Teaching Note on Convertible Bonds

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1. Introduction

Convertible bonds combine the features of bonds and stocks in one instrument. It is a bond that gives the holder the right to "convert" or exchange the par amount of the bond for common shares of the issuer at some fixed ratio during a particular period. As bonds, they have some characteristics of fixed income securities. Their conversion feature also gives them features of equity securities.

2. Features of Convertible Bond

2.1 An example of a simple convertible bond

On Sep 2003, Primus Telecom issued the following convertible bond.

Size:	US$ 110 million
Term:	7 years
Redemption date:	15 Sep 2010
Nominal value:	US$ 1000
Interest coupon:	3.75%
Conversion price:	US$ 9.3234
Conversion ratio:	107.257
Market price at issue:	100
Bloomberg Ticker	PRTL 3.75 09/10

This is the most elementary example of convertible bond. The bond has a nominal (or par) value of $1000. The market price is always quoted as percentage of the nominal value, which means you have to pay $1000 to buy this bond at issue. Like a straight bond, it pays you coupon semi-annually, so each coupon payment will be $1000*3.75\%/2 = \$18.75$. In addition, it allows you to exchange the bond for 107.2570 shares anytime before maturity, which is 09/15/10. If the bond is not converted, it will be redeemed at par on maturity. Finally, the conversion price is equal to the nominal value divided by conversion rate.

2.1 Main complications

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- *Call and put features*

Many of the convertible bonds are also callable by the issuer on a set of pre-specified dates, which may lead to "forced conversion". Consider a callable convertible bond where the issuer has the option to call the bond at par tomorrow. However, the conversion value of the bond is $110. In this case, the investor would be forced to convert the bond into shares worth $110 before the call date. The call feature is an option with the issuer, and it will decrease the value of the convertible bond.

To make things more complicated, there are "protected" calls or "soft" calls where the bond can be called only if the share price (or the average share price over the past 20 days) is above a certain barrier.

Some convertible bonds may also have put features that allow the buyer to put back the bonds to the issuer. This is buyer's option and it would increase the value of the convertible bond. If the call and put features occur simultaneously, priority is given according to the prospectus.

- *Refix Clauses*

In the early 1990s, Japanese corporations began to issue CBs with "refix Clauses". In its simplest form, it changes the conversion ratio subject to the share price level on certain days between issue and expiry. Suppose on one of these days the share price drops by 20%, then the refix clause may increase the conversion ratio by 20%. This feature makes it attractive to investor and will increase the value of the convertible bond.

- *Conversion proceeds*

There are complications on conversion proceeds. First, some bonds can be converted into a combination of shares and cash. In most cases, the conversion number as well as the cash amount varies as a function of time.

Second, when a buyer converts and receives shares, these shares may either be distributed from existing stock or new shares just issued. In the latter case, there is a dilution effect – the same company issues more shares.

Third, shares receive upon conversion may be denominated into a different currency. For example, the underlying shares of US dollar convertible bonds may be traded in Japanese yen. Buyers of this type of convertible bonds are also exposed to currency risk.

3. Convertible Bond Market

As of the year 2000, the global convertible securities market has reached an approximate value of $470 billion. The US convertible market makes up about $160 billion, relatively small as compared to the US stock market valued over $10 trillion. Approximately 500

US companies with about 550 actively traded convertible securities comprise the majority of that $160 billion market value[2].

The convertible bond market has been increasing in size. Figure 1 shows the number of new convertible bond issues and the dollar proceeds in US in the past five years. On average, there are about 250 convertible bond issues in US with total proceeds of about 34 billion dollars every year. This is a steady growth from an average of 12 billion dollars from 1984 to 1994.

