Cash Flow, Consumption Risk, and the Cross-section of Stock Returns

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ABSTRACT

I link an asset’s risk premium to two characteristics of its underlying cash flow: covariance and duration. Using empirically novel estimates of both cash flow characteristics based exclusively on accounting earnings and aggregate consumption data, I examine their dynamic interaction in a two-factor cash flow model and find that they are able to explain up to 82% of the cross-sectional variation in the average returns on size, book-to-market, and long-term reversal-sorted portfolios for the period 1964 to 2002. This finding highlights the importance of fundamental cash flow characteristics in determining the risk exposure of an asset.

Differences in expected return across assets are determined by differences in the assets’ exposure to systematic risk. This key insight in financial economics is reflected in the standard consumption-based asset pricing model (CCAPM, see Rubinstein (1976), Lucas (1978), and Breeden (1979)). CCAPM predicts that an asset’s consumption beta—a measure of the comovement between asset return and aggregate consumption—determines its expected return. Yet despite its intuitive appeal, many early empirical tests have not supported this prediction (see Breeden, Gibbons, and Litzenberger (1989)). Beginning with the seminal work of Bansal and Yaron (2004), advances in our understanding of “long-run” consumption risk may validate this theoretical position after all. Indeed, papers such as Parker and Julliard (2005), Bansal, Dittmar, and Lundblad (2005), Jagannathan and Wang (2007), Hansen, Heaton, and Li (2008) and Bansal, Dittmar, and Kiku (2009) point out that systematic consumption risk, if measured over longer horizons, is able to explain cross-sectional variation in expected return.

This paper presents further evidence consistent with the implications of the consumption-based asset pricing model. While many papers evaluate exposure

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to risk using returns and prices, this paper shows that differences in risk premia can be directly linked to fundamental cash flow characteristics. Pinning risk premia to cash flows instead of prices or returns has two advantages. In the short run, price may temporarily deviate from its “fair” value due to mispricing and liquidity events. More importantly, in typical rational expectations asset pricing models, prices (and returns) are set by expectations about future cash flows. Cash flows therefore more directly underlie risk premia. Given this, I break down the role that cash flow plays and identify two of its important characteristics. The contribution of this paper is to analyze both characteristics simultaneously for the first time, revealing their interactions and how these interactions dynamically impact returns.

The first characteristic of cash flow is the degree of its comovement with consumption, the cash flow covariance. In the consumption-based models of Abel (1999) and Bansal and Yaron (2004), this type of covariance ultimately determines stocks’ exposure to systematic risk. Recent empirical work by Bansal, Dittmar, and Lundblad (2005) shows that the cash flow beta measuring such comovement explains 62% of the cross-sectional variation in risk premia across various assets. The second important cash flow characteristic is the temporal pattern of cash flow, that is, whether the asset pays more cash flow in the future or now. I call this characteristic cash flow duration. In recent work, both Lettau and Wachter (2007) and Dechow, Sloan, and Soliman (2004) fundamentally link duration risk to stock returns.1

Empirically, the impact of these characteristics on the cross-sectional variation of risk premia can be largely captured by the following two-factor cash flow model:

$$E_t[R_{t+1} - R_f] = \gamma_0 + \gamma_1 Cov^i + \gamma_2 Cov^i \times Dur^i,$$

where $\gamma_0, \gamma_1 > 0$ and $\gamma_2 < 0$. (1)

The variables $Cov$ and $Dur$ denote the relative cash flow covariance and duration measures, respectively. They are defined relative to the aggregate consumption portfolio. For an asset with positive (negative) $Cov$, its cash flow comoves more (less) with aggregate consumption than aggregate consumption itself. For an asset with positive (negative) $Dur$, its cash flow duration is higher (lower) than the duration of the aggregate consumption portfolio. By construction, the aggregate consumption portfolio will have zero $Cov$ and $Dur$ and will have an expected return equal to $\gamma_0$. Consistent with Bansal, Dittmar, and Lundblad (2005), cash flow covariance has a first-order impact on the cross-sectional variation of risk premia. Higher cash flow covariance leads to a higher

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1 The role of equity duration has been highlighted in the literature. Early literature such as Jagannathan and Viswanathan (1988) and Connor and Korajczyk (1989) demonstrates that the replication of a stock’s cash flows in a dynamic world requires bonds; therefore, bond factors contribute to the correct pricing of equities. Using a case study and regression analysis, Cornell (1999, 2000) reemphasizes the importance of equity duration risk in determining a firm’s cost of capital. The “cash flow duration” in this paper is a purely cash flow-based measure and is different from the equity duration previously analyzed, which typically uses price information.
risk premium, much as higher beta leads to higher return in CAPM. Consistent with the intuition in Lettau and Wachter (2007), cash flow duration also affects the risk premium. Interestingly, when there is large cross-sectional variation in cash flow covariance, cash flow duration provides additional explanatory power through a second-order interaction term with cash flow covariance. In explaining the return difference of two assets with very different cash flow durations (such as value and growth stock portfolios), it is therefore imperative to account for cash flow duration.

For analytical tractability and easy economic interpretation, I highlight the intuition behind the two-factor cash flow model in a simple economy where both characteristics are modeled as high frequency short-run properties of the cash flow. The empirical estimates of Cov and Dur, although identifying their theoretical counterparts up to scaling factors, are estimated in a novel way using exclusively long-run accounting earnings and aggregate consumption data. Measuring earnings and consumption over the long run alleviates problems caused by short-term earnings management (Jones (1991) and Teoh, Welch, and Wong (1998)), short-term consumption commitment, or delayed response (Parker and Julliard (2005) and Jagannathan and Wang (2007)).

With these measures of Cov and Dur, I show that the two-factor cash flow model explains more than 80% of the cross-sectional variation in risk premia in 60 size-sorted, book-to-market-sorted, and long-term reversal portfolios. The duration measure adds more than 20% additional explanatory power. I focus on stock portfolios sorted on size, book-to-market ratio, and past long-term return as cross-sectional variation in these stock characteristics generates large and persistent cross-sectional variation in risk premia that is particularly challenging for many common benchmark models (see Fama and French (1992) and De Bondt and Thaler (1985)). In the empirical section, I also validate the two-factor cash flow model using several robustness tests and compare its performance to alternative benchmark models.

The empirical cash flow covariance in this paper, measured using long-run consumption data, is similar to the “long-run” cash flow beta considered by Bansal, Dittmar, and Lundblad (2005). However, while they show that the cash flow beta is capable of explaining the short-term return momentum, this paper is the first, to my knowledge, to empirically show that cash flow covariance, measured properly using long-run earnings data, also helps to explain the long-run return reversal. In a theoretical model, Yang (2007) shows that when the cash flow growth rate of a stock contains two time-varying components that are exposed to long-run consumption risk, both short-term return momentum and long-run reversal can be explained. My empirical finding corroborates Yang’s prediction and thus provides further support for the long-run consumption risk model of Bansal and Yaron (2004).

As a supplementary exercise motivated by the long-run risk model of Bansal and Yaron (2004), I also directly model the cash flow covariance as exposure to both long-run and short-run consumption risk in a simple economy and derive the same two-factor cash flow model. Details are provided in the Internet Appendix available at http://www.afajof.org/supplements.asp.
A. Other Related Literature

The “cash flow risk” of an asset has been actively researched in recent work, with the related papers falling into two groups. Papers in the first group focus on the time series implications of cash flow risk. For example, Bansal and Yaron (2004) model long-run risk and find that it can explain many time-series properties of financial markets. Menzly, Santos, and Veronesi (2004) analyze the predictability of dividend growth and asset return in an economy where assets have mean-reverting cash flows. Papers in the second group focus on the cross-sectional implications of cash flow risk. For example, Brennan and Xia (2006) study the pricing of equity strips in an Intertemporal CAPM (ICAPM) framework. Santos and Veronesi (2004) decompose the CAPM beta into a discount beta and a cash flow beta and examine which one dominates using cross-section return data. Santos and Veronesi (2005) extend Menzly, Santos, and Veronesi (2004) to explain simultaneously the time-series properties of the market portfolio and the value premium in the cross-section. Hansen, Heaton, and Li (2008) analyze the risk-return trade-off between cash flow risk and the long-run return of a security. Cohen, Polk, and Vuolteenaho (2008) empirically analyze a cash flow-based CAPM beta computed as the covariance between portfolio and market cash flows and show that it explains the difference in price levels. Finally, Bansal, Dittmar, and Lundblad (2005) show that a cash flow beta measure alone explains 62% of the cross-sectional variation in risk premia across various assets.

This paper’s focus falls with the second group; however, with the exception of Bansal, Dittmar, and Lundblad (2005), few papers have empirically tested the cross-sectional implications of cash flow risk in relation to aggregate consumption. This paper contributes to this line of research by showing, both theoretically and empirically, that the temporal pattern of cash flow as measured by Dur has additional explanatory power for the cross-section of stock returns.

The remainder of the paper is organized as follows. Section I derives the two-factor cash flow model in a simple economy to highlight its intuition. Section II discusses the empirical estimation of the cash flow characteristics. Section III provides empirical support for the two-factor cash flow model. Section IV briefly summarizes the findings. Appendix A contains detailed proofs. Appendix B discusses the relation between dividends and earnings as cash flow measures.

I. Two-Factor Cash Flow Model

This section describes a mean-reverting cash flow share process in a simple economy, derives the exact expression for the risk premium on a stock and develops the two-factor cash flow model as an approximation. The simple economy is arguably restrictive and unlikely to capture all the ways in which fundamental cash flow characteristics of a stock determine its risk premium. Nevertheless, it highlights the key intuition behind the two-factor cash flow model.
A. The Cash Flow Process

In order to model cash flow covariance and duration parsimoniously, I consider a simple mean-reverting cash flow share process. Let $S_i^t$ denote the share of asset $i$’s cash flow ($D$) relative to aggregate consumption ($C$) at time $t$, or

$$S_i^t = \frac{D_i^t}{C_t}.$$ 

Take the log to get $d_i^t = s_i^t + c_t$.\(^3\) I then assume that the log cash flow share follows an AR(1) process:

$$s_{i,t+1}^t = (1 - \phi)s_i^t + \phi s_{i,t+1} + \lambda_i \Delta c_{t+1} + \varepsilon_{i,t+1}$$

where $\varepsilon$ is independent of $w$ and $t$.

The key assumption is that the cash flow share is stationary and mean-reverting, as captured by an AR(1) process. A similar assumption is made in Menzly, Santos, and Veronesi (2004) and Santos and Veronesi (2004). The mean-reverting cash flow share process ensures a stationary steady state where no single asset dominates. Arguably, the AR(1) assumption about cash flow share may not extend to individual stock and certain sector portfolios. However, I show that it is a reasonable assumption for the cash flows in buy-and-hold stock portfolios constructed based on size, book-to-market ratio, and past long-term stock return.

Define $z_i^t$ as $s_i - s_i^t$. Then, given (2), the cash flow growth rate can be written as

$$\Delta d_{i,t+1} = \Delta c_{t+1} + \Delta s_{i,t+1}^t = \Delta c_{t+1} + (1 - \phi)z_i^t + \lambda_i w_{t+1} + \varepsilon_{i,t+1}^t,$$

where $\varepsilon$ is independent of $w$ and $t$.

The cash flow characteristics of interest are now each captured by one parameter of the model. Parameter $\lambda_i$ measures the contemporaneous covariance between innovations in cash flow share and innovations in aggregate consumption growth ($\Delta c_{t+1} - E_t[\Delta c_{t+1}] = w_{t+1}$), and is a measure of relative cash flow covariance. For an asset with larger $\lambda$, its cash flow varies more with the aggregate consumption innovation, resulting in more cash flow covariance. Parameter $z_i^t$ is a relative cash flow duration measure. A higher $z_i^t$ also results in a higher expected cash flow growth rate in the future as in (3), leading to more cash flow (as a share of aggregate consumption) being paid out in the future. The term $z_i^t$ is purely cash flow-based, which is different from the usual fixed-income Macaulay duration. The latter is price-based, meaning that it measures

\(^3\) Throughout the paper lower case is used to denote the log of the original variable unless otherwise defined.
the change in price as a result of a change in discount rate. However, the two duration measures are related in that they both capture the temporal pattern of cash flows, making “duration” a reasonable label for \( z_t \).

