Coaxial Cable: It’s not just a piece of wire.

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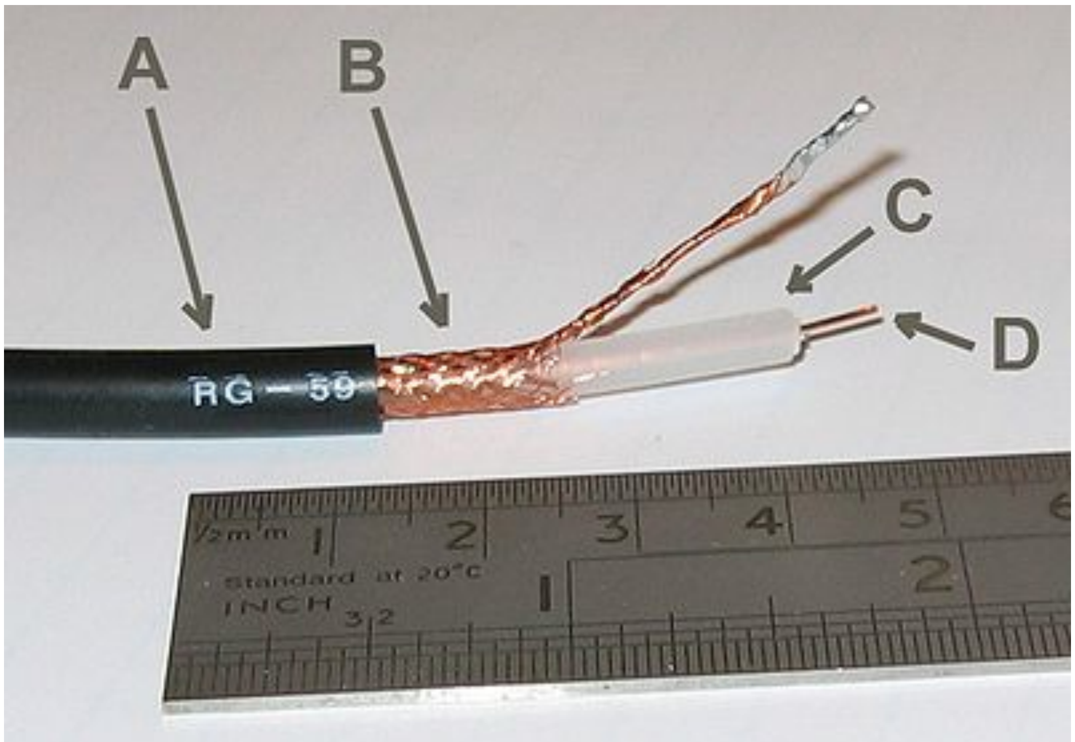
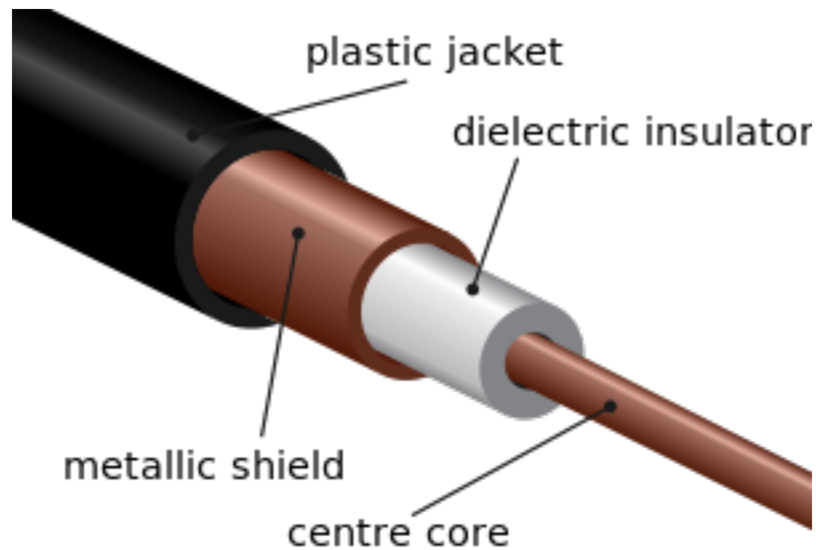
For PHYS40441

