

Math 60790 (Numerical PDEs), Spring 2008

Homework 3 (Due May 7, 2008)

Consider the initial boundary value problem

$$\begin{cases} u_t(x,t) = u_{xx}(x,t), & \text{for } -1 < x < 1, \ 0 < t \leq T; \\ u(x,0) = \cos(\pi x), & -1 < x < 1; \\ u(-1,t) = -e^{-\pi^2 t}, & 0 < t \leq T; \\ u(1,t) = -e^{-\pi^2 t}, & 0 < t \leq T. \end{cases}$$

This problem has the exact solution $u(x,t) = e^{-\pi^2 t} \cos(\pi x)$.

Solve this problem by the cG(1)dG(1) method. Use a uniform spatial mesh with

$h = \frac{1}{10}, \frac{1}{20}, \frac{1}{40}, \frac{1}{80}, \frac{1}{160}$ and $\frac{1}{320}$, respectively. **Choose a uniform time step**

k **such that** $k = h^2$. **Compute the** L^∞ **error at** $T=1$, **which is**

$\|e(x,T)\|_{L^\infty} = \max_{1 \leq i \leq M} |u(x_i,T) - U(x_i,T)|$. **Make a table for the errors. Compute the**

numerical orders of accuracy during mesh refinement and include them in the table.

Use **double precision** for your codes. Attach a print-out of your codes.