Given the simple cash flow process, assets differ from each other only in terms of the cash flow covariance and duration. As a result, the cross-sectional variation in risk premia should also be a function of these two cash flow characteristics. To understand the exact relationship between risk premium and the cash flow characteristics, I need to first specify the dynamics for aggregate consumption growth and the stochastic discount factor (SDF) in the economy.

B. The Economy

I assume that the log aggregate consumption growth in the economy follows an ARMA(1,1) process:

\[
\Delta c_{t+1} = \mu_c (1 - \rho_1) + \rho_1 \Delta c_t + w_{t+1} - \rho_2 w_t,
\]

\[
w_t \sim N(0, \sigma_w^2).
\]

(4)

The same assumption on consumption growth is made in Campbell (1999), Bansal and Yaron (2000), and Bansal, Dittmar, and Lundblad (2002), among others. When \( \rho_2 = 0 \), the ARMA(1,1) process in (4) reduces to an AR(1) as considered in Mehra and Prescott (1985). Overall, the simple economy is a special case of Bansal and Yaron (2004).

To make the algebra simpler and allow the economic intuition to come through more clearly, I further assume the Epstein and Zin (1989) recursive utility for the agent, with elasticity of intertemporal substitution \( \psi = 1 \). I then follow Hansen, Heaton, and Li (2008) who show the log SDF in such an economy can be written as

\[
m_{t+1} = \log \delta - \Delta c_{t+1} + (1 - \gamma) \frac{1 - \rho_2 \delta}{1 - \rho_1 \delta} w_{t+1} - \frac{1}{2} \left[ \frac{(1 - \gamma)(1 - \rho_2 \delta)}{1 - \rho_1 \delta} \sigma_w \right]^2,
\]

(5)

where \( \gamma \) and \( \delta \) denote the coefficient of risk aversion and the time discount factor, respectively. An interesting case studied by Bansal and Yaron (2000) is where \( \rho_1 \) is close to one and slightly greater than \( \rho_2 \). In this case, although the consumption growth very closely resembles an i.i.d. process, the persistent expected consumption growth rate leads to a larger consumption risk premium \(((\gamma - 1) \frac{1 - \rho_2 \delta}{1 - \rho_1 \delta}) \) and a potential solution to the equity risk premium puzzle.

C. Expected Return

In this section, I derive an exact expression for expected excess stock return in the simple economy. This expression can then be used to motivate the two-factor cash flow model.

Just like a bond, a stock can be considered as a portfolio of claims on future cash flow payments. These cash flow claims are also known as equity strips.
Denote $P_{n,t}^i$ as the time $t$ price of an equity strip that pays a cash flow $D_{t+n}^i$ at time $t + n$. I first compute the price and one-period expected (excess) return for each equity strip. The results are summarized in the following proposition (see Appendix A for the proof):

**PROPOSITION 1:** The price-to-cash flow ratio of an equity strip is

$$\frac{P_{n,t}^i}{D_t^i} = \exp\left[A_i(n) + (1 - \phi^n)z_t^i\right],$$

where $A_i(n)$ is defined in Appendix A. The risk premium for each equity strip is

$$RP_i(n) = \log E_t\left[R_{n,t+1}^i/R_f^t\right] = \left(1 + \phi^{n-1}\lambda_i^z\right)\left[1 + (\gamma - 1)^{1 - \rho_2\delta}/(1 - \rho_1\delta)\right] \sigma_w^2. \quad (6)$$

The expected (excess) return of a stock is just the value-weighted average of the expected (excess) returns of all equity strips:

$$E_t\left[R_{t+1}^i/R_f^t\right] = \sum_{n=1}^{\infty} w_i(n) \exp[RP_i(n)]$$

$$w_i(n) = \frac{\exp\left[A_i(n) + (1 - \phi^n)z_t^i\right]}{\sum_{n=1}^{\infty} \exp\left[A_i(n) + (1 - \phi^n)z_t^i\right]}. \quad (7)$$

The risk premium of each equity strip comes from the contemporaneous covariance between cash flow growth and the SDF. It depends on $\lambda_i^z$ and varies across stocks. Intuitively, an equity strip with higher cash flow covariance, as measured by a positive $\lambda_i^z$, has a higher risk premium, holding maturity $n$ constant. For an equity strip with infinite maturity ($n = \infty$), the risk premium becomes $[1 + (\gamma - 1)^{1 - \rho_2\delta}/(1 - \rho_1\delta)] \sigma_w^2$ much as if the cash flow share has no contemporaneous covariance with consumption growth. This is due to the mean-reversion in cash flow share that causes the impact of cash flow covariance to decrease with maturity. As a result, for $\lambda_i^z > 0 (\lambda_i^z < 0)$, the risk premium decreases (increases) in $n$. The relationship between the risk premium and maturity therefore depends on the cash flow covariance $\lambda_i^z$, consistent with the model in Brennan and Xia (2006). In contrast, the risk premium of the equity strip always decreases in its maturity in the calibrated model of Lettau and Wachter (2007) as they do not allow cash flow covariance to vary across stocks.4

Cash flow duration affects risk premia via its interaction with cash flow covariance. When cash flow covariance is positive, the risk premium of an

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4 In the calibrated model of Lettau and Wachter (2007), shocks in price of risk are not correlated with shocks in dividend growth or shocks in expected dividend growth. In addition, cash flow covariance is not directly modeled. While not precisely analogous, such specification is like choosing a positive constant for cash flow covariance in my model. As a result, the risk premium is decreasing in maturity for individual equity strips and high duration stocks will have lower expected returns.
Figure 1. Exact and approximate annual risk premia as functions of cash flow duration and covariance in the simple economy. The left figure plots the risk premium (\(E_t[R_{t+1} - R_{ft}]\)) calculated using the exact solution in (7). The right figure plots the risk premium approximated by the two-factor cash flow model in (13). The parameters (at a monthly frequency) used for the simple economy are: \(\gamma = 5, \rho_1 = 0.965, \rho_2 = 0.851, \delta = 0.998, \sigma_w = 0.01, \phi = 0.98, \sigma_e = 0.02, \) and \(rf = 0.0025.\)

An interesting feature of the model is that the risk premium on each equity strip does not vary over time as shown in equation (6) in the simple economy. The present value weight received by cash flows with different maturities in equation (7), however, does depend on the cash flow duration, which is time-varying. This dependence ultimately yields a time-varying risk premium on the equity (and not the equity strip).

In a more general economy where the elasticity of intertemporal substitution \(\psi\) is not equal to one and additional shocks in the economy are priced,
additional risk premium terms will appear in (6) that only vary with maturity \( n \). Their presence provides a channel for cash flow duration to affect the risk premium on an asset, independent of any interaction with cash flow covariance. Consequently, cash flow duration \( Dur \) could enter the cash flow model of (1) as a separate third factor. However, when cross-sectional variation in cash flow covariance in the economy is large, both calibration and empirical results show the first two factors (\( Cov \) and \( Cov \times Dur \)) to dominate this third \( Dur \) factor in explaining the cross-sectional variation in risk premia.

D. Two-Factor Cash Flow Model, an Approximation

This section presents the two-factor cash flow as an approximation of the true expected (excess) return in equation (7), and highlights its intuition.\(^5\)

The true expected (excess) return (7) can be approximated as

\[
E_t[R_{t+1}^i - R_f t] \approx \sum_{n=1}^{\infty} w^i(n) R P^i(n). \tag{8}
\]

Consider a linear approximation of the risk premium on individual equity strip \( RP^i(n) \) around some fixed maturity \( n^* \):

\[
RP^i(n) \approx RP^i(n^*) + RP^i_n(n^*) (n - n^*),
\]

where \( RP^i_n(n^*) \) denotes the first derivative of \( RP^i_n \) with respect to \( n \), evaluated at \( n^* \). Equation (8) then becomes

\[
E_t[R_{t+1}^i - R_f t] \approx RP^i(n^*) + RP^i_n(n^*) \left[ \sum_{n=1}^{\infty} w^i(n)n - n^* \right]. \tag{9}
\]

Direct computation shows that

\[
RP^i(n^*) = a_0 + a_1 \lambda^i, \tag{10}
\]

\[
RP^i_n(n^*) = a_2 \lambda^i, \tag{11}
\]

\[
\sum_{n=1}^{\infty} w^i(n)n - n^* \approx a_3 z^i, \tag{12}
\]

where \( a_0 \) to \( a_3 \) are constants.\(^6\) Equation (10) states that the risk premium is directly related to the relative cash flow covariance measure (\( a_1 > 0 \)). The intuition is clear: a higher cash flow covariance should lead to a higher return,

\(^5\) The Internet Appendix available at http://www.afajof.org/supplements.asp contains an alternative derivation of the two-factor cash flow using the usual return-based beta representation framework, which also relates the cash flow characteristics directly to the standard return-based consumption beta.

\(^6\) Their expressions and the details of computation can be found in the Internet Appendix available at http://www.afajof.org/supplements.asp.
much as a high beta stock should earn a higher return. Equation (11) highlights the fact that the relationship between the risk premium and the maturity of an equity strip depends on its cash flow covariance, $\lambda$. If $\lambda$ is positive, then the risk premium on the equity strip decreases with maturity, leading to a negative $RP^i(n^*)$. The reverse logic holds for negative $\lambda$. In other words, $RP^i(n^*)$ is negatively related to $\lambda$ ($a_2 < 0$). Finally, recall that the Macaulay duration is defined in the fixed income literature as present value-weighted time. The term $\sum_n w^i(n)n - n^*$ can then be easily interpreted as a relative duration, thus explaining (12). Substituting (10), (11), and (12) into (9) gives us the two-factor cash flow model:

$$Et\left[R^{i}_{t+1} - Rf_{t}\right] \approx \gamma_0 + \gamma_1 \lambda^i + \gamma_2 (z^i \lambda^i).$$

Figure 1 gives us a sense of the performance of the two-factor cash flow model (13) as an approximation of the true expected (excess) return model (7). The left and right graphs plot the expected (excess) returns under the true model and the two-factor approximation, respectively, for a range of cash flow covariances and durations. The two-factor cash flow model seems to do a reasonable job describing the cross-sectional variation in risk premia in the simple economy.

II. Measuring Cash Flow Duration and Covariance

Having described the two-factor model and its intuition in a simple model, I proceed to empirically measure the cash flow characteristics and test their relation to the risk premium.

I measure cash flow duration and covariance using only aggregate consumption and firm cash flow data. Thus, like recent studies of cash flow consumption risk, this paper directly ties risk premia to fundamentals rather than to return-based risk factors. However, the empirical procedure in this paper, differs from previous studies of cash flow consumption risk in two important ways. First, while previous studies estimate the cash flow of a portfolio by rebalancing the portfolio over time, I instead adopt a buy-and-hold procedure similar to that used in Cohen, Polk, and Vuolteenaho (2008) that is more appropriate for my empirical analysis. Whenever a portfolio is formed, I hold its composition constant and trace out its cash flow over time. By keeping the composition of the portfolio unchanged, it becomes possible to estimate cash flow duration even for a portfolio that pays very little dividends at portfolio formation (such as the current growth portfolio). In addition, the resulting cash flow will not depend on stock prices at the time of rebalancing, which are potentially subject to mispricing.