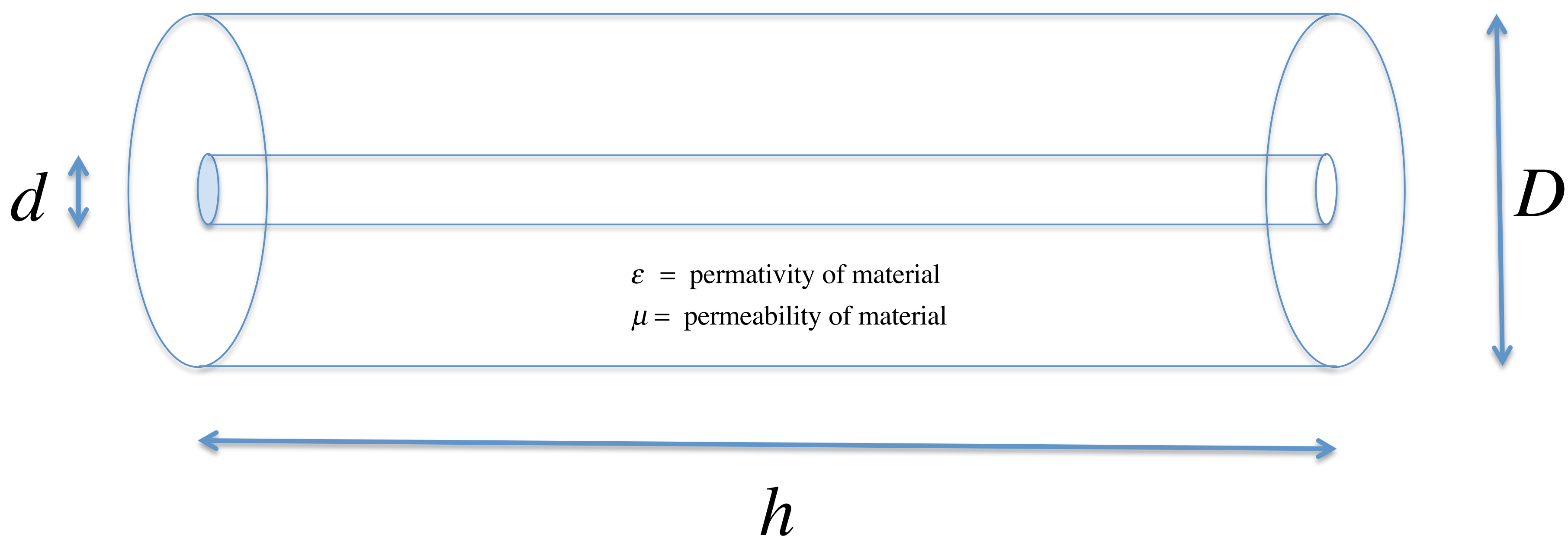
Coaxial Cable

Coaxial cable From Wikipedia, the free encyclopedia





Capacitance and Inductance per Unit Length



$$\left(\frac{C}{h}\right) = \frac{2\pi\epsilon}{\ln(D/d)} = \frac{2\pi\epsilon_0\epsilon_r}{\ln(D/d)}$$

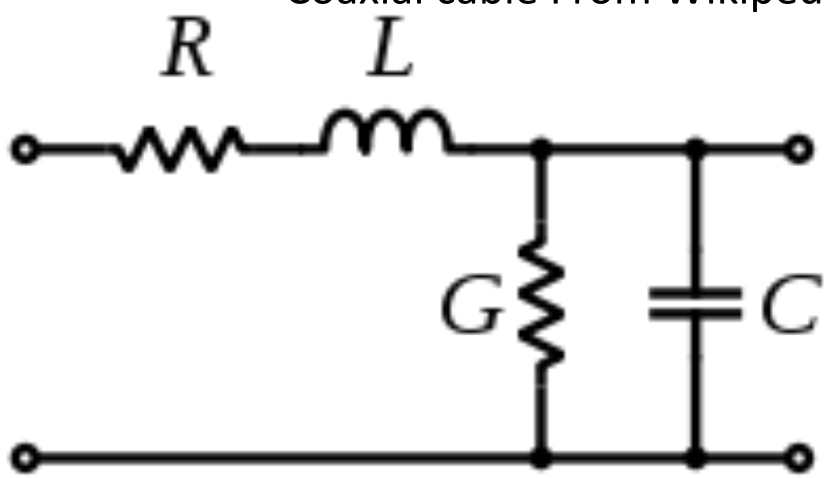
Typically derived in your basic physics text.

