experience a bad health shock, it costs you L to recover and you are returned to new
• Many situations where
  – health shocks generate large expenses
  – And the expenses may not return you to normal
  – AIDS, stroke, diabetes, etc.

• Insure catastrophic events
  – Large but rare risks
• As we will see, many of the insurance contracts we see do not fit these characteristics – they pay for small predictable expenses and leave exposed catastrophic events
• In these cases, the health shock has fundamentally changed life.
• We can deal with this situation in the expected utility model with adjustment in the utility function
• “State dependent” utility
  – $U(y)$ utility in healthy state
  – $V(y)$ utility in unhealthy state

• Typical assumption
  – $U(Y) > V(Y)$
    • For any given income level, get higher utility in the healthy state
  – $U'(Y) > V'(Y)$
    • For any given income level, marginal utility of the next dollar is higher in the healthy state

Note that:
• At $Y_1$,
  – $U(Y_1) > V(Y_1)$
  – $U'(Y_1) > V'(Y_1)$
  – Slope of line $aa >$ slope of line $bb$
• Notice that slope line $aa =$ slope of line $cc$
  – $U'(Y_1) = V'(Y_2)$
What does this do to optimal insurance

• $E(u) = (1-p)U[Y - pq - t] + pV[Y-L+q-pq-t]
• Again, let's set $t=0$ to make things easy
• $\frac{dE(u)}{dq} = (1-p)(-p)U'[Y-pq] + p(1-p)V'[Y-L+q+pq] = 0$
• $U'[Y-pq] = V'[Y-L+q-pq]$

Just like in previous case, we equalize marginal utility across the good and bad states of the world

• Recall that
  - $U'(y) > V'(y)$
  - $U'(y) = V'(y)$ if $y_1 > y_2$
• Since $U'[Y-pq] = V'[Y-L+q-pq]$
• In order to equalize marginal utilities of income, must be the case that
  $[Y-pq] > [Y-L+q-pq]$

Income in healthy state > income in unhealthy state

Do not fully insure losses. Why?
  - With insurance, you take $ from the good state of the world (where MU of income is high) and transfer $ to the bad state of the world (where MU is low)
  - Do not want good money to chance bad

Allais Paradox

• Which gamble would you prefer
  - 1A: $1$ million w/ certainty
  - 1B: (.89, $1$ million), (0.01, $0$), (0.1, $5$ million)
• Which gamble would you prefer
  - 2A: (.89, $0$), (0.11, $1$ million)
  - 2B: (.9, $0$), (0.10, $5$ million)
• 1st gamble:
  • \( U(1) > 0.89U(1) + 0.01U(0) + 0.1U(5) \)
  • \( 0.11U(1) > 0.01U(0) + 0.1U(5) \)

• Now consider gamble 2
  • \( 0.9U(0) + 0.1U(5) > 0.89U(0) + 0.11U(1) \)
  • \( 0.01U(0) + 0.1U(5) > 0.11U(1) \)

• Choice of Lottery 1A and 2B is inconsistent with expected utility theory