This paper also differs from previous studies in its measurement of cash flow. Previous studies measure cash flow using dividend (including share repurchase) data. There are potential problems associated with dividends. Some firms are expected to pay no dividends for a long time into the future and most firms tend to keep a stable dividend payout policy in the near future. Other empirical difficulties associated with working with dividend data are also highlighted in a review article by Campbell (2000). Taking these concerns
into consideration, I use instead the theoretically equivalent, but empirically better behaved, earnings data to estimate cash flow characteristics. The relationship between dividend data and earnings data is discussed in Vuolteenaho (1999), who makes use of the accounting clean surplus identity to show that (see Appendix B for details)

\[
\sum_{n=0}^{\infty} \rho^n \Delta d^i(t, n + 1) = \sum_{n=0}^{\infty} \rho^n e^i(t, n + 1) - \frac{\kappa}{1 - \rho} - \xi^i_t,
\]

(14)

where \( e \) is the accounting return or log of one plus ROE (Return on book equity) and \( \xi \) is the log cash flow-to-book equity ratio. The terms \( \kappa \) and \( \rho \) are constants defined in Appendix B. The notation \( x^i(t, n + 1) \) represents the variable of interest \( x \) \( n + 1 \) years after portfolio formation for portfolio \( i \) formed in year \( t \). Equation (14) essentially states that if we look at the infinite horizon, cash flow and earnings data contain the same information. The summations involving \( n \) are actually summations over time as \( n \) indexes the years since portfolio formation.

Recall that in the previous section I model cash flow duration and covariance as high frequency cash flow characteristics. Here I base the empirical measures of cash flow duration and covariance on long-run earnings data because the equivalence between earnings data and cash flow data only holds over a long horizon. Moreover, the empirical long-run earnings-based measures are closely related to their theoretical counterparts. It can be shown that the empirical long-run earnings-based measures identify their theoretical counterparts \( \lambda^i \) and \( z^i_t \) up to scaling factors in the simple economy.\(^7\) Long-run earnings-based measures have an additional advantage. At the annual horizon, earnings data may suffer from the discretionary choices of management on the timing of accruals and other types of accounting manipulations as documented in Jones (1991) and Teoh, Welch, and Wong (1998) among others. The impact of such earnings management will be significantly reduced if earnings are “smoothed” over much longer horizons as implicitly incorporated in (14) and the empirical cash flow measures.

Having established my empirical procedure, the next two sections describe in detail the cash flow duration and covariance measures.

A. Cash Flow Duration

The cash flow duration for a given portfolio (say the “growth” portfolio) is constructed as follows. For each year in the sample (say \( t = 1990 \)), consider firms in the growth portfolio in that year. For this set of firms, I construct an ex post duration \( \text{Dur}^i_t \) by following their earnings over years after formation and using the formula

\[
\text{Dur}^i_t = \sum_t e^i - \frac{\kappa}{1 - \rho} - \xi^i_t - E_t[\Sigma_t \Delta e],
\]

(15)

\(^7\) Details can be found in the Internet Appendix available at http://www.afajof.org/supplements.asp.
where $\Sigma^e_t = \sum_{n=0}^{\infty} \rho^n e^i(t, n + 1)$ and $\Sigma^\Delta c_t = \sum_{n=0}^{\infty} \rho^n \Delta c_{t+n+1}$. The term $\Sigma^e_t$ measures the discounted sum of all future accounting returns, $\xi^i_t$ measures the log portfolio cash flow-to-book equity ratio at portfolio formation, and $E_t[\Sigma^\Delta c_t]$ measures the time $t$ expectation of the discounted sum of all future consumption growth. The expectation is computed under the assumption that consumption growth follows an ARMA(1,1) process. It does not, however, generate cross-sectional variation in cash flow duration. Intuitively, $\Sigma^e_t$ measures the discounted sum of all future accounting returns, $\xi^i_t$ measures the log portfolio cash flow-to-book equity ratio at portfolio formation, and $E_t[\Sigma^\Delta c_t]$ measures the time $t$ expectation of the discounted sum of all future consumption growth.

To estimate the term $\Sigma^e_t$, I break it into two terms, namely, a finite summation term and the terminal value term:

$$\Sigma^e_t = \sum_{n=0}^{N-1} \rho^n e^i(t, n + 1) + \sum_{n=N}^{\infty} \rho^n E_t[e^i(t, n + 1)].$$

The terminal value term is estimated as

$$\sum_{n=N}^{\infty} \rho^n E_t[e^i(t, n + 1)] = \frac{\rho^N}{1-\rho} \bar{e}_t.$$ 

In turn, $\bar{e}_t$ is estimated as the time-series average of $\{e^i(t, n), n = 1, \ldots, N\}$.

Once the ex post cash flow durations of the growth portfolio are estimated for each year in my sample, I construct a time-series average across all years to obtain the average cash flow duration for the growth portfolio. Put another way, the portfolio cash flow duration measure ($\text{Dur}_t^i$) is defined as the times-series average of $\text{Dur}_t^i$. I also consider an ex ante portfolio duration measure ($\tilde{\text{Dur}}_t^i$) by projecting $\Sigma^e_t$ on a set of time $t$ measurable cash flow instruments.

### B. Cash Flow Covariance

Cash flow covariance $\lambda^i$ can be identified by regressing $\sum_{n=0}^{\infty} \rho^n [e^i(t, n + 1) - \Delta c_{t+n+1}]$ (long-run accounting returns) on $\sum_{n=0}^{\infty} \rho^n w_{t+n+1}$ (long-run consumption innovations). I denote the regression coefficient $Cov^i$. Empirically, I replace the infinite summation by a finite summation. I fit an ARMA(1,1) process on $\Delta c_{t+1}$ and the residual terms are the estimates of $\{w_t\}$.

Although $\lambda^i$ is modeled theoretically as the contemporaneous covariance between cash flow share and consumption innovation, it is empirically estimated by regressing long-run accounting returns on long-run consumption innovations. I use long-run consumption innovations since short-term consumption growth is problematic due to measurement error, consumption commitment, or delayed response. Recent studies show that cross-sectional variation in expected excess returns can be explained if consumption growth over a longer horizon is used (see Daniel and Marshall (2004), Parker and Julliard (2005),...
and Jagannathan and Wang (2007)). Finally, the summation used in the long-run measures also helps to alleviate possible measurement errors from seasonal adjustment in consumption data.

III. Empirical Results

This section tests empirically the relationship between cash flow characteristics and expected (excess) return as captured by the two-factor cash flow model.

A. Data and Portfolio Construction

Quarterly log aggregate consumption (c) data are used. I measure $\Delta c_t$ annually (fourth quarter to fourth quarter) to match the cash flow data series. In addition, since investors are more likely to make consumption and investment decisions together during the fourth quarter, fourth quarter to fourth quarter consumption growth better explains cross-sectional stock returns, as shown in Jagannathan and Wang (2007).

Since I examine annual returns and the cash flow characteristics are empirically measured using long-run earnings data, I construct test portfolios by sorting stocks along several characteristics that generate persistent cross-sectional dispersion in average stock returns. These characteristics include the firm’s market value (size), book-to-market ratio, and past long-run returns. Every June starting from 1964, I group all stocks issued by industrial firms in NYSE, Amex and NASDAQ into 10 size-sorted portfolios and 10 book-to-market-sorted portfolios. Fama and French (1992), among many others document that small stocks and stocks with high book-to-market ratios earn higher returns that cannot be explained by the CAPM. I then sort these stocks into 10 long-term reversal portfolios based on their past 3-year returns 1 year prior to portfolio formation. De Bondt and Thaler (1985) document that past winners tend to underperform relative to past losers over the long run. I do not consider momentum portfolios because the cross-sectional variation in average returns across momentum portfolios decreases significantly after 6 months, rendering the long-run earnings-based cash flow measures less relevant.

I record the first year annual returns of the portfolios after their formation. Following Shumway (1997), I assign a return of −0.3 to firms delisted.

8 Consumption data from 1951Q2 to 2005Q1 are kindly made available by Sydney Ludvigson at her website—http://www.econ.nyu.edu/user/ludvigsons/. Detailed information on the data construction can be found in the Appendix of Lettau and Ludvigson (2001a).

9 I exclude financial firms (SICCD in [6000, 6999]) and utilities companies (SICCD in [4900, 4999]). I follow Fama and French’s (1996) procedure in measuring the book equity of a firm. The book-to-market ratio in June of year $t$ is book equity for the fiscal year ending in calendar year $t - 1$, divided by market equity at the end of December of year $t - 1$. Size is defined as market equity at the end of June of year $t$. To avoid potential data errors and extreme outliers, I exclude stocks whose book-to-market ratios exceed the 99th percentile or fall below the 1st percentile. In addition, I exclude stocks whose book-to-market ratios are negative.
for performance related reasons (delisting code is 500 or in [520, 584]). Table I presents descriptive statistics on the 30 portfolios. The sampling period is from 1964 to 2002. Consistent with previous literature, sorting on size, book-to-market ratio, and past long-run return generates sizable dispersion in average portfolio returns during this period.

Table I
Descriptive Statistics of 30 Portfolios

Every June, I construct 10 book-to-market (BM-sorted) portfolios, 10 size (ME-sorted) portfolios, and 10 reversal (PastRet-sorted) portfolios. The portfolio characteristics at formation and annual returns during the first year after portfolio formation are reported. All values are time-series averages across the 1964 to 2002 period. Market Equity (ME) is measured in millions. BM denotes the book-to-market ratio and PastRet denotes the past 3-year (prior 2 to 4 years) return. I also directly test the validity of the AR(1) assumption on the cash flow share for the 30 portfolios. I first fit an AR(1) process for the cash flow share and compute the residuals. I then test whether these residuals violate the white noise condition using the Ljung-Box (LB) Q test. Both the LB Q test statistics and the associated p-values are reported. In addition, I test the stationarity of the cash flow share using the Augmented Dickey-Fuller test with a constant and a lag of one. The t-values are reported (** means the hypothesis can be rejected at the 99% confidence level and * means the hypothesis can be rejected at the 95% confidence level). The cash flow share in year t is computed as the log of the ratio between the portfolio cash flow (sum of common dividend and common share repurchase) and aggregate consumption during year t.

Panel A: 10 Book-to-Market (BM-Sorted) Portfolios

<table>
<thead>
<tr>
<th></th>
<th>Growth</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ME</td>
<td>1,955.2</td>
<td>1,334.0</td>
<td>992.4</td>
<td>889.4</td>
<td>640.1</td>
<td>561.5</td>
<td>458.0</td>
<td>356.1</td>
<td>269.6</td>
<td>143.8</td>
</tr>
<tr>
<td>BM</td>
<td>0.16</td>
<td>0.29</td>
<td>0.41</td>
<td>0.52</td>
<td>0.64</td>
<td>0.77</td>
<td>0.92</td>
<td>1.11</td>
<td>1.37</td>
<td>1.86</td>
</tr>
<tr>
<td>PastRet</td>
<td>1.550</td>
<td>1.248</td>
<td>1.035</td>
<td>0.807</td>
<td>0.659</td>
<td>0.537</td>
<td>0.413</td>
<td>0.298</td>
<td>0.167</td>
<td>0.019</td>
</tr>
<tr>
<td>Return</td>
<td>0.108</td>
<td>0.100</td>
<td>0.126</td>
<td>0.124</td>
<td>0.123</td>
<td>0.144</td>
<td>0.136</td>
<td>0.161</td>
<td>0.163</td>
<td>0.174</td>
</tr>
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</table>

Panel B: 10 Size (ME-Sorted) Portfolios

<table>
<thead>
<tr>
<th></th>
<th>Small</th>
<th>2</th>
<th>3</th>
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<th>7</th>
<th>8</th>
<th>9</th>
<th>Big</th>
</tr>
</thead>
<tbody>
<tr>
<td>ME</td>
<td>4.3</td>
<td>10.4</td>
<td>19.0</td>
<td>31.6</td>
<td>52.1</td>
<td>87.6</td>
<td>151.7</td>
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<td>651.6</td>
<td>6184.6</td>
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<tr>
<td>BM</td>
<td>1.10</td>
<td>0.98</td>
<td>0.93</td>
<td>0.87</td>
<td>0.82</td>
<td>0.77</td>
<td>0.73</td>
<td>0.68</td>
<td>0.63</td>
<td>0.58</td>
</tr>
<tr>
<td>PastRet</td>
<td>0.046</td>
<td>0.272</td>
<td>0.396</td>
<td>0.526</td>
<td>0.620</td>
<td>0.731</td>
<td>0.766</td>
<td>0.826</td>
<td>0.823</td>
<td>0.784</td>
</tr>
<tr>
<td>Return</td>
<td>0.272</td>
<td>0.190</td>
<td>0.173</td>
<td>0.148</td>
<td>0.151</td>
<td>0.148</td>
<td>0.138</td>
<td>0.136</td>
<td>0.138</td>
<td>0.116</td>
</tr>
</tbody>
</table>