$$\left(\frac{L}{h}\right) = \frac{\mu}{2\pi}\ln(D/d) = \frac{\mu_0\mu_r}{2\pi}\ln(D/d)$$

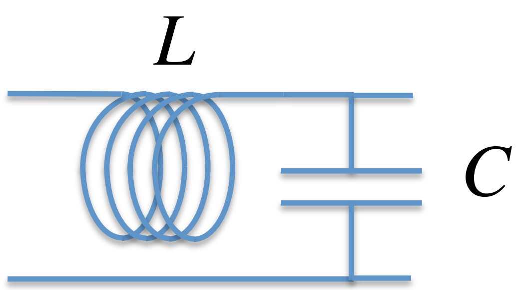
From Wikipedia, the free encyclopedia

Schematic Representation

Coaxial cable From Wikipedia, the free encyclopedia



L=total inductance
C= total capacitance
R= total resistance (small –> zero)
G= total leakage resistance
(very large –> infinite)



Simplified Schematic

This is not a very good model of a transmission line.

Reminders from Basic Physics Courses

$$V_{drop} = RI$$

$$V_{drop} = \frac{Q}{C}$$

$$V_{drop} = L\frac{dI}{dt}$$

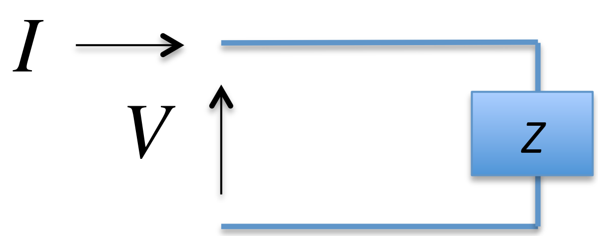
For AC circuits:

$$Z_R = R$$

$$Z_C = \frac{1}{j\omega C}$$

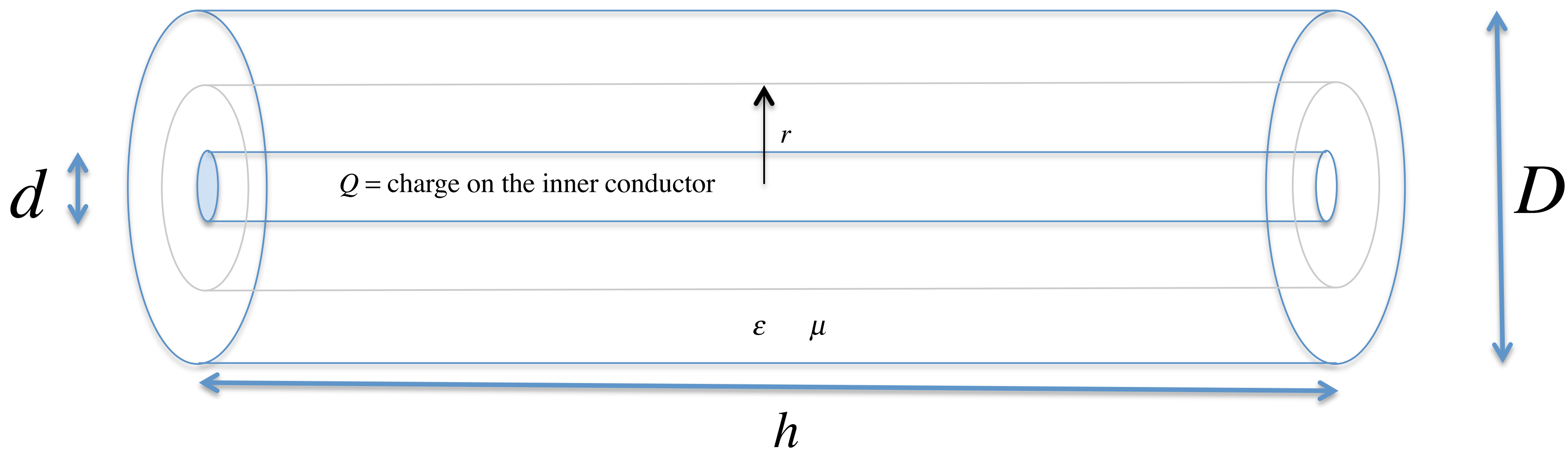
$$Z_L = j\omega L$$

$$V = Z\ I$$



$$Z = \frac{V}{I}$$

Derive the Capacitance of a Cylinder



Used Gauss' Law and an imaginary surface to calculate the E field in the cylinder.

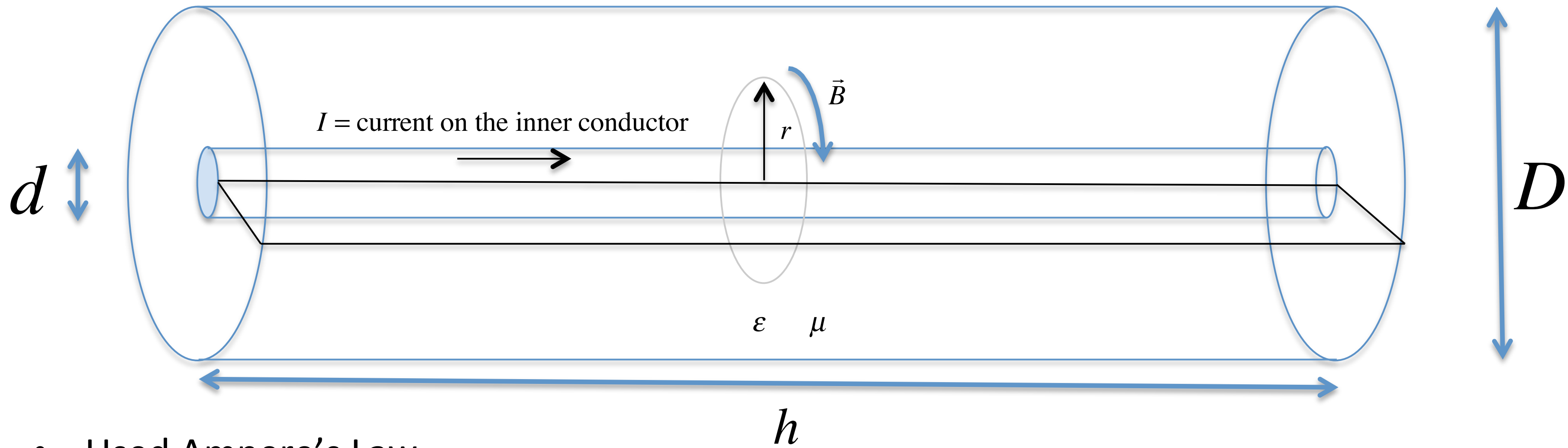
$$\int \vec{E} \bullet d\vec{a} = \frac{Q}{\varepsilon} \qquad E 2\pi r h = \frac{Q}{\varepsilon} \qquad E = \frac{Q}{\varepsilon 2\pi r h}$$

Voltage difference is the integral of the E field.

$$V = \int_{r_1}^{r_2} E\,dr = \frac{Q}{\varepsilon 2\pi h} \int_{d/2}^{D/2} \frac{1}{r} dr = \frac{Q}{\varepsilon 2\pi h}\left[\ln(D/2) - \ln(d/2)\right] = \frac{Q}{\varepsilon 2\pi h} \ln\left(\frac{D}{d}\right)$$

$$Q = CV \qquad C = \frac{Q}{V} = \frac{2\pi\varepsilon h}{\ln\left(\frac{D}{d}\right)} \qquad \frac{C}{h} = \frac{2\pi\varepsilon}{\ln(D/d)}$$

