Problem 1.1

2. Show that the function \( f(x) = x \sin(1/x) \) with \( f(0) = 0 \), is continuous at 0 but not differentiable at 0.

16. If the series for \( \ln(x) \) is truncated after the term involving \( (x - 1)^{1000} \) and is then used to compute \( \ln(2) \), what bound on the error can be given?

26. Derive the Taylor series with remainder term for \( \ln(1 + x) \) about 1 (expansion point). Derive an inequality that gives the number of terms that must be taken to yield \( \ln(4) \) with error less than \( 2^{-m} \), where \( m \) is some given positive integer.

35. How many terms are required in the series \( e = \sum_{k=0}^{\infty} \frac{1}{k!} \) to give \( e \) with an error of at most \( 6/10 \) unit in the 20th decimal place?

Problem 1.2

6. For the pair \( (x_n, \alpha_n) \), is it true that \( x_n = O(\alpha_n) \) as \( x \to \infty \)?

a. \( x_n = 5n^2 + 9n^3 + 1, \alpha_n = n^2 \)

b. \( x_n = 5n^2 + 9n^3 + 1, \alpha_n = 1 \)

c. \( x_n = \sqrt{n+3}, \alpha_n = 1 \)

d. \( x_n = 5n^2 + 9n^3 + 1, \alpha_n = n^3 \)

e. \( x_n = \sqrt{n+3}, \alpha_n = 1/n \)

8. The expression \( e^h, (1 - h^4)^{-1}, \cos(h), \) and \( 1 + \sin(h^3) \) all have the same limit as \( h \to 0 \). Express each in the following form with the best integer values of \( \alpha \) and \( \beta \). \( f(h) = c + O(h^\alpha) = c + o(h^\beta) \)

28. Prove that \( x_n = x + o(1) \) if and only if \( \lim_{n \to \infty} x_n = x \).

30. For fixed \( n \), show that \( \sum_{k=0}^{n} x^k = \frac{1}{(1 - x)} + O(x^n) \) as \( x \to 0 \).