Panel C: 10 Reversal (PastRet-Sorted) Portfolios

<table>
<thead>
<tr>
<th></th>
<th>Winner</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>Loser</th>
</tr>
</thead>
<tbody>
<tr>
<td>ME</td>
<td>2,457.4</td>
<td>2,733.0</td>
<td>2,646.1</td>
<td>2,272.8</td>
<td>1,867.7</td>
<td>1,293.3</td>
<td>1,122.9</td>
<td>868.8</td>
<td>564.5</td>
<td>286.5</td>
</tr>
<tr>
<td>BM</td>
<td>0.51</td>
<td>0.63</td>
<td>0.72</td>
<td>0.78</td>
<td>0.84</td>
<td>0.91</td>
<td>0.97</td>
<td>1.03</td>
<td>1.08</td>
<td>1.11</td>
</tr>
<tr>
<td>PastRet</td>
<td>3.174</td>
<td>1.361</td>
<td>0.913</td>
<td>0.642</td>
<td>0.442</td>
<td>0.274</td>
<td>0.115</td>
<td>0.045</td>
<td>0.224</td>
<td>0.501</td>
</tr>
<tr>
<td>Return</td>
<td>0.093</td>
<td>0.099</td>
<td>0.125</td>
<td>0.135</td>
<td>0.134</td>
<td>0.124</td>
<td>0.130</td>
<td>0.149</td>
<td>0.138</td>
<td>0.166</td>
</tr>
</tbody>
</table>

Dickey-Fuller test with a constant and a lag of one. The associated portfolio returns during this period.
For each year $t$, after I form the portfolios, I hold them for 10 years. All accounting cash flow data are converted to real terms using the Personal Consumption Expenditure (PCE) deflator. I record the return on equity ($ROE$) of each portfolio from year $t + 1$ to year $t + 10$. Following Vuolteenaho (2002), I define $ROE_t$ as aggregate portfolio earnings measured according to U.S. Generally Accepted Accounting Principles (GAAP) (COMPSTAT data item 172) at year $t$ divided by aggregate portfolio book equity at year $t - 1$. For firms that disappear due to delisting, merger, or acquisition, I assume we invest the proceeds from such events in the original portfolios that contained the disappeared firms. In this way, the portfolio $ROE$ will not be altered. At the portfolio level, the $ROE$ is well above $-1$; I can therefore compute $e = \log(1 + ROE)$ without risking the number in brackets becoming negative or too close to zero. Finally, I compute the portfolio cash flow-to-book equity ratio at portfolio formation, which is used later in computing cash flow duration. This ratio is defined as the aggregate portfolio common dividend plus common share repurchase of the portfolio formation year divided by the aggregate portfolio book equity of the portfolio formation year. Common dividend is measured using COMPSTAT data item 21. Following Grullon and Michaely (2002), common share repurchase is defined as expenditure on the purchase of common and preferred stocks (data item 115) minus any reduction in the book value of preferred stock.

I directly test the stationarity of the log cash flow share using the Augmented Dickey-Fuller test with a constant and a lag of one. The log cash flow share in year $t$ is computed as the log of the ratio between the portfolio cash flow (sum of common dividend and common share repurchase) and aggregate consumption during year $t$. The sampling period is again from 1964 to 2002, so I have a time series of 38 cash flow shares for each of the 30 portfolios. The alternative hypothesis on the existence of a unit root is rejected for most of the 30 portfolios (24 out of 30, see Table I). Using the Ljung-Box $Q$ test, I also test the AR(1) assumption on the log cash flow share by examining whether the AR(1) residuals are white noises. The $p$-values associated with the tests are higher than 0.05 for almost all the portfolios (29 out of 30), which means that the AR(1) assumption cannot be rejected. In conclusion, the AR(1) assumption imposed on the log cash flow share seems to be reasonable at least at the portfolio level.

**B. Portfolio Cash Flow Characteristics**

Table II breaks down the average cash flow duration for the 30 portfolios. I choose $N = 7$ when estimating the future earnings component $\Sigma_{t}^{ci}$. Similar estimates are obtained if $N$ is 5 or 10. Due to the 7-year holding period of a portfolio, the sampling period is reduced from 1964 to 2002 to 1964 to 1995. The $t$-values are computed using GMM standard errors, which account for both cross-sectional and time-series error correlations using the Newey and West (1987) formula of seven lags. I report the differences in the cash flow duration estimates between extreme portfolios in the last two columns.
Table II
Average Cash Flow Duration Measures of 30 Portfolios

I measure average cash flow duration (Dur) as the time-series average of \( \sum_{t=0}^{T} \frac{e_t}{V_t} - \sum_{t=0}^{T} \xi_t - E_t[\Sigma_t] \). The future earnings component (\( \Sigma_t^{f} \)) measures the average sum of discounted future accounting returns since portfolio formation (the cutoff \( N \) is chosen to be seven). The current payout component (\( \xi_t \)) measures the average log cash flow-to-book equity ratio at portfolio formation. \( E_t[\Sigma_t] \) is estimated under the assumption that consumption growth follows the ARMA(1,1) process. The \( t \)-values are computed using GMM standard errors, which account for both cross-sectional and time-series error correlations, with Newey and West (1987) formula of seven lags.

The last two columns report the differences in the cash flow duration estimates between extreme portfolios.

### Panel A: 10 Book-to-Market (BM-Sorted) Portfolios

<table>
<thead>
<tr>
<th>Growth</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>Value</th>
<th>(1−10)</th>
<th>(1−3)–(8−10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Future earnings</td>
<td>3.49</td>
<td>3.02</td>
<td>2.68</td>
<td>2.49</td>
<td>2.23</td>
<td>2.05</td>
<td>1.79</td>
<td>1.64</td>
<td>1.30</td>
<td>1.09</td>
<td></td>
</tr>
<tr>
<td>Current payout</td>
<td>−2.40</td>
<td>−2.48</td>
<td>−2.59</td>
<td>−2.67</td>
<td>−2.71</td>
<td>−2.81</td>
<td>−2.88</td>
<td>−3.02</td>
<td>−3.14</td>
<td>−3.47</td>
<td></td>
</tr>
<tr>
<td>Dur</td>
<td>1.47</td>
<td>1.08</td>
<td>0.84</td>
<td>0.75</td>
<td>0.52</td>
<td>0.44</td>
<td>0.24</td>
<td>0.23</td>
<td>0.01</td>
<td>0.13</td>
<td>1.34</td>
</tr>
<tr>
<td>t-value</td>
<td>7.92</td>
<td>15.31</td>
<td>12.88</td>
<td>13.28</td>
<td>3.86</td>
<td>2.97</td>
<td>1.53</td>
<td>1.53</td>
<td>0.05</td>
<td>0.75</td>
<td>4.54</td>
</tr>
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</table>

### Panel B: 10 Size (ME-Sorted) Portfolios

<table>
<thead>
<tr>
<th>Small</th>
<th>2</th>
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<th>7</th>
<th>8</th>
<th>9</th>
<th>Big</th>
<th>(1−10)</th>
<th>(1−3)–(8−10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Future earnings</td>
<td>0.54</td>
<td>0.85</td>
<td>1.04</td>
<td>1.10</td>
<td>1.27</td>
<td>1.34</td>
<td>1.63</td>
<td>1.87</td>
<td>1.98</td>
<td>2.34</td>
<td></td>
</tr>
<tr>
<td>Current payout</td>
<td>−4.01</td>
<td>−3.80</td>
<td>−3.74</td>
<td>−3.57</td>
<td>−3.47</td>
<td>−3.33</td>
<td>−3.26</td>
<td>−3.14</td>
<td>−3.02</td>
<td>−2.66</td>
<td></td>
</tr>
<tr>
<td>Dur</td>
<td>0.08</td>
<td>0.20</td>
<td>0.34</td>
<td>0.22</td>
<td>0.28</td>
<td>0.22</td>
<td>0.45</td>
<td>0.57</td>
<td>0.56</td>
<td>−0.48</td>
<td>−0.36</td>
</tr>
<tr>
<td>t-value</td>
<td>0.16</td>
<td>0.50</td>
<td>1.05</td>
<td>0.67</td>
<td>0.95</td>
<td>0.62</td>
<td>2.05</td>
<td>3.28</td>
<td>4.17</td>
<td>4.33</td>
<td>−1.21</td>
</tr>
</tbody>
</table>

### Panel C: 10 Reversal (PastRet-Sorted) Portfolios

<table>
<thead>
<tr>
<th>Winner</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>Loser</th>
<th>(1−10)</th>
<th>(1−3)–(8−10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Future earnings</td>
<td>2.61</td>
<td>2.46</td>
<td>2.44</td>
<td>2.26</td>
<td>2.14</td>
<td>2.17</td>
<td>2.03</td>
<td>1.96</td>
<td>1.86</td>
<td>1.57</td>
<td></td>
</tr>
<tr>
<td>Current payout</td>
<td>−2.90</td>
<td>−2.75</td>
<td>−2.72</td>
<td>−2.73</td>
<td>−2.73</td>
<td>−2.77</td>
<td>−2.78</td>
<td>−2.90</td>
<td>−3.07</td>
<td>−3.41</td>
<td></td>
</tr>
<tr>
<td>Dur</td>
<td>1.10</td>
<td>0.78</td>
<td>0.73</td>
<td>0.57</td>
<td>0.44</td>
<td>0.52</td>
<td>0.38</td>
<td>0.40</td>
<td>0.50</td>
<td>0.54</td>
<td>0.56</td>
</tr>
<tr>
<td>t-value</td>
<td>12.71</td>
<td>7.34</td>
<td>5.65</td>
<td>3.85</td>
<td>2.15</td>
<td>2.99</td>
<td>1.92</td>
<td>1.65</td>
<td>2.18</td>
<td>1.86</td>
<td>2.43</td>
</tr>
</tbody>
</table>

Panel A contains average duration estimates for 10 book-to-market-sorted portfolios. The average cash flow duration measure Dur decreases almost monotonically with book-to-market. As expected, growth stocks have higher cash flow durations than value stocks. The difference in Dur between two extreme portfolios is 1.34 and highly significant. The high duration of growth stocks is largely driven by higher earnings in the future. Both the future earnings component (average \( \Sigma_t^{f} \)) and the current accounting payout rate (average \( \xi_t \)) decrease in book-to-market. The larger average payout rate for growth stocks is mainly driven by their small book values. Since the future earnings component decreases faster than the current payout rate as we move from growth stocks to value stocks, the impact of higher future earnings dominates, resulting in higher average cash flow durations for growth stocks. As Cohen, Polk, and Vuolteenaho (2008) point out, a bias may partly contribute to this pattern as I sort stocks according to book-to-market ratio. To the extent we underestimate
the book equity value of a stock, we tend to sort that stock to growth portfolios. This error in measurement will increase the future earnings component, resulting in an increase in the duration measure. Even if this bias is accounted for, the pattern will not likely change given the significant difference in average duration measures between extreme growth and value stocks (robust $t$-value above 4.5).

Panel B contains average duration estimates for 10 size-sorted portfolios. In general, small stocks have lower average cash flow duration than big stocks. This is because most of the traded small stocks are “distressed” stocks and their cash flows, if any, are expected to decrease in the future. Both future earnings and current payout rate increase in size, but the future earnings component increases more rapidly. Evidently, the impact of future earnings again dominates. Lastly, the difference in $Dur$ between two extreme size-sorted portfolios is calculated to be 0.48 (in absolute terms). Sorting on size, then, induces a smaller spread in cash flow duration than sorting on book-to-market.

Panel C contains average duration estimates for 10 reversal portfolios. In general, past losers have lower average cash flow duration than past winners. This pattern is mainly driven by a difference in the future earnings component: Past winners have higher future earnings than past losers. The dispersion in the current payout rate is much smaller across the 10 portfolios. Overall, the difference in $Dur$ between two extreme reversal portfolios is 0.56 and significant ($t$-value = 2.43).