Derive the Inductance of a Cylinder



- Used Ampere's Law and an imaginary path to calculate the B field in the cylinder.

$$\oint \vec{B} \bullet d\vec{\ell} = \mu I \qquad B 2\pi r = \mu I \qquad B = \frac{\mu I}{2\pi r}$$

- Determine the flux through the imaginary loop.

$$\phi = \int_{r_1}^{r_2} \vec{B} \bullet d\vec{a} = \int_{d/2}^{D/2} \frac{\mu I}{2\pi r} h\, dr = \frac{\mu I}{2\pi} h\left[\ln(D/2) - \ln(d/2)\right] = \frac{\mu I}{2\pi} h \ln\left(\frac{D}{d}\right)$$

- Inductance is proportional to the induced EMF for a change in current.

$$V_{drop} = -EMF = \frac{d\phi}{dt} = L\frac{dI}{dt} = \left[h\frac{\mu}{2\pi}\ln\left(\frac{D}{d}\right)\right]\frac{dI}{dt}$$

$$L = h\frac{\mu}{2\pi}\ln\left(\frac{D}{d}\right) \qquad \frac{L}{h} = \frac{\mu}{2\pi}\ln\left(\frac{D}{d}\right)$$

Characteristic Impedance of a Coaxial Cable

$$Z_0 = \sqrt{\frac{L/h}{C/h}} = \sqrt{\frac{L}{C}} = \sqrt{\frac{\frac{\mu}{2\pi}\ln(D/d)}{\frac{2\pi\varepsilon}{\ln(D/d)}}} = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}\ln(D/d)$$

$$v = \frac{1}{\sqrt{\mu\varepsilon}}$$

Speed of EM wave in a medium.

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

Speed of EM wave in a vacuum.

Index of Refraction

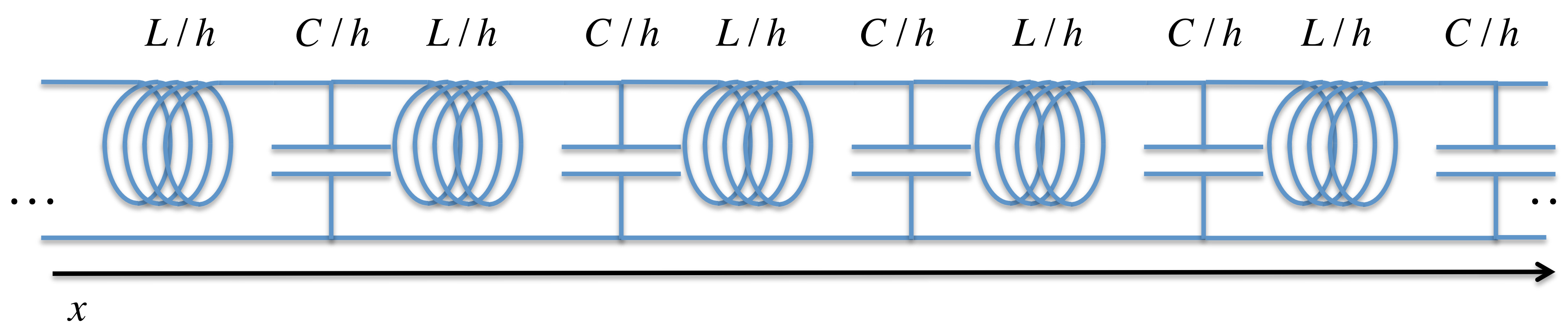
$$\frac{c}{v} = \frac{\frac{1}{\sqrt{\mu_0\varepsilon_0}}}{\frac{1}{\sqrt{\mu\varepsilon}}} = \sqrt{\frac{\mu}{\mu_0}\frac{\varepsilon}{\varepsilon_0}}$$

