Problem 1.1

2. Show that the function $f(x) = xsin(1/x)$ with $f(0) = 0$, is continuous at 0 but not differentiable at 0.

16. If the series for $ln(x)$ is truncated after the term involving $(x-1)^{1000}$ and is then used to compute $ln(2)$, what bound on the error can be given?

26. Derive the Talyor series with remainder term for $ln(1+x)$ about 1 (expansion point). Derive an inequality that gives the number of terms that must be taken to yield $ln(4)$ with error less than 2^{-m}, where m is some given positive integer.

35. How many terms are required in the series $e = \sum_{k=0}^{\infty} \frac{1}{k!}$ to give e with an error of at most 6/10 unit in the 20th decimal place?

Problem 1.2

6. For the pair (x_n, α_n), is it true that $x_n = \mathcal{O}(\alpha_n)$ as $x \to \infty$?

a. $x_n = 5n^2 + 9n^3 + 1$, $\alpha_n = n^2$

b. $x_n = 5n^2 + 9n^3 + 1$, $\alpha_n = 1$

c. $x_n = \sqrt{(n+3)}$, $\alpha_n = 1$

d. $x_n = 5n^2 + 9n^3 + 1$, $\alpha_n = n^3$

e. $x_n = \sqrt{(n+3)}$, $\alpha_n = 1/n$

8. The expression e^h, $(1-h^4)^{-1}$, $cos(h)$, and $1 + sin(h^3)$ all have the same limit as $h \to 0$. Express each in the following form with the best integer values of α and β. $f(h) = c + \mathcal{O}(h^\alpha) = c + o(h^\beta)$

28. Prove that $x_n = x + o(1)$ if and only if $\lim_{n\to\infty} x_n = x$.

30. For fixed n, show that $\sum_{k=0}^{n} x^k = 1/(1-x) + \mathcal{O}(x^n)$ as $x \to 0$.