Table III contains the cash flow covariance for the 30 portfolios. I replace the infinite sum in measuring long-run accounting returns and consumption innovations by a finite sum up to $N$. I present estimates for $N = 5, 7, $ and $10$. Due to the need to hold portfolios $N$ years ahead, the sampling period associated with each $N$ spans from 1964 to 2002 $− N$. The estimates of $Cov$ are obtained using overlapping OLS regressions. As with $Dur$, I compute robust $t$-values for $Cov$ using GMM standard errors, which account for both cross-sectional and time-series error correlations, with the Newey and West (1987) formula of $N$ lags. For all choices of $N$, value stocks, small stocks, and past losers have higher cash flow covariance measures than growth stocks, large stocks, and past winners, accordingly, consistent with cash flow covariance being an important determinant of risk premia. A large $N$ makes the estimate closer to its theoretical analog. On the other hand, a large $N$ accumulates measurement error in cash flow data and reduces the sample size, making the estimates empirically less accurate. Due to this trade-off, I choose the cash flow covariance estimates associated with $N = 7$ for the main cross-sectional analysis and I also verify the results to be very similar when cash flow covariance is measured with $N$ equal to 5 or 10.

C. Cross-sectional Analysis

In this section, I empirically examine the two-factor cash flow model (13) in the cross-section. To achieve higher statistical power, I adopt finer sorts by sorting sample stocks into 20 portfolios using size, book-to-market, and past
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Table III
Cash Flow Covariance Measures of 30 Portfolios

I regress $\sum_{i=1}^{N} \rho_i (\Delta C_i (t+1) - \Delta C_{i, t+1})$ on $\sum_{i=1}^{N} \rho_i \Delta C_{i, t+1}$ for each of the 30 portfolios, where $\rho_i$ are consumption growth innovations. The regression coefficient $Cov^i$ measures the cash flow covariance for portfolio $i$. I repeat the regressions for different horizons ($N_i$), and the associated sampling period for each $N_i$ is from 1964 to 2002 – $N_i$. I report the OLS estimates in the first row and the associated $t$-values below. The $t$-values are computed using GMM standard errors, which account for both cross-sectional and time-series error correlations, with Newey and West (1987) formula of $N$ lags. The last two columns report the differences in the cash flow covariance estimates between extreme portfolios.

Panel A: 10 Book-to-Market (BM-Sorted) Portfolios

<table>
<thead>
<tr>
<th>Growth</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>Value (1–10)</th>
<th>(1–3)–(8 10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N = 5</td>
<td>-3.78</td>
<td>-2.54</td>
<td>-2.00</td>
<td>-1.38</td>
<td>-0.50</td>
<td>-0.61</td>
<td>0.13</td>
<td>0.00</td>
<td>1.25</td>
<td>0.98</td>
</tr>
<tr>
<td>N = 7</td>
<td>-4.39</td>
<td>-3.33</td>
<td>-2.50</td>
<td>-1.81</td>
<td>-0.20</td>
<td>-0.96</td>
<td>0.43</td>
<td>0.32</td>
<td>1.04</td>
<td>0.79</td>
</tr>
<tr>
<td>N = 10</td>
<td>-4.39</td>
<td>-4.15</td>
<td>-2.89</td>
<td>-2.95</td>
<td>-0.74</td>
<td>-1.48</td>
<td>-0.50</td>
<td>-0.22</td>
<td>0.29</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>-1.54</td>
<td>-2.27</td>
<td>-3.08</td>
<td>-5.09</td>
<td>-0.91</td>
<td>-1.96</td>
<td>-1.32</td>
<td>-0.33</td>
<td>0.57</td>
<td>0.13</td>
</tr>
</tbody>
</table>

Panel B: 10 Size (ME-Sorted) Portfolios

<table>
<thead>
<tr>
<th>Small</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>Big (1–10)</th>
<th>(1–3)–(8 10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N = 5</td>
<td>1.07</td>
<td>0.19</td>
<td>-0.53</td>
<td>0.25</td>
<td>-0.45</td>
<td>0.38</td>
<td>0.24</td>
<td>-0.71</td>
<td>-1.05</td>
<td>-0.64</td>
</tr>
<tr>
<td>N = 7</td>
<td>0.54</td>
<td>0.15</td>
<td>-0.35</td>
<td>0.15</td>
<td>-0.28</td>
<td>0.21</td>
<td>0.16</td>
<td>-0.68</td>
<td>-1.13</td>
<td>-0.82</td>
</tr>
<tr>
<td>N = 10</td>
<td>3.16</td>
<td>0.96</td>
<td>0.24</td>
<td>0.67</td>
<td>0.27</td>
<td>0.73</td>
<td>0.91</td>
<td>-0.37</td>
<td>-0.76</td>
<td>-0.85</td>
</tr>
<tr>
<td></td>
<td>1.18</td>
<td>0.52</td>
<td>0.12</td>
<td>0.32</td>
<td>0.14</td>
<td>0.36</td>
<td>0.55</td>
<td>-0.34</td>
<td>-0.87</td>
<td>-1.25</td>
</tr>
<tr>
<td></td>
<td>4.82</td>
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<td>2.66</td>
<td>2.85</td>
<td>2.03</td>
<td>1.97</td>
<td>1.57</td>
<td>-0.17</td>
<td>-0.47</td>
<td>-1.41</td>
</tr>
<tr>
<td></td>
<td>1.22</td>
<td>0.68</td>
<td>0.85</td>
<td>1.14</td>
<td>0.95</td>
<td>1.23</td>
<td>0.94</td>
<td>-0.17</td>
<td>-0.71</td>
<td>-4.63</td>
</tr>
</tbody>
</table>

Panel C: 10 Reversal (PastRet-Sorted) Portfolios

<table>
<thead>
<tr>
<th>Winner</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>Loser (1–10)</th>
<th>(1–3)–(8 10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N = 5</td>
<td>-3.08</td>
<td>-1.21</td>
<td>-0.74</td>
<td>-1.24</td>
<td>1.41</td>
<td>-0.10</td>
<td>0.00</td>
<td>0.96</td>
<td>0.59</td>
<td>0.12</td>
</tr>
<tr>
<td>N = 7</td>
<td>-3.97</td>
<td>-2.04</td>
<td>-1.27</td>
<td>-1.33</td>
<td>0.94</td>
<td>-0.12</td>
<td>0.00</td>
<td>0.46</td>
<td>0.36</td>
<td>0.06</td>
</tr>
<tr>
<td>N = 10</td>
<td>-5.80</td>
<td>-1.16</td>
<td>-0.87</td>
<td>-1.71</td>
<td>1.32</td>
<td>0.18</td>
<td>-0.19</td>
<td>1.10</td>
<td>1.14</td>
<td>1.32</td>
</tr>
<tr>
<td></td>
<td>-5.99</td>
<td>-1.57</td>
<td>-2.26</td>
<td>-1.95</td>
<td>0.92</td>
<td>0.22</td>
<td>-0.16</td>
<td>0.51</td>
<td>0.63</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>-5.15</td>
<td>-1.54</td>
<td>-1.41</td>
<td>-2.06</td>
<td>1.63</td>
<td>0.05</td>
<td>-0.15</td>
<td>1.01</td>
<td>0.56</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td>-8.36</td>
<td>-1.56</td>
<td>-5.93</td>
<td>-6.40</td>
<td>2.07</td>
<td>0.13</td>
<td>-0.23</td>
<td>0.49</td>
<td>0.46</td>
<td>0.37</td>
</tr>
</tbody>
</table>

long-run return characteristics to obtain 60 test portfolios in total. I then repeat the earlier procedure to compute the cash flow duration and covariance measures for each of the 60 portfolios. In the cross-section, I first estimate the unconditional version of the two-factor cash flow model:

$$E[R_{t+1} - R_f] = \gamma_0 + \gamma_1 Cov^i + \gamma_2 Cov^i \times Dur^i,$$  (16)

where I use the average duration measure ($Dur^i$). This allows for a direct comparison with other common pricing models that do not allow for time-varying portfolio characteristics. As the next period’s earning surprise is empirically correlated with the next period’s return innovation, and since I estimate $\Sigma_i$ using forward-looking earnings data, such correlation might introduce a bias in the cross-sectional analysis. Since both $Dur^i$ and $E[R_{t+1} - R_f]$ are time-series averages, earnings surprises will be averaged out across time and the bias should be small. I also estimate the ex ante measure of cash flow duration—$Dur^i$.
using only currently observable instrumental variables. When I replace $\hat{D}_{ur}^i$ with $D_{ur}^i$ (the time-series average of $D_{ur}^i$) in the cross-sectional regression, the results hardly change. In the next section, I use the ex ante measure of cash flow duration and other alternative duration measures that are time-varying in the cross-sectional analysis.

I use the GMM approach for the cross-sectional analysis, where I stack moment conditions in both the time-series and cross-sectional regressions in a one-stage GMM system, similar to the procedure discussed in Cochrane (2001) and Bansal, Dittmar and Lundblad (2002). The moment conditions are chosen such that the estimates of $\gamma_0$, $\gamma_1$, and $\gamma_3$ are identical to those obtained using OLS cross-sectional regressions. The associated robust $t$-values are computed using GMM standard errors. The covariance matrix of the moment conditions is computed using the Newey and West (1987) formula of seven lags. The resulting robust $t$-values thus account for error correlation both cross-sectionally and in the time series. In particular, they adjust for estimation errors in both $Cov$ and $Dur$ in the time-series regression. For comparison, I also run the more commonly used Fama and MacBeth (1973) regression and report the associated $t$-values. The Fama-MacBeth regression produces identical point estimates.

I estimate three alternative return-based models as benchmarks. They are:

1. the standard CAPM:
   \[
   E[R_{t+1}^i - R_f^i] = \gamma_0 + \gamma_1 \beta_{MKT}^i;
   \]

2. the standard Consumption-based CAPM estimated using returns:
   \[
   E[R_{t+1}^i - R_f^i] = \gamma_0 + \gamma_1 \beta_{\Delta c}^i;
   \]

and (3) the Fama and French (1993) three-factor model: \(^{10}\)
   \[
   E[R_{t+1}^i - R_f^i] = \gamma_0 + \gamma_1 \beta_{MKT}^i + \gamma_2 \beta_{SMB}^i + \gamma_3 \beta_{HML}^i.
   \]

The cross-sectional regression results are presented in Table IV. The cash flow covariance measure alone explains 58% of the cross-sectional variation in expected excess returns. While cash flow covariance estimated using long-run consumption data has been shown to be able to explain short-term return momentum (Bansal, Dittmar, and Lundblad (2005)), this paper to my knowledge, is the first to empirically show that a long-run cash flow covariance, measured properly, also helps to explain the long-run return reversal.

It is interesting that cash flow covariance measured using long-run consumption data is able to explain both short-term return momentum and long-run return reversals. One potential explanation is given by Yang (2007), who models two time-varying components of exposure to the long-run consumption risk in the cash flow growth rate of a stock—a fast mean-reverting component and a slow mean-reverting component. The cash flow innovation is positively correlated with one component while negatively correlated with the other. As a

\(^{10}\) I thank Ken French for providing data on the three factors.
Table IV
Cross-sectional Regressions

Results of cross-sectional regressions of average excess returns on the 60 portfolios on cash flow duration and covariance measures are presented in Panel A. The coefficient estimates are obtained from OLS regressions. However, the robust t-values are computed using GMM standard errors which account for both cross-sectional and time-series error correlations, with Newey and West (1987) formula of seven lags. The one-stage GMM estimation is carried out by stacking moment conditions of both time-series regressions and cross-sectional regressions. For comparison, I also run the standard Fama-MacBeth regressions and report the associated (FM) t-values. Results of cross-sectional regressions of alternative models are presented in Panel B as benchmarks. I report both Fama-MacBeth t-values and Shanken t-values which account for errors in the estimates of factor loadings. Finally, both $R^2$ and adjusted-$R^2$ of the regressions are reported. The sampling period is 1964 to 1995.