Figure 1:
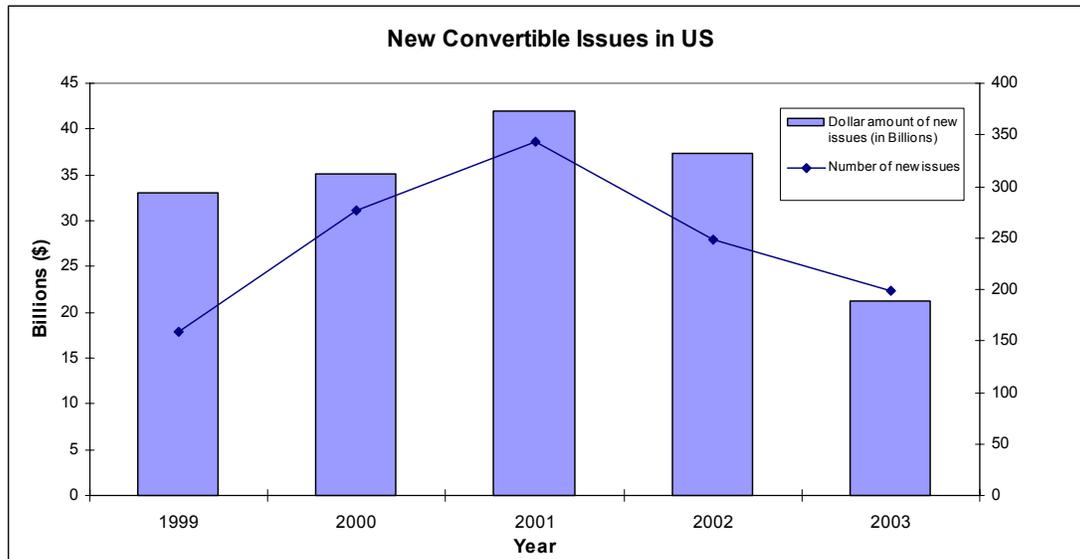
Source: SDC Platinum

3.1 Issuers and investors of convertible bonds

In the US, with the exception of the largest issues, convertible bonds do not have active market makers. Though they may be exchange-listed, most convertible bonds are traded by appointment away from the exchange floor. The convertible desks in brokerage firms bring buyers and sellers together and this process can take a few minutes or several days depending on various factors such as order size and market conditions.

The convertible bond market place embraces a broad universe of companies. In an analysis of 311 US companies having actively traded convertible bonds in Jan 2000, Noddings, Christoph and Noddings found[3]:

- 58% of companies are in small-cap category (market cap less than 1.25 billion), 27% are in medium cap category (market cap ranging from 1.25 billion to 10.5

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[2] The numbers are taken from Noddings, Christoph and Noddings (2001), the convertible securities include equity warrants, convertible bonds, and convertible preferred stocks.
[3] For detailed analysis and more information on individual firms, please refer to Noddings, Christoph and Noddings (2001), pg 51 -65.

billion) and the remaining 15% are large cap companies (market cap above 10.5 billion).

- Only 17% of the companies have the Standard & Poor's stock rating of B+ and above and only 21% of the companies have the Standard & Poor's bond rating of BBB and above.

- A large portion of convertible bonds are issued by smaller firms in high-growth industries such as computers, electronics, health care, internet, and semiconductor.

The above evidences is, to a large extent, consistent with theories on why firms issue convertible bonds, see Brennan and Schwartz (1988), Brennan and Kraus (1987), Mayers (1998) and Stein (1992). Brealey & Myers's textbook on corporate finance gives an excellent summary.

Convertible bond helps to resolve some conflicts between equity and debt holders. Shareholder can hurt debt holder by taking more risk or issuing senior debt. To compensate for this risk, debt holders will charge a very high interest rate, which may give shareholder incentives to take even more risk and eventually destroy firm value. However, this problem is alleviated in the case of convertible bond since debt holders may also become shareholders. From an informational point of view, issuing convertible bond signals management's confidence in the company and leads to less price discount due to asymmetric information.

Convertible bond issue can be regarded as a contingent issue of equity. If a company's investment opportunity expands, its stock price is likely to increase, leading to conversion. Thus the company gets fresh equity when it is most needed for expansion.

Small and growth firms are typically less known and have more expansion opportunities. Therefore, it is not surprising to see they are the main issuers of convertible bonds. In addition, the relatively low coupon rate on convertible bonds may also be attractive to small growth firms facing heavy cash constraints.

On the buying side, there are two main classes of investors[4]:

- Money managers. They look at the universe of convertibles and pick issuers whose stock seems to be favorable. They purchase the convertible if they are bullish on the underlying equity. Examples include convertible fund managers, fixed-income managers, risk-averse equity managers, income-oriented equity managers, and so on.

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[4] We use the classification in Nelken (1997).

- Arbitrage specialists. They identify misalignment between the equity market and the convertibles and take advantage of the relative mis-pricing by longing the equity and shorting the convertible or vice versa. They hedge their positions constantly and are less concerned about the positive outlook for the equity.