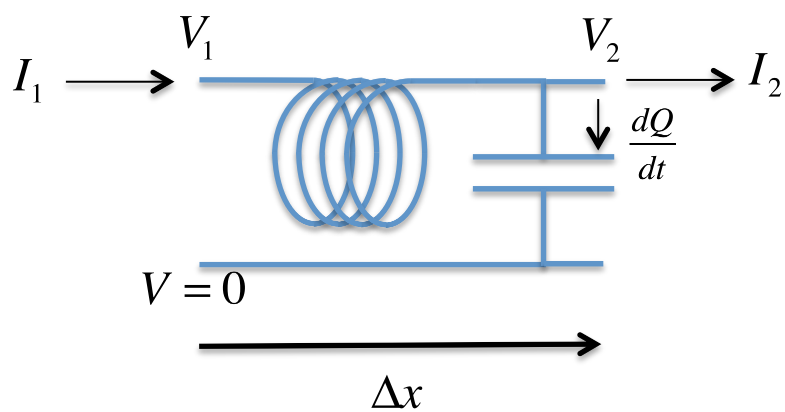
Common Types

type	impedance ohms	core	Dielectric Type	Dielectric VF	Dielectric in	Dielectric mm	OD in	OD mm	shields	comments	max attenuation @ 750 MHz dB/100 ft
RG-56/U	48	1.4859 mm					0.308	7.82	Dual braid shielded	Rated to 8000 volts, rubber dielectric	
RG-58/U	50	0.81 mm	PE	0.66	0.116	2.9	0.195	5.0	single	Used for radiocommunication and amateur radio, thin Ethernet (10BASE2) and NIM electronics, Loss 1.056 dB/m @ 2.4 GHz. Common.[19]	13.104[16]
RG-59/U	75	0.64 mm	PE	0.66	0.146	3.7	0.242	6.1	single	Used to carry baseband video in closed-circuit television, previously used for cable television. In general, it has poor shielding but will carry an HQ HD signal or video over short distances.[20]	9.708[16]
RG-59A/U	75	0.762 mm	PF	0.78	0.146	3.7	0.242	6.1	single	Similar physical characteristics as RG-59 and RG-59/U, but with a higher velocity factor.	8.9@700MHz[21]

What it is the characteristic impedance of a transmission line?

We need a better model!





$$V_1 - V_2 = \frac{L}{h}\Delta x \frac{dI}{dt}$$

$$\Delta V = V(x_2) - V(x_1)$$

$$\frac{\Delta V}{\Delta x} = -\frac{L}{h}\frac{dI_1}{dt}$$

$$\lim_{\Delta x \to 0}\frac{\Delta V}{\Delta x} = \frac{\partial V}{\partial x} = -\frac{L}{h}\frac{dI}{dt}$$

1

$$I_1 - I_2 = \frac{dQ}{dt} = \frac{C}{h}\Delta x \frac{dV_2}{dt}$$

$$\Delta I = I(x_2) - I(x_1)$$

$$\frac{\Delta I}{\Delta x} = -\frac{C}{h}\frac{dV_2}{dt}$$

$$\lim_{\Delta x \to 0}\frac{\Delta I}{\Delta x} = \frac{\partial I}{\partial x} = -\frac{C}{h}\frac{dV}{dt}$$

2

Two Coupled Diff. Eq.s

Solve for voltage and current.

1 $$\frac{\partial V}{\partial x} = -\frac{L}{h}\frac{dI}{dt}$$

2 $$\frac{\partial I}{\partial x} = -\frac{C}{h}\frac{dV}{dt}$$

$$\frac{\partial}{\partial x}\frac{\partial V}{\partial x} = -\frac{L}{h}\frac{d}{dt}\frac{\partial I}{\partial x} = --\frac{L}{h}\frac{C}{h}\frac{d}{dt}\frac{dV}{dt}$$

Wave Equation

$$\frac{\partial^2 V}{\partial x^2} - \left(\frac{L}{h}\frac{C}{h}\right)\frac{d^2 V}{dt^2} = 0$$

Traveling wave solution for V

$$V = Ae^{i(kx-\omega t)}$$

$$k^2 = \omega^2 \frac{L}{h}\frac{C}{h}$$

$$k = \pm\omega\sqrt{\frac{L}{h}\frac{C}{h}}$$

$$\left|\frac{\omega}{k}\right| = v = \frac{1}{\sqrt{\frac{L}{h}\frac{C}{h}}}$$

Find the current for a positive going wave.