Panel A: Cross-sectional Regression of Cash Flow Models

<table>
<thead>
<tr>
<th></th>
<th>Intercept</th>
<th>Cov</th>
<th>Dur x Cov</th>
<th>Dur</th>
<th>$R^2 / \text{adj } R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>One Factor:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coefficient</td>
<td>0.087</td>
<td>0.019</td>
<td></td>
<td></td>
<td>0.583</td>
</tr>
<tr>
<td>FM t-value</td>
<td>2.67</td>
<td>3.79</td>
<td></td>
<td></td>
<td>0.576</td>
</tr>
<tr>
<td>Robust t-value</td>
<td>2.60</td>
<td>3.31</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Two Factors:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coefficient</td>
<td>0.075</td>
<td>0.037</td>
<td>−0.024</td>
<td></td>
<td>0.818</td>
</tr>
<tr>
<td>FM t-value</td>
<td>2.43</td>
<td>4.04</td>
<td>−3.29</td>
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<td>0.812</td>
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<tr>
<td>Robust t-value</td>
<td>2.23</td>
<td>3.40</td>
<td>−2.57</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Three Factors:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coefficient</td>
<td>0.083</td>
<td>0.036</td>
<td>−0.026</td>
<td>−0.018</td>
<td>0.821</td>
</tr>
<tr>
<td>FM t-value</td>
<td>2.89</td>
<td>3.67</td>
<td>−3.95</td>
<td>−0.72</td>
<td>0.811</td>
</tr>
<tr>
<td>Robust t-value</td>
<td>2.14</td>
<td>3.43</td>
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<td>−0.44</td>
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</tbody>
</table>

Panel B: Cross-sectional Regression of Alternative Models

<table>
<thead>
<tr>
<th></th>
<th>Intercept</th>
<th>Δc</th>
<th>MKT</th>
<th>SMB</th>
<th>HML</th>
<th>$R^2 / \text{adj } R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CAPM:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coefficient</td>
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<td>0.142</td>
<td></td>
<td></td>
<td></td>
<td>0.269</td>
</tr>
<tr>
<td>FM t-value</td>
<td>−1.15</td>
<td>2.02</td>
<td></td>
<td></td>
<td></td>
<td>0.269</td>
</tr>
<tr>
<td>Shanken t-value</td>
<td>−0.89</td>
<td>1.50</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>CCAPM:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coefficient</td>
<td>0.004</td>
<td>0.021</td>
<td></td>
<td></td>
<td></td>
<td>0.279</td>
</tr>
<tr>
<td>FM t-value</td>
<td>0.12</td>
<td>2.77</td>
<td></td>
<td></td>
<td></td>
<td>0.279</td>
</tr>
<tr>
<td>Shanken t-value</td>
<td>0.07</td>
<td>1.51</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>FF Three Factors:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coefficient</td>
<td>−0.011</td>
<td>0.064</td>
<td>0.058</td>
<td>0.032</td>
<td></td>
<td>0.514</td>
</tr>
<tr>
<td>FM t-value</td>
<td>−0.38</td>
<td>1.60</td>
<td>2.23</td>
<td>1.23</td>
<td></td>
<td>0.497</td>
</tr>
<tr>
<td>Shanken t-value</td>
<td>−0.33</td>
<td>1.19</td>
<td>1.50</td>
<td>0.84</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

result, sorting on past returns is likely to generate a positive spread in the fast mean-reverting component, explaining the short-run return momentum, and a negative spread in the slow mean-reverting component, thus explaining the long-run return reversal. These two components are not directly modeled in my paper, but it is conceivable that my cash flow covariance, measured using
long-run cash flow on a seven-year buy-and-hold portfolio, is likely to capture the slow mean-reverting component empirically. In contrast, the cash flow beta in Bansal, Dittmar, and Lundblad (2005), estimated using cash flow on quarterly rebalanced portfolio, is more likely to capture the fast mean-reverting component empirically. Overall, the empirical finding that cash flow covariance also helps to explain the long-run return reversal provides additional support for the long-run risk model of Bansal and Yaron (2004).

While cash flow covariance does a reasonable job explaining the cross-sectional variation in risk premia, cash flow duration also provides additional explanatory power. After adding in a second factor, $Cov \times Dur$, which contains cash flow duration, $Cov$ remains significant. However, the second factor is also significant, which means that cash flow duration still has additional explanatory power. The $R^2$ from the addition of this second factor increases to 82%. The incremental $R^2$ of adding the second factor is 24% (the incremental adjusted-$R^2$ is also about 24%). The coefficient on the second factor $Cov \times Dur$ is negative, consistent with the theoretical prediction and simulation result—that is, cash flow duration increases expected return when cash flow covariance is negative and reduces expected return when cash flow covariance is positive, as demonstrated in Figure 1. Finally, I add $Dur$ as a third factor to test whether it provides any additional explanatory power on a stand-alone basis. This term has a negative coefficient, consistent with both Dechow, Sloan, and Soliman (2004) and Lettau and Wachter (2007), who find that high duration leads to lower stock returns. However, this negative coefficient is not significant; the additional explanatory power of the factor $Dur$ in terms of incremental $R^2$ is less than 1%.

Overall, the two-factor cash flow model, compared to the standard CCAPM or the one- or three-factor models, seems to capture most of the cross-sectional variation in risk premia. With just one factor—covariance—the cash flow model yields a higher $R^2$ and a more significant consumption risk premium than the standard CCAPM in which consumption beta is estimated using returns. The cash flow model with only cash flow covariance, which can stand in as a first-order approximation of the true model, explains a reasonably large portion of the cross-sectional variation in expected excess returns, confirming the findings in Bansal, Dittmar, and Lundblad (2005). Figure 2 contains a graphic representation of this. However, this figure also shows that the relationship between realized and fitted expected excess returns is still somewhat nonlinear, largely due to the omission of the second-order effect. The second-order effect is well captured by the second factor $Cov \times Dur$, an interaction term between cash flow covariance and duration. If I account for this second-order term explicitly in a two-factor cash flow model, the relationship between realized and fitted expected excess returns becomes linear. The two-factor cash flow model performs better than the three-factor model in terms of yielding a higher $R^2$ and more significant risk premium. The two-factor cash flow model accounting for cash flow duration well explains the expected excess returns of all 60 portfolios, including the smallest size portfolio and lowest book-to-market portfolio, where all other models considered slightly falter.
Figure 2. Realized and fitted excess returns of 60 portfolios. b1 and b20 are the extreme growth and value portfolios, respectively. s1 and s20 are the smallest and biggest portfolios, respectively. r1 and r20 are the portfolios with the highest and lowest past return, respectively. Details of the cross-sectional regressions are in Table IV.
D. Monte Carlo Analysis

The robust t-values estimated in the cross-sectional analysis are derived using asymptotic statistics and could potentially be imprecise due to the small sample size. To examine the finite-sample empirical distribution for various key parameters such as the risk premia on the cash flow characteristics and the cross-sectional regression $R^2$, I follow the procedure in Bansal, Dittmar, and Lundblad (2005) and conduct two Monte Carlo experiments. These experiments show that the empirical results reflect economic content rather than random chance.

The first Monte Carlo experiment is conducted under the alternative hypothesis that the two-factor model is incorrect. I simulate 10,000 time series of long-run consumption innovations ($\sum_{n=0}^{7} \rho^n w_{t+n+1}$) modeled as i.i.d. with a standard deviation matched to the data. I then regress the observed long-run accounting returns ($\sum_{n=0}^{7} \rho^n [e^i(t, n + 1) - \Delta c_{t+n+1}]$) on the simulated long-run consumption innovations to obtain the cash flow covariance measures. I also simulate 10,000 cross-sections of 60 cash flow durations from a multivariate normal distribution with a zero mean vector and a covariance matrix matched to its empirical counterpart. Finally, I estimate the two-factor cash flow model using the observed average excess returns on the 60 portfolios, the estimated cash flow covariances, and the simulated cash flow durations. By construction, the population values of cash flow covariances and durations and the risk premia terms should all be zeros. Consequently, the $R^2$ of the cross-sectional regression should also be zero.

The result of the first experiment is presented in Panel A of Table V. The risk premia are estimated with sizable errors, but their distributions are centered at the population values of zeros. The point estimate of the risk premium on Cov of 0.037 exceeds the 95th percentile of its empirical distribution. The point estimate of the risk premium on Cov × Dur of −0.024 also exceeds the 95th percentile of its empirical distribution. The cross-sectional $R^2$ and adjusted-$R^2$ both exceed the 95th percentiles of their corresponding empirical distributions. These results indicate that if the two-factor cash flow model is incorrect, observing the magnitudes of risk premia and the cross-section $R^2$ that I find in data would be unlikely. The two-factor cash flow model is therefore unlikely a result of random chance but reflects economic content.

The second Monte Carlo experiment is conducted under the null hypothesis that the two-factor model is correct. I simulate 10,000 cross-sections of 60 portfolio cash flow durations from a multivariate normal distribution where the mean vector and the covariance matrix are matched to their empirical counterparts. To simulate the cash flow covariances, consider the regression used to estimate them:

$$\sum_{n=0}^{7} \rho^n [e^i(t, n + 1) - \Delta c_{t+n+1}] = constant^i + Cov^i \sum_{n=0}^{7} \rho^n w_{t+n+1} + err^i.$$
Table V
Monte Carlo Analysis

This table reports the empirical distribution of the estimated parameters in the cross sectional analysis. Panel A reports the parameter distributions under the alternative hypothesis that the two-factor model is incorrect. 10,000 samples are simulated under the assumption that cash flow covariances and durations are zeros in the population. Panel B reports the parameter distributions under the null hypothesis that the two-factor model is correct. 10,000 samples are simulated under the assumption that the cash flow characteristics and risk premia in the population are equal to their empirical counterparts.

Panel A: Under the “Alternative” Hypothesis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Under “Alter”</th>
<th>2.5%</th>
<th>5.0%</th>
<th>10.0%</th>
<th>50.0%</th>
<th>90.0%</th>
<th>95.0%</th>
<th>97.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk Premia on $Cov(\gamma_1)$</td>
<td>0.000</td>
<td>-0.030</td>
<td>-0.027</td>
<td>-0.024</td>
<td>0.000</td>
<td>0.023</td>
<td>0.027</td>
<td>0.030</td>
</tr>
<tr>
<td>Risk Premia on $Cov \times Dur(\gamma_2)$</td>
<td>0.000</td>
<td>-0.019</td>
<td>-0.015</td>
<td>-0.011</td>
<td>0.000</td>
<td>0.011</td>
<td>0.015</td>
<td>0.020</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.000</td>
<td>0.005</td>
<td>0.010</td>
<td>0.024</td>
<td>0.207</td>
<td>0.483</td>
<td>0.535</td>
<td>0.574</td>
</tr>
<tr>
<td>Adj $R^2$</td>
<td>-0.035</td>
<td>-0.030</td>
<td>-0.025</td>
<td>-0.011</td>
<td>0.179</td>
<td>0.465</td>
<td>0.518</td>
<td>0.559</td>
</tr>
</tbody>
</table>

Panel B: Under the “Null” Hypothesis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Under “Null”</th>
<th>2.5%</th>
<th>5.0%</th>
<th>10.0%</th>
<th>50.0%</th>
<th>90.0%</th>
<th>95.0%</th>
<th>97.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk Premia on $Cov(\gamma_1)$</td>
<td>0.037</td>
<td>0.003</td>
<td>0.011</td>
<td>0.016</td>
<td>0.027</td>
<td>0.038</td>
<td>0.042</td>
<td>0.045</td>
</tr>
<tr>
<td>Risk Premia on $Cov \times Dur(\gamma_2)$</td>
<td>-0.024</td>
<td>-0.030</td>
<td>-0.028</td>
<td>-0.025</td>
<td>-0.017</td>
<td>-0.008</td>
<td>0.000</td>
<td>0.009</td>
</tr>
<tr>
<td>$R^2$</td>
<td>1.000</td>
<td>0.113</td>
<td>0.208</td>
<td>0.348</td>
<td>0.666</td>
<td>0.784</td>
<td>0.804</td>
<td>0.819</td>
</tr>
<tr>
<td>Adj $R^2$</td>
<td>1.000</td>
<td>0.082</td>
<td>0.180</td>
<td>0.325</td>
<td>0.654</td>
<td>0.777</td>
<td>0.797</td>
<td>0.813</td>
</tr>
</tbody>
</table>

I simulate 10,000 time series of consumption growth innovations $\{w_t\}$ from an i.i.d. normal distribution with zero mean and a standard deviation matched to the data. Each time series of simulated consumption growth innovations is then converted to a time series of long-run consumption growth innovations $\left(\sum_{n=0}^{\infty} \rho^nw_{t+n+1}\right)$. I also simulate 10,000 time series of the error terms $err^i$ from a multivariate normal distribution with zero mean vector and the covariance matrix matched to its empirical counterpart. Each time series of simulated $err^i$, combined with the estimated intercept terms (constant$^i$), the estimated cash flow covariance $Cov^i$, and the simulated long-run consumption innovations, generates a time series of simulated long-run accounting returns for portfolio $i$. These simulated long-run accounting returns are then regressed on the simulated long-run consumption innovations to estimate the simulated cash flow covariances. Finally, 10,000 cross-sections of average excess returns are simulated from the two-factor model with the error term drawn from a normal distribution matched to its empirical distribution.