3.2 Convertible bond as an asset class

It is not appropriate to treat the convertible bond as a separate asset class for two reasons. First, convertible bonds share features of both stock and bond. Second, the convertible bond market is small as compared to other asset classes. In fact, both Ibbotson Associates' 6 asset classes and William F. Sharpe Associates' 12 asset classes do not categorize convertible securities as an asset class.

A more practical approach is to divide convertible securities into subclasses and treat each subclass as an alternative investment to other asset class. For example, we can group convertible securities according to their sensitivity to stock and bond markets[5]:

- High-Yield Convertibles – typically trading at substantial discount to par, they are alternatives to straight corporate bonds.
- Core Convertibles – typically trading near par, they are alternatives to a stock/bond balanced approach.
- Low-Premium Convertibles – typically trading well above par, they are alternatives to common stocks.

4. Valuation of Convertible Bond

Convertible bonds combine the features of bonds and stocks in one instrument and its price will be affected by both interest rates and share prices. Take an example of the simple convertible bond - PRTL 3.75 09/10, discussed before, we first consider two extreme cases:

(1) When share price of Primus Telecom is very small relative to the conversion price 9.32, the convertible bond is very unlikely to be converted and therefore it is effectively a straight bond and can be evaluated using the standard bond pricing formula.

(2) When the share price is very high relative to the conversion price, the convertible bond will certainly be converted to shares. The convertible bond price will be the conversion value, which is the share price times the conversion ratio.

In Figure 2, we plot out the price of the convertible bond as the function of the share price. The solid line corresponds to the conversion value in percentage, and it is linearly increasing in the share price. The horizontal dashed line corresponds to the price of a

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[5] This classification is taken from Noddings, Christoph and Noddings (2001), pg 203.

straight bond and it is not affected by the share price. In general, since you have the option to make the convertible bond either a bond or stock, its value should be at least the value of the bond or the stock. The maximum value of the stock and the bond serves as a lower bound for the price of the convertible bond which is represented by a dotted convex curve. In fact, the price of the convertible bond is strictly higher than the lower bound except in the two extreme cases.

The conversion feature resembles a call option on the stock. As the volatility of stock increases, the conversion feature will be more valuable and convertible bond price will increase. In fact, the convertible bond price curve in Figure 2 will be less convex.

Figure 2:

Convertible bond price diagram
(PRTL 3.75 09/10)
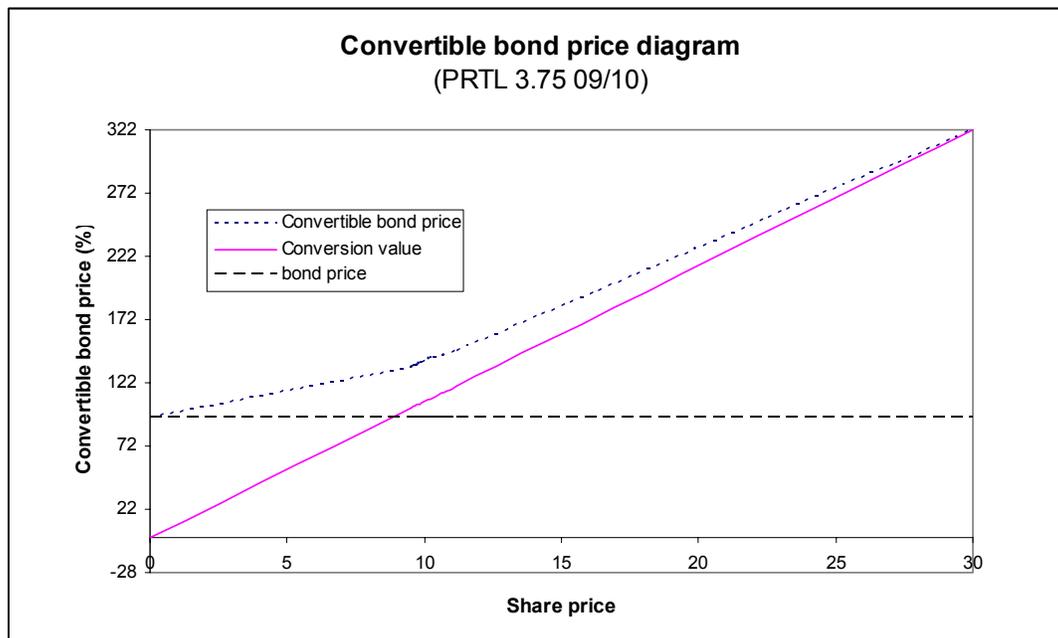
The convertible bond price will also be affected by interest rate. In a low interest rate environment, the straight bond price is high, pushing up the horizontal line in Figure 2, which will also increase the price of the convertible bond.