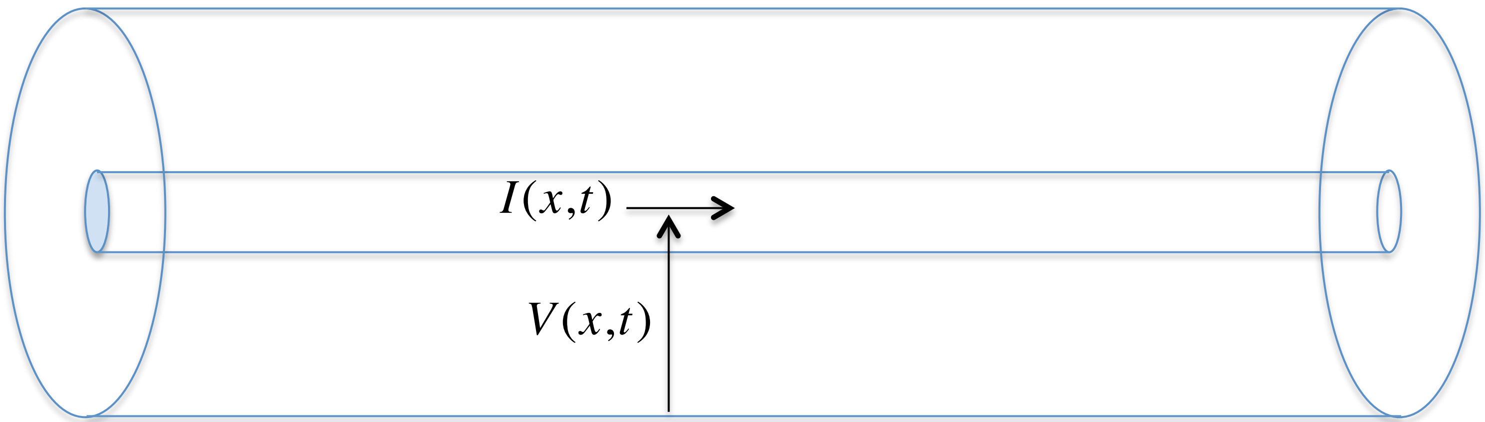
$$\frac{dI}{dt} = -\frac{1}{L/h}\frac{\partial V}{\partial x} = -\frac{1}{L/h} ikAe^{i(kx-\omega t)}$$

$$I = \int \frac{dI}{dt}dt = -\frac{1}{L/h} ikA\int e^{i(kx-\omega t)}\,dt = \frac{1}{L/h}\frac{-ik}{-i\omega}Ae^{i(kx-\omega t)} = \frac{1}{L/h}\sqrt{\frac{L}{h}\frac{C}{h}}Ae^{i(kx-\omega t)}$$

$$I(x,t) = \sqrt{\frac{C/h}{L/h}}Ae^{i(kx-\omega t)}$$

What voltage and current do we use to calculate the characteristic impedance?

The voltage and current at any particular point along the transmission line.



Assume $V = 0$ on the outside conductor.

$$Z_0 = \frac{V(x,t)}{I(x,t)} = \frac{Ae^{i(kx-\omega t)}}{\sqrt{\frac{C/h}{L/h}}Ae^{i(kx-\omega t)}} = \sqrt{\frac{L/h}{C/h}} = \sqrt{\frac{L}{C}}$$

$$\left(\frac{C}{h}\right) = \frac{2\pi\epsilon}{\ln(D/d)} = \frac{2\pi\epsilon_0\epsilon_r}{\ln(D/d)}$$

$$\left(\frac{L}{h}\right) = \frac{\mu}{2\pi}\ln(D/d) = \frac{\mu_0\mu_r}{2\pi}\ln(D/d)$$

$$v = \frac{1}{\sqrt{\frac{L}{h}\frac{C}{h}}} = \frac{1}{\sqrt{\mu\varepsilon}}$$

Demonstration