The result of the second experiment is presented in Panel B of Table V. Due to the estimation errors associated with the cash flow covariance and duration...
in the time series, the corresponding median risk premium estimates are biased away from their population values and towards zero. Consequently, the $R^2$ estimates are also biased down from their population value of one. The fact that estimates are biased towards zero even under the null hypothesis suggests that our empirical risk premium and $R^2$ estimates are likely conservative. Finally, I find that the risk premium estimate on $Cov$ is mostly positive and that on $Cov \times Dur$ is mostly negative. This indicates that despite the large time-series imprecision, one should still be able to recover in data the true pricing relation as suggested by the two-factor cash flow model.

E. Robustness and Additional Diagnostics

I run several robustness checks on the cash flow models and present the results in Table VI. Although the two-factor cash flow model explains the return on the smallest portfolio particularly well, I verify that the smallest portfolio does not drive the result. Panel A reports the results of the test in which I exclude the smallest-sized portfolio from the cross-sectional analysis. Although the $R^2$ on the two-factor cash flow model decreases, it is still reasonably high at 71.4%. More importantly, the incremental $R^2$ when moving from the one-factor model to the two-factor model is still 18% (incremental adjusted-$R^2$ is about 17%), indicating the incremental explanatory power of cash flow duration is not driven by the smallest portfolio. In addition, risk premia on both cash flow factors are still significant.\(^\text{11}\)

In Panels B and C, I report the results of the cross-sectional analysis when cash flow covariance ($Cov$) is measured with a holding horizon of 5 years and 10 years ($N = 5$ and $N = 10$, respectively). The results are very similar to the benchmark case where $N = 7$. The risk premia on both cash flow factors are significant and cash flow duration provides roughly 20% additional explanatory power in terms of incremental $R^2$ (or adjusted-$R^2$).

\(^{11}\) I also test the two-factor cash flow model on book-to-market and size double-sorted portfolios. Similar to other benchmark models, the two-factor cash flow model is not able to explain the average return on the small growth portfolio. Once the small growth portfolio is excluded, the two factor model has an $R^2$ of above 80% and the risk premium terms are significant and assume the correct signs. There are several potential reasons why the two-factor model fails on the small-growth portfolio. First, the mean-reverting cash flow share assumption may be violated for the small-growth stocks. Second, the function of equity strip's risk premium on its maturity might be very convex for the small-growth portfolio. When approximating the true equity risk premium in (7), a “cash flow convexity” term might be needed in addition to the cash flow duration. In addition, I test the two-factor model on 15 industry portfolios. The mean-reverting cash flow share assumption is not a good assumption for the industry portfolios (rejected for 8 out of the 15 portfolios), making the estimation of the cash flow characteristics imprecise. Although the two cash flow characteristic factors assume risk premia of the correct signs and do a reasonable job in describing the cross-sectional variation in stock excess returns (adjusted-$R^2$ is above 0.5), their risk premia are not statistically significant. Detailed results on industry portfolios can be found in the Internet Appendix available at http://www.afajof.org/supplements.asp.
Table VI
Cash Flow Models: Robustness Results

Panel A reports the cross-sectional regression results using 59 test portfolios (excluding the smallest portfolio). Panel B reports the cross-sectional regression results where Cov is measured using a holding horizon of 5 years \((N = 5)\). Panel C reports the cross-sectional regression results where Cov is measured using a holding horizon of 10 years \((N = 10)\). Panel D reports the cross-sectional regression results where Dur is measured on an ex ante basis. Specifically, each year from 1965 to 1999, I compute the future earnings component \(\sum t_i^e\) for each of the 60 test portfolios and regress it on a set of instruments in a panel data setting: \(\sum t_i^e = \beta X + u\), and \(X = [e_t, DIV_t, SG_t]\), where \(e\) is the log \((1+ROE)\), DIV is the log current book dividend yield, and SG is the sales growth. Variables with upper bars are cross-sectionally demeaned so there is no constant term in the regression. Given the estimate of \(\beta\), an “ex ante” duration measure is defined as \(\hat{D}_{urt} = \hat{\beta} X_t - \frac{\xi_t}{\gamma} e_t - \Sigma_E / \Sigma c_t\). Panel E reports the cross-sectional regression results where Dur is measured using the dividend yield \((D/P)\). Finally, Panel F reports the cross-sectional regression results where Dur is measured using the consensus equity analyst’s long-term earnings growth forecast \((LTG)\). In Panels D, E, and F, the first two rows report regression results where the (time-series) average duration measure is used. Since Dur, D/P and LTG are all free from forward-looking bias, I also allow them to be time-varying in the regression and the corresponding results are reported in the last two rows in Panels D, E, and F. The sampling period is from 1964 to 1995 except in Panel F \((LTG\ is available only from 1982\). Robust \(t\)-values are reported in italics below the coefficient estimates and both \(R^2\s and adjusted-\(R^2\s of the regressions are reported.

<table>
<thead>
<tr>
<th>Panel A: Excluding the Smallest Portfolio (s1)</th>
<th>Panel B: Measuring Cov with (N = 5)</th>
<th>Panel C: Measuring Cov with (N = 10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>Cov &amp; Cov \times Dur</td>
<td>(R^2/adj R^2)</td>
</tr>
<tr>
<td>Coefficient</td>
<td>0.084 0.014</td>
<td>0.538</td>
</tr>
<tr>
<td>Robust t-value</td>
<td>2.59 2.78</td>
<td>0.530</td>
</tr>
<tr>
<td>Coefficient</td>
<td>0.077 0.029 −0.017</td>
<td>0.714</td>
</tr>
<tr>
<td>Robust t-value</td>
<td>2.49 2.81 −2.20</td>
<td>0.703</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>Cov &amp; Cov \times Dur</td>
<td>(R^2/adj R^2)</td>
</tr>
<tr>
<td>Coefficient</td>
<td>0.073 0.028 −0.025</td>
<td>0.751</td>
</tr>
<tr>
<td>Robust t-value</td>
<td>2.46 3.66 −2.63</td>
<td>0.742</td>
</tr>
<tr>
<td>Coefficient</td>
<td>0.083 0.027 −0.018</td>
<td></td>
</tr>
<tr>
<td>Robust t-value</td>
<td>2.06 3.20 −2.43</td>
<td></td>
</tr>
</tbody>
</table>
I address the forward-looking problem associated with the cash flow duration estimation in Panel D by estimating an ex ante measure of cash flow duration, $\hat{\text{Dur}}^i_t$, as $E_t[\Sigma^e_i] - \frac{\kappa}{1 - \rho} - \xi^i_t - E_t[\Sigma^\Delta_i]$. To directly compute $E_t[\Sigma^e_i]$, I apply a predictive regression. Specifically, each year from 1965 to 1995, I compute $\Sigma^e_i$ for each of my 60 portfolios and regress them on a set of instruments $X$ in a balanced panel setting:

$$\Sigma^e_i = \beta X + u.$$ (17)

Variables with upper bars are cross-sectionally demeaned so there is no constant term in (17). I avoid choosing variables that contain price information, as one of the main objectives of this paper is to measure risk using only accounting cash flow information. The variables I include in the vector $X$ are: log current ROE ($e_t = \log (1 + ROE_t)$), log current book dividend yield ($DIV_t = \log (1 + D_t/B_t)$), and percentage sales growth from year $t - 1$ to year $t$ ($SG_t = Sales_t/Sales_{t-1} - 1$). The three variables turn out to explain a large portion of the cross-sectional variation in $\Sigma^e_i$ with an $R^2$ of above 0.8 over the full sample. I also repeat the same panel regression in two subsamples and obtain qualitatively similar results. The regression coefficients $\beta$ on all three variables are positive and significant. Since ROE is persistent, high current earnings are likely to be associated with high earnings in the near future, resulting in a positive coefficient on $e_t$. The positive coefficient on $DIV_t$ is consistent with the empirical relation in Table II: $\Sigma^e_i$ is positively correlated with book dividend yield $\xi^i_t$, since firms facing more growth opportunities tend to pay less dividends. Finally, the coefficient on $SG_t$ is also positive since higher sales growth indicates greater growth potential in the future. Once I estimate the regression (17), I can compute the ex ante cash flow duration measure (up to a constant) for each of the 60 portfolios as

$$\hat{\text{Dur}}^i_t = \hat{\beta} X^i_t - \frac{\kappa}{1 - \rho} - \xi^i_t - E_t[\Sigma^\Delta_i].$$

If I replace the previous cash flow duration measure $\text{Dur}^i_t$ with the time-series average of the ex ante cash flow duration measure $\hat{\text{Dur}}^i_t$ in the cross-sectional analysis, the results are qualitative very similar as reported in the first two rows of Panel D, although the significance level on the risk premium terms and the $R^2$ are slightly reduced, potentially due to the omission of other useful instruments in the prediction equation (17). I also allow duration to be time-varying in the cross-sectional analysis (the number of cross-sectional moment conditions increases from one to $T$). The results are again very similar as evident in the last two rows of Panel D.

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12 Finally, I also examine the rolling window estimates of cash flow duration measure. At each year, I reestimate (17) using data from 1965 through the current year, so the duration measure $\hat{\text{Dur}}_t$, is only computed using information available at year $t$. For this calculation, I start my portfolio construction at year 1975 so I have enough data to compute reasonably reliable estimates of $\hat{\text{Dur}}_t$, even during early years. Such rolling window estimation provides similar cash flow duration measures for the 60 portfolios.
Intuitively, dividend yield is inversely related to cash flow duration. However, since dividend yield is computed using price, which reflects all types of risk, it may not be a "clean" measure of cash flow duration. For instance, if I were to replace $\text{Dur}$ with $D/P$—the dividend yield where $D$ includes both common dividend payout and common share repurchase—the dividend yield does not provide too much additional explanatory power as shown in Panel E. In addition, the risk premium on the second factor, $\text{Cov} \times D/P$, is associated with the wrong sign. It is marginally insignificant when the time-series average $D/P$ is used and becomes insignificant once I allow for time-variation in $D/P$.

The last alternative measure of cash flow duration is the consensus long-term earnings growth forecast (LTG) issued by equity analysts. High LTG indicates analyst optimism on the expected future earnings growth of the company and therefore is likely associated with higher cash flow duration. The consensus LTG is collected from I/B/E/S on June and then averaged at the portfolio level. Due to the availability of LTG, the sampling period is shortened to 1982 to 1995. Once I replace $\text{Dur}$ with LTG, I find qualitatively similar results in Panel F. The risk premium on the second factor, $\text{Cov} \times \text{LTG}$, is negative and significant although the $R^2$ becomes much smaller (0.43) potentially due to the reduced sample size. In addition, once time-varying LTG is allowed, the risk premium on the second factor, $\text{Cov} \times \text{LTG}$, becomes marginally insignificant ($t$-value $=-1.82$), again likely a result of a much smaller sample size.