In addition, as explained before, the call feature lowers the price of the convertible bond and the put feature make the convertible bond more expensive.

4.1 Brief discussion of the pricing methods

- *Break-even analysis*

The earliest method compares the relative advantage of owning the convertible versus owning the underlying share directly. This analysis divides the initial conversion

premium (convertible price – conversion value at issue) by the difference between coupon rate and dividend yield. The resulting number can be interpreted as the number of years required for the yield differential to compensate for the initial conversion premium. This method is obviously ad-hoc and does not account for other option features. It only serves as a reasonable check on convertible bond: convertible bonds with break-even period of less than three years are generally considered acceptable investments.

- *Bond plus equity option approach*

A more sophisticated approach treats the convertible bond as a straight bond plus a call option on the stock. These two components are then valued separately using standard methods. The price of the convertible bond is simply the sum of bond price and option price. This method is not exact, since the exercise price on the equity option is not fixed.

- *Multi-factor model*

The most advanced models consider the stochastic nature of interest rate, credit spread and stock price (potentially also foreign exchange rate if conversion proceeds are in different currencies) at the same time. However, multi-factor models are more difficult to build and implement.

For more references on the valuation of convertible bond, please refer to the Appendix I.

4. A Worked-out Example

In this section, we take a specific example of a convertible bond with reasonable complexity, and discuss the pricing and hedging of this bond.

4.1 Bond Specification

Detail of the bond can be found in Bloomberg or the prospectus attached. Main features are summarized below.

Issuer:	St. Mary Land & Exploration Company
Issue date:	3/15/02
Term:	20 years
Redemption date:	3/15/22
Nominal value:	US$ 1000
Interest coupon[6]:	5.75%
Conversion price:	US$ 26
Conversion ratio:	38.4615
Call option:	Yes
Call date:	3/20/07

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[6] The bond also has contingent interest feature which we ignore in this illustrative example.

Call price:	100
Softcall barrier	No
Put option:	Yes
Put date:	3/20/07, 3/15/12, 3/15/17
Put price:	100
Market price at issue:	100
Bloomberg Ticker	SM 5.75 03/15/22

Therefore, it is a convertible bond that is both callable and putable.

4.2 Pricing

For the simplicity of illustration, we assume a flat term structure for the risk free interest rate. In addition, we assume both risk free rate and credit spread are constant across time. In another words, we do not model interest rate and credit risk. In this example, we are concerned with hedging the convertible bond for a short period of a week, so constant interest rate and credit spread assumption is reasonable.

We model the stock price dynamics using a binomial tree, which can be built in the usual way:

```
              Su
             /
   S -------
             \
              Sd
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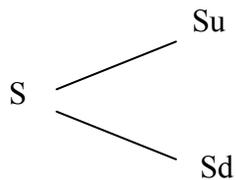
Here S denotes the stock price, and $u = \exp(\sigma\sqrt{\Delta t})$, d = 1/u, where σ is the volatility and Δt is the length of time interval for each step. The risk neutral probability for an up move is (r-q-d)/ (u-d), where r and q denote the risk free rate and dividend yield over Δt respectively.

The price of a convertible bond at the end of the binomial tree (or step n+1) is:
P_{n+1} = max (100, conversion value) +last coupon payment

We can then work backward on the tree to compute the convertible price now. At any node of the tree, we compute the price using the following steps:

(1) Compute $P_i = [P_{i+1}^u p + P_{i+1}^d (1-p)]/(1+R)$, where subscript u (d) denotes a subsequent up (down) move and R denotes the discount rate.
(2) If there is call feature and the current step coincides with call date and the stock price on the current node exceeds the softcall barrier. Denote X = max (call price, conversion value), if $P_i > X$, then set $P_i = X$..
(3) If there is put feature and the current step coincides with put date, set P_i = max (P_i, put price).

(4) Set P_i = max (P_i, conversion value).
(5) If the current step coincides with coupon payment date, add the coupon payment to the price.

The above procedure has been coded in excel spreadsheet: CBcalculator.xls. Open it and key in all relevant inputs as in Figure 3. The riskfree rate of 4% is interpolated from Treasury curve on the pricing date. Number of Steps field refers to the number of steps on the binomial tree. Since this convertible bond does not have softcall feature, we set the value of the Softcall field to be zero. After all inputs are keyed in, click on the yellow compute button.

Figure 3

	A	B	C	D	E	F	G	H	I	J
1										
2	Convertible Bond Calculator				Compute					
3										
4	Pricing Date	1/9/2004			Convertible Bond Price	132.89				
5					Delta	2.6213				
6	Bond information				Striaght Bond Price	90.17				
7					Conversion Value	111.69				
8	Maturity Date	3/15/2022								
9	Face Value	1000								
10	Conversion Ratio	38.4615			Call Features			Put Features		
11	Coupon (%)	5.75								
12	Frequency	2	Semi-annual		Softcall Barrier	0				
13										
14					Call Date	Call Price		Put Date	Put Price	
15	Stock Information					3/20/2007	100	3/20/2007	100	
16								3/15/2012	100	
17	Current Price	29.04						3/15/2017	100	
18	Dividend Yield	0.02								
19	Volatility	0.4								
20										
21	Pricing Information									
22										
23	Risk Free Rate	0.04								
24	Credit Spread	0.03								
25	Number of Steps	100								
26										

We found the price of the convertible bond to be 132.886, slightly lower than the actual price of 133.875. It is higher than both the price of an otherwise identical straight bond (90.17) and the conversion value (111.69) as explained before. The result will improve if we calibrate the credit spread and volatility as discussed later. The delta measures the sensitivity of convertible price to the stock price. In this example, it equals to 2.6213, which means if stock price increases by $1, the convertible bond price will increase by $26.213 (since the face value of the bond is $1000). This number will help in hedging.

4.3 Estimation of parameters

Credit spread and stock volatility are two parameters that are not directly available. In practice, we can use the credit spread of a similar bond with the same credit rating and use option implied volatility if call or put is traded on the stock. In this illustrative example, we choose to estimate these two parameters using historical prices of the