As a model misspecification check, I include portfolio characteristics as additional variables in the cross-sectional analysis using a Fama-MacBeth regression approach (see Jagannathan and Wang (1998) on using portfolio characteristics to detect model misspecification). The characteristics chosen are size and book-to-market after log transformation. The results are presented in Table VII. I find that the portfolio characteristics are not significant in the cross-sectional regressions of the two-factor cash flow model (Panel A), so model misspecification is unlikely. In contrast, both size and book-to-market are significant in the cross-sectional regressions for the commonly used Fama-French three-factor model, indicating possible model misspecification (Panel B), consistent with previous research (see Daniel and Titman (1997) and Lettau and Ludvigson (2001b) for example). Finally, when I put the Fama-French three factor loadings and the two cash flow factors in one cross-sectional regression as a "horse race," the two cash flow factors seem to drive out the loadings on the Fama-French factors.

To summarize the results, the cash flow models estimated using pure cash flow data sufficiently explain the cross-sectional variation of expected excess returns of book-to-market, size, and long-term reversal portfolios. An interaction term involving cash flow duration captured using the variable $\text{Dur}$ has additional explanatory power on top of covariance risk. A two-factor cash flow model that accounts for the cash flow duration performs better than most of the commonly used models estimated using returns. Furthermore, it is not likely to suffer from model misspecification.
Table VII
Diagnostic Cross-sectional Regressions
I report results on additional cross-sectional regressions using the Fama-MacBeth methodology. I include common characteristics of the 60 portfolios in the regressions in Panels A and B. Log (ME) denotes the log of market value of equity and Log (BM) denotes the log of book-to-market ratio. Both $R^2$s and adjusted-$R^2$s of the regressions are reported. Panel C reports the results of a regression where cash flow characteristics and three-factor betas are included at the same time. The sampling period is 1964 to 1995.

Panel A: Cash Flow Models with Portfolio Characteristics

<table>
<thead>
<tr>
<th>Intercept</th>
<th>Cov</th>
<th>Cov × Dur</th>
<th>log (ME)</th>
<th>log (BM)</th>
<th>$R^2$/adj $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient</td>
<td>0.101</td>
<td>0.033</td>
<td>−0.022</td>
<td>−0.004</td>
<td>0.828</td>
</tr>
<tr>
<td>FM $t$-value</td>
<td>1.34</td>
<td>5.84</td>
<td>−4.30</td>
<td>−0.47</td>
<td>0.819</td>
</tr>
<tr>
<td>Coefficient</td>
<td>0.077</td>
<td>0.036</td>
<td>−0.025</td>
<td>0.011</td>
<td>0.820</td>
</tr>
<tr>
<td>FM $t$-value</td>
<td>2.56</td>
<td>3.37</td>
<td>−4.15</td>
<td>0.54</td>
<td>0.810</td>
</tr>
</tbody>
</table>

Panel B: FF Three-Factor Model with Portfolio Characteristics

<table>
<thead>
<tr>
<th>Intercept</th>
<th>MKT</th>
<th>SMB</th>
<th>HML</th>
<th>log (ME)</th>
<th>log (BM)</th>
<th>$R^2$/adj $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient</td>
<td>0.167</td>
<td>0.027</td>
<td>0.006</td>
<td>0.013</td>
<td>−0.020</td>
<td>0.549</td>
</tr>
<tr>
<td>FM $t$-value</td>
<td>3.49</td>
<td>0.72</td>
<td>0.21</td>
<td>0.46</td>
<td>−3.20</td>
<td>0.516</td>
</tr>
<tr>
<td>Coefficient</td>
<td>−0.006</td>
<td>0.102</td>
<td>0.029</td>
<td>−0.070</td>
<td>0.074</td>
<td>0.637</td>
</tr>
<tr>
<td>FM $t$-value</td>
<td>−0.19</td>
<td>2.54</td>
<td>1.17</td>
<td>−2.34</td>
<td>4.85</td>
<td>0.611</td>
</tr>
</tbody>
</table>

Panel C: Cash Flow Model versus FF Three-Factor Model

<table>
<thead>
<tr>
<th>Intercept</th>
<th>Cov</th>
<th>Cov × Dur</th>
<th>MKT</th>
<th>SMB</th>
<th>HML</th>
<th>$R^2$/adj $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient</td>
<td>0.059</td>
<td>0.034</td>
<td>−0.021</td>
<td>0.015</td>
<td>0.008</td>
<td>−0.014</td>
</tr>
<tr>
<td>FM $t$-value</td>
<td>1.91</td>
<td>6.35</td>
<td>−5.29</td>
<td>0.40</td>
<td>0.32</td>
<td>−0.53</td>
</tr>
</tbody>
</table>

IV. Conclusion
This paper links the cross-sectional variation in assets’ returns directly to the cross-sectional variation in their fundamental cash flow characteristics. In particular, I examine two such characteristics: Cov (covariance—how cash flow varies with aggregate consumption), and Dur (duration—whether cash flow occurs further in the future). Their impact on the cross-sectional variation in expected excess return can be largely captured by a two-factor cash flow model with the two factors being Cov and Cov × Dur. The intuition behind such a model is illustrated in a simple economy where a portfolio’s cash flow as a share of aggregate consumption is mean-reverting.

Empirically, cash flow covariance and duration are estimated using only long-run consumption and accounting earnings data for 60 book-to-market-sorted, size-sorted and long-term reversal portfolios. I show that cash flow covariance alone is able to explain about 60% of the cross-sectional variation in risk premia across the 60 portfolios. The finding that cash flow covariance measured using long-run consumption data helps to explain long-run return reversal provides
new empirical support for the long-run risk model of Bansal and Yaron (2004). Cash flow duration provides additional explanatory power of about 20% in terms of incremental $R^2$ through its interaction with cash flow covariance. Overall, the two-factor cash flow model incorporating both cash flow characteristics is able to explain 82% of the cross-sectional variation in risk premia.

This paper provides empirical support that fundamental cash flow characteristics including both covariance and duration are important in understanding the difference in risk premia across assets. In addition, the two-factor cash flow model provides a new way to estimate a financial asset’s risk exposure and can be used in the cost-of-capital calculation even in the absence of price or return information.

**Appendix A: Proof of Proposition 1**

To compute the price of an equity strip, $P_{n,t}^i$, I make use of the fact that

$$E_t[M_{t+1}P_{n-1,t+1}^i] = P_{n,t}^i,$$

which implies

$$E_t \left[ M_{t+1} \frac{D_{t+1}^i P_{n-1,t+1}^i}{D_{t}^i} \right] = \frac{P_{n,t}^i}{D_{t}^i}. \quad (A1)$$

Conjecture that

$$\frac{P_{n,t}^i}{D_{t}^i} = \exp \left[ A^i(n) + B(n)z_{t}^i \right]. \quad (A2)$$

Since $P_{0,t}^i = D_{t}^i$, we have $A^i(0) = B(0) = 0$.

Equations (A1) and (A2) imply

$$E_t \left[ \exp \left\{ m_{t+1} + \Delta s_{t+1}^i + \Delta c_{t+1} + A^i(n-1) + B(n-1)z_{t+1}^i \right\} \right] = \exp \left\{ A^i(n) + B(n)z_{t}^i \right\}.$$

Under the assumptions on aggregate consumption (4), the SDF (5) and the cash flow process (3), evaluating the expectation, and matching terms involving $z_{t}^i$, we have

$$(1 - \phi) + B(n - 1)\phi = B(n).$$

Solving the difference equation with initial condition $B(0) = 0$, we have

$$B(n) = 1 - \phi^n.$$
Matching constants, it follows that

\[ A^i(n) = A^i(n - 1) + \log \delta - \frac{1}{2} \left[ \frac{(1 - \gamma)(1 - \rho_2 \delta)}{1 - \rho_1 \delta} \sigma_w \right]^2 + \frac{1}{2} \left[ \frac{\phi^{n-1} \lambda_i \sigma_w}{1 - \rho_1 \delta} + \frac{(1 - \gamma)(1 - \rho_2 \delta) \lambda_i \sigma_w}{1 - \rho_1 \delta} \right] + \frac{1}{2} \left( \phi^{n-1} \sigma^i \right)^2, \]

where \( \sigma_i^i \) is the standard deviation of \( \varepsilon_i \). Therefore, \( A^i(n) \) can also be solved iteratively given the initial condition as

\[ A^i(n) = n \log \delta + \frac{(\lambda_i \sigma_w)^2 + (\sigma_i)^2}{2} \frac{1 - \phi^{2n}}{1 - \phi^2} + \frac{(1 - \gamma)(1 - \rho_2 \delta) \lambda_i \sigma_w^2}{1 - \rho_1 \delta}. \]

Define the return on individual cash flow claim as

\[ R^i_{n,t+1} = \frac{P^i_{n-1,t+1}}{P^i_{n,t}} = \frac{P^i_{n-1,t+1}}{P^i_{n,t}} / \frac{D^i_{t+1}}{D^i_t} \]

and take the log:

\[ r^i_{n,t+1} = A^i(n - 1) - A^i(n) + B(n - 1) z^i_{t+1} - B(n) z^i_t + \Delta s^i_{t+1} + \Delta c_{t+1}. \]

The one-period innovation is

\[ r^i_{n,t+1} - E_t[r^i_{n,t+1}] = (1 + \phi^{n-1} \lambda_i) \sigma_t + \phi^{n-1} \varepsilon^i_{t+1}. \]

Therefore,

\[ \log E_t[R^i_{n,t+1} / \bar{R}_{f_t}] = (1 + \phi^{n-1} \lambda_i) \left[ 1 + (\gamma - 1) \frac{1 - \rho_2 \delta}{1 - \rho_1 \delta} \right] \sigma^2_w. \]

**Appendix B: Moving from Cash Flows to Earnings**

The clean-surplus identity implies

\[ B^i_{t+1} = B^i_t + X^i_{t+1} - D^i_{t+1}, \]

where \( B^i, X^i, \) and \( D^i \) denote firm \( i \)'s book value of equity, earnings, and cash flow, respectively. Therefore, log accounting return can be written as

\[ e^i_{t+1} = \log \left( \frac{B^i_{t+1} + D^i_{t+1}}{B^i_t} \right) = \log \left( 1 + \frac{X^i_{t+1}}{B^i_t} \right) = \log (1 + ROE^i_{t+1}). \]

Denoting the log cash flow-to-book equity ratio as \( \xi^i_t = d^i_t - b^i_t \), we have

\[ e^i_{t+1} = \log \left( \exp \left( - \xi^i_{t+1} \right) + 1 \right) + \Delta d^i_{t+1} + \xi^i_t. \]
Consider the log-linear approximation of (B2) first proposed by Vuolteenaho (1999):

$$e_t^{i+1} \approx \kappa - \rho \xi_t^{i+1} + \Delta d_t^{i+1} + \xi_t^i,$$

where $\rho = \frac{1}{1 + D/B}$ such that $D/B$ denotes the average book dividend yield. The term $\rho$ is chosen to be 0.95, which corresponds to an average book dividend yield of 5.26%, close to its historical value. The constant $\kappa$ is related to $\rho$ by

$$\kappa = -(1-\rho) \log(1-\rho) - \rho \log(\rho).$$

With the choice of $\rho = 0.95$, we have $\kappa = 0.1985$. Rearrange (B2) to get

$$\Delta d_t^{i+1} = e_t^{i+1} - \kappa + \rho \xi_t^{i+1} - \xi_t^i.$$ 

Direct computation shows that

$$\sum_{n=0}^{\infty} \rho^n \Delta d(t, n+1) = \sum_{n=0}^{\infty} \rho^n e(t, n+1) - \frac{\kappa}{1-\rho} - \xi_t^i,$$

where I assume

$$\lim_{n \to \infty} \rho^n \xi_t^{i+n} = 0.$$

REFERENCES


Lettau, Martin, and Sydney Ludvigson, 2001b, Resurrecting the (C)CAPM: A cross-sectional test when risk premia are time-varying, *Journal of Political Economy* 109, 1238–1287.