convertible bond. The prices of this convertible bond from 12/22/03 to 12/31/03 are given as:

Date	CB Price	Stock Price
12/31/2003	132.5	28.5
12/30/2003	133.875	29.05
12/29/2003	133.875	29.05
12/26/2003	132.625	28.55
12/24/2003	132.125	28.37
12/23/2003	132	28.35
12/22/2003	132.125	28.4

Enter the information in the CBpricing Sheet, then compute the model price of the convertible bond on those dates. This can be done by calling the function CBprice().

To do that, click the fx button in the formula bar, select function CBprice from the User Defined category (Figure 4, left panel). Hit OK, then a second window will pop up, showing all the necessary inputs to the function CBprice (Figure 4, right panel), just key them in and hit OK, we have the model convertible price.

Figure 4:
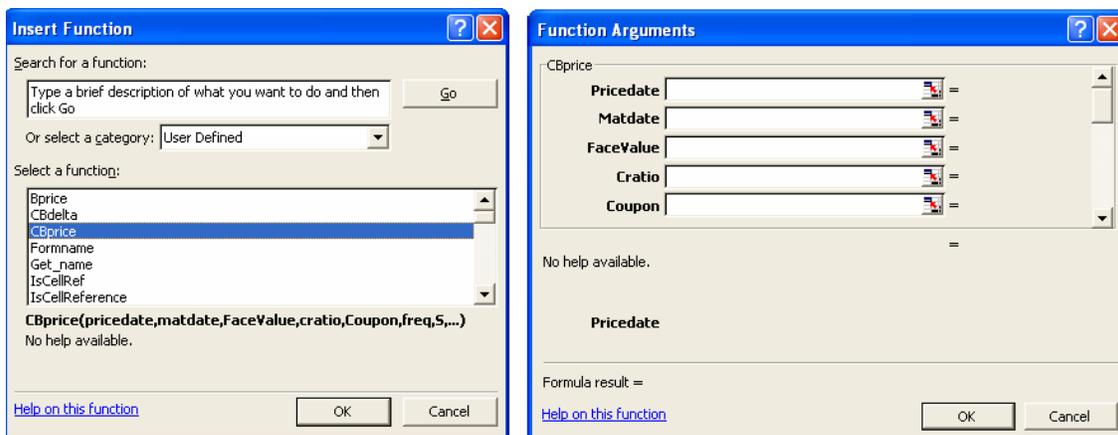
After computing the model prices on all dates, we can also calculate the pricing error defined as the difference of actual price and model price. Finally, we sum all the squared pricing errors to get a single measure of pricing performance over this short period of time.

Now we can choose values of credit spread and volatility to minimize the sum of squared pricing errors. This is achieved by using solver. Open the solver screen, and key the following as in Figure 5:

This optimization exercise may take a few minutes since it involves building and computing from a 100-step binomial tree dozen times. The estimated credit spread is 212 bp and the volatility is 0.37. The sum of squared pricing errors over the 7 days is only 0.0113 so the fitted pricing model prices the convertible bond almost perfectly.

Date	CB Price	Stock Price	Model Price	Squared error	Sum of squared error
12/31/2003	132.5	28.5	132.449	0.0026	0.0113
12/30/2003	133.875	29.05	133.885	0.0001	
12/29/2003	133.875	29.05	133.884	0.0001	
12/26/2003	132.625	28.55	132.576	0.0024	
12/24/2003	132.125	28.37	132.104	0.0005	
12/23/2003	132	28.35	132.051	0.0026	
12/22/2003	132.125	28.4	132.180	0.0031	

Figure 5:
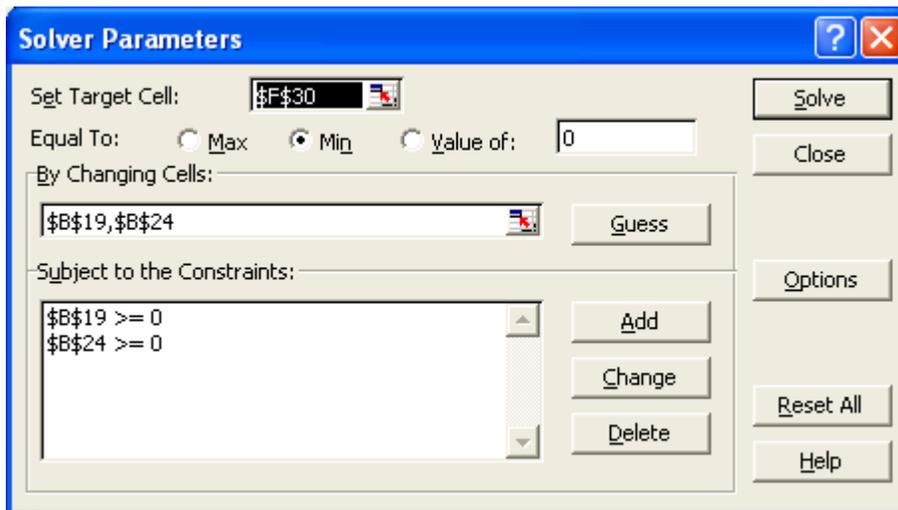
4.4 Hedging

Assume we bought the convertible bond at 12/31/03 at the price of 132.5, we want to hedge the convertible bond against changes in stock price over the week from 1/2/2004 to 1/9/2004. After all the previous steps, this task becomes easy.

At 12/31/03, we can compute the delta of the convertible bond to be 2.612 using the estimated credit spread and volatility. To hedge the convertible bond, we need to short sell 26.12 units of shares for every unit of convertible bond (since the face value of the bond is $1000). The profit and loss can be calculated as follows (Numbers are in percentage of face value):

Date	CB Price	Stock Price	Change in CB position	Change in Stock position (26.12 shares)	Net Change
12/31/2003	132.5	28.5			

11

1/2/2004	133.875	29.03	1.375	-1.38436	-0.00936
1/5/2004	134.5	29.25	0.625	-0.57464	0.05036
1/6/2004	134.25	29.15	-0.25	0.2612	0.0112
1/7/2004	133.875	29.05	-0.375	0.2612	-0.1138
1/8/2004	133.375	28.83	-0.5	0.57464	0.07464
1/9/2004	133.875	29.04	0.5	-0.54852	-0.04852
Total changes			1.375	-1.41048	-0.03548

The hedge is relatively successful over this short period of time. Although the convertible bond price changes by 1.375% of the face value, the hedged portfolio value only changes by 0.035% of the face value.

5. Conclusion

Convertible bond share features of both bond and stock. In addition, it may also include call / put features and other complications. The US convertible market is small relative to stock market and most of the convertible bonds are issued by small growth companies with low credit rating. Therefore, it should be treated with caution and managed with prudence. It offers profitable opportunities for knowledgeable investors, both institutional